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Plato's Platonism

We can divide Plato's views on mathematics into their epistemic and metaphysical components.
The *Timaeus*, *Phaedo*, and *Republic* focus mostly on the metaphysical aspects of Plato's view, contrasting the sensible world, the mathematical world, and the world of the forms.
The *Theaetetus* and *Meno* discuss the epistemic aspect of Plato's view, most importantly his claim that knowledge is recollection, and that it is not perception.

I. Plato's Metaphysics

Plato's most detailed discussion of the metaphysics of mathematics comes in the section of the *Republic* on the divided line.
But, we see the distinction between the world of being and the world of becoming in the *Timaeus*.
The world of being is eternal and true.
The world of becoming is created and transient.
What we perceive with sensation is constantly in flux, and so merely the world of becoming.
The world of becoming is like the shadows on the wall of the cave.
The world of being is the real world, outside the cave, and it will be stable.

Consider the claim that I am in Clinton, NY.
It is true now, it was false some time ago, and it will be false again (I hope).
Plato takes seriously the Parmenidean worries about the transitory nature of the sensible world.
Real truth must be eternal, and immutable.
Thus, Plato says that as being is to becoming so is truth to belief.
The world of being is apprehended by reason.

Note the underlying Pythagoreanism in *Timaeus*:

> The world has been framed in the likeness of that which is apprehended by reason and mind and is unchangeable...

Plato's requirement that true claims must be eternal and stable is the source of his criticism of geometers, in Book VII of the *Republic*.

> [Geometry] is in direct contradiction with the language employed in it by its adepts...Their language is most ludicrous, though they cannot help it, for they speak as if they were doing something and as if all their words were directed toward action. For all their talk is of squaring and applying and adding and the like, whereas in fact the real object of the entire study is pure knowledge (*Republic* 527a).

When we talk about adding, or squaring, or performing other mathematical operations, it looks as if we are changing some object, altering it by an action.
But, mathematical objects must be stable and unchanging.
So, the active language of mathematicians is misleading.

Plato's attempt to avoid misleadingly active language in mathematics also appears in the *Phaedo* discussion of causation.
Socrates starts with the assumption of the forms, which we saw implicitly in *Timaeus*.
The forms are used as causes: things are tall because they participate in tallness.
We see that objects in the world of becoming participate in both of opposite forms.
But the forms themselves never do.
The forms are universals, and uniform.

In contrast, mathematical objects are particulars.
Still, mathematical objects are like the forms in not participating in opposites.
Things are odd or even because of their participation in particular forms.
But three can never be even, although it is not a form itself of odd, it is not itself the opposite of even.
So, mathematical objects are like the forms in their eternal existence and properties.

We can make a house larger, by adding to it.
Then the house participates more in the form of largeness, and less in the form of smallness.
But we can not make a number odd, or square, by performing an addition or other operation.
Mathematical objects have their own natures.

> Suppose...that we add one to one. You would surely avoid saying that the cause of our getting two is the addition, or in the case of a divided unit, the division. You would loudly proclaim that you know of no other way in which any given object can come into being except by participation in the reality peculiar to its appropriate universal, and that in the cases which I have mentioned you recognize no other cause for the coming into being of two than participation in duality, and that whatever is to become two must participate in this, and whatever is to become one must participate in unity (*Phaedo* 101b-c).

We may distinguish between mathematicians that focus on processes or actions with mathematical objects and those who focus on their eternal natures.
The former are engineers, and scientists, looking at the applications of mathematics.
The latter are pure mathematicians, examining the eternal mathematical truths.
In the *Philebus*, Plato calls the former ordinary men and the latter philosophers.
But the true philosopher, the person of highest contemplation of truth, is distinguished from the mathematician in the *Republic*.

It is understanding, rather than pure intellect, which grasps mathematics.
Besides the *Philebus*'s distinction between mathematics and philosophy, Plato provides two arguments to the conclusion that mathematics is not of the highest order of reasoning.
The first argument is that the geometer uses diagrams, whereas the dialectician (or philosopher) uses only ideas.
Since diagrams are only perceivable through the senses, they are associated with the world of becoming, rather than the world of being.
The second argument is that the mathematician makes assumptions, but does not trace everything back to first principles.
The geometer has to start with assumptions, and can never get rid of them.

> Students of geometry and reckoning and such subjects first postulate the odd and the even and the various figures and three kinds of angles and other things akin to these in each branch of a

> science, regard them as known, and, treating them as absolute assumptions, do not deign to render any further account of them to themselves or others, taking it for granted that they are obvious to everybody (*Republic*, 510 c)

In contrast, the philosopher works without indispensable assumptions.

> By the other section of the intelligible I mean that which the reason itself lays hold of by the power of dialectic, treating its assumptions not as absolute beginnings but literally as hypotheses, underpinnings, footings, and springboards so to speak, to enable it to rise to that which requires no assumption and is the starting point of all, and after attaining to that again taking hold of the first dependencies from it, so to proceed downward to the conclusion, making no sue whatever of any object of sense but only of pure ideas moving on through ideas to ideas and ending with ideas (*Republic* 511b-c).

In Book VII, Plato uses an analogy of a line divided into four segments.
The top two represent the intelligible world (world of being) and the bottom two represent the sensible world (world of becoming).
The lower portion of the line is divided into images and those things of which they are images.
The upper portion of the line is divided into mathematical objects and forms.
Mathematical objects may thus be seen as images, in some sense, of the superior forms.

Note that Plato elevates arithmetic above geometry, in the second selection from the *Republic*.
There is a question in Plato scholarship about whether there are two kinds of mathematical objects.
The first are ideal numbers and geometrical ideas, forms of mathematical objects.
The second are mathematical numbers, and geometrical figures, which would pattern themselves after the ideal numbers and geometric ideas.
These latter objects are the objects of logistike, which was the ancient Greek science of calculation.
The former are closer to the forms.

It is a question for serious scholarship whether Plato's distinction between the philosopher's number and the common number (in the *Philebus*) corresponds to the distinction between mathematical objects and forms (in the *Republic*), or between the intelligible and sensible realms.

II. Plato's Epistemology

We see some aspects of Plato's mathematical epistemology in the *Republic*, where he names the changes in the soul which arise from apprehension of objects of each division of the divided line: intellection or reason (forms); understanding (mathematics); belief (sensory objects); picture thinking or conjecture (shadows).
But, Plato's doctrine of recollection is the central element in his account of all objects of intellect, whether the forms or mathematical objects.

In the *Theaetetus*, Plato argues that knowledge is not perception, but making judgments.
These judgments may be made on the basis of our perceptions, but maybe otherwise, as well.
Sensation can give us only the raw data for thought.
There must be a mind, independent of the body, to unify and compare the input from the senses.

> Socrates: Now take sound and color. Have you not, to being with, this thought which includes both at once - that they both *exist*?
> Theaetetus: I have.
> Socrates: And, further, that each of the two is *different* from the other and the *same* as itself?
> Theaetetus: Naturally.
> Socrates: And again, that both together are *two*, and each of them is *one*?
> Theaetetus: Yes.
> Socrates: And also you can ask yourself whether they are *unlike* each other or *alike*?
> Theaetetus: No doubt.
> Socrates: Then through what organ do you think all this about them both? What is common to them both cannot be apprehended either through hearing or through sight.
> ...
> Theaetetus: There is no special organ at all for [perceiving existence and nonexistence, likeness and unlikeness, sameness and difference, unity and numbers in general as applied to them, and even and odd] as there is for the others. It is clear to me that the mind in itself is its own instrument for contemplating the common terms that apply to everything (*Theaetetus* 185a-e).

In the *Meno*, Socrates argues explicitly that knowledge is recollection by showing how a slave boy, with no knowledge of mathematics, can be brought to understand a mathematical theorem by mere questioning.
We can see that the knowledge is inside of the slave boy, since he was only asked questions, and not taught.
It is difficult for me to interpret the *Meno* charitably.
I do not want to impute stupidity to Plato, but, it seems awfully difficult to avoid it here.
Consider Socrates' key question.

> Now does this line going from corner to corner cut each of these squares in half? (Meno 84e)

Socrates seems clearly to be teaching the boy, even if his instruction is in the form of a question.
The most charitable reading we can give Plato is that the slave boy needs to have some innate ability to recognize the mathematical truth when it is presented.
Showing that such an ability is innate would be a significant accomplishment, but it would be weaker than Plato's claim that all knowledge is already inside of us at birth.

Some people think that the doctrine of recollection in the *Meno* depends on the doctrine of the eternal soul.
The argument might go in the other direction.
If Plato could show that the slave could only be described as having recollected the theorem, then he would have evidence that the soul is eternal.

III. A Summary of Plato's Central Mathematical Theses

Knowledge must be of eternal objects. (*Phaedo, Republic*)
The world we receive in sense perception, the sensible world, is constantly changing. (*Timaeus, Phaedo*)
So, we can not have knowledge of the world we receive in sense perception. (*Theaetetus*)
Our best explanation of the changes in the sensible world involves the interactions of eternal forms. (*Phaedo, Republic*)

So, there is a sensible realm and an intelligible realm. (*Republic*)
We do not receive mathematical objects via sense perception, so they must belong to the intelligible realm. (*Republic*)
Mathematical objects undergo some sorts of changes, so they can not be perfectly eternal and unchanging forms. (*Republic*)
The intelligible realm is known via recollection. (*Phaedo, Meno*)

Here's a random, but interesting question: Are there important differences between Plato's accounts of geometry and arithmetic?

IV. Aristotle's criticisms of Plato, and the forms

We will now look briefly at a couple of Aristotle's criticisms of Plato's forms.
It's worth looking briefly at Aristotle's criticisms of the forms for two reasons.
First, mathematical objects are closely related to the forms.
While the precise relation between mathematical objects and the forms is obscure, mathematical objects are like the forms in that they are inhabitants of the intelligible, rather than sensible, realm.
Mathematical objects are particulars, rather than universals.
But, the forms sometimes have particular aspects as well, though not concrete particular aspects; a recent book argues that they are exclusively abstract particulars and not universals at all.
Second, Aristotle's criticisms of the forms leads him to his independent account of mathematics.

The criticisms Aristotle makes of Plato's work are not of our selections, mostly.
They are mostly not even of anything in the dialogues, but of teachings in the Academy that may never have been written down.
Let's look at Aristotle's criticism of four of Plato's arguments for the existence of forms.

P1. The argument from the sciences
P2. The one over many
P3. The object for thought
P4. Causation

Aristotle's criticisms of the forms were reportedly presented in extended form in a lost work called *On Ideas*.
The passages from *Metaphysics* in which Aristotle discusses the forms are unsatisfyingly abridged.

> According to the arguments from the existence of the sciences there will be Forms of all things of which there are sciences, and according to the argument that there is one attribute common to many things there will be Forms even of negations, and according to the argument that there is an object for thought even when the thing has perished, there will be Forms of perishable things; for we can have an image of these (*Metaphysics* I.9: 990b12-14).

P1, the argument from the sciences, appears to be the claim that the objects of a science must be stable and constant.
Since the objects of the sensible world are always changing, and we have some knowledge, our knowledge must be of eternal forms.
Aristotle's criticism of P1, I think, is that sciences are supposed to explain the sensible world.

But, if the objects of knowledge were forms, which are not in the sensible world, then the sciences are impotent in their central purpose.

> It must be held to be impossible that the substance and that of which it is the substance should exist apart; how, therefore, can the Ideas, being the substances of things, exist apart? (*Metaphysics* I.9: 991b1-3).

Against P2, Aristotle worries about the preponderance of forms.
There is some confusion about whether Plato thinks that all commonalities are explicable by forms.
Plato's theory of forms uses the forms as universals, of commonalities.
If two things are both tall, there is some property that they share, tallness.
If two things are beautiful, there is some property that they share.
Plato reifies these properties, takes them to be objects.
The question is whether there are forms of all commonalities, or only some.
On a narrow view, only some commonalities have corresponding forms.
The argument from the sciences seems to indicate a narrow view.
On a broad view, all commonalities lead to forms.
For any many, there is a one.
It is typical to say that early Plato accepted the broad view, but later Plato moved to a narrower view.
One problem with the broad view is that the forms are supposed to be perfect and unchanging.
If there were forms for every commonality, there would have to be forms of muddiness, and feces, say.
But, those don't seem like the kinds of things that could be forms.
Aristotle specifically doubts about there are forms of negations, and of perishable things.

The object for thought argument, P3, is the clearest of the three arguments in the *Metaphysics* paragraph.
We can think about something that is not present, or even existent.
Indeed, you and I can share the content of our thought about, say, James Brown.
We can not account for the content of our thought by appeal to our individual ideas; those are idiosyncratic and not shared.
What is shared is the form.
Again, Aristotle seems concerned that P3 yields too many forms, and illegitimate ones; we can think of mud even when there is none.

Lastly, against P4, Aristotle denies that the forms are useful explications of causes.
Any explanation in terms of forms will need to be supplemented by reference to an efficient cause.

> Above all one might discuss the question what on earth the Forms contribute to sensible things, either to those that are eternal or to those that come into being and cease to be. For they cause neither movement nor any change in them...All other things cannot come from the Forms in any of the usual senses of 'from'. And to say that they are patterns and the other things share them is to use empty words and poetical metaphors. For what is it that works, looking to the Ideas? Anything can either be, or become, like another without being copied from it, so that whether Socrates exists or not a man might come to be like Socrates; and evidently this might be so even if Socrates were eternal. And there will be several patterns of the same thing, and therefore several Forms, e.g. animal and two-footed and also man himself will be Forms of man. Again, the Forms are patterns not only of sensible things, but of themselves too, e.g. the Form of genus will be a genus of Forms; therefore the same thing will be pattern and copy (*Metaphysics* I.9: 991a9-31).

In addition to the criticisms of using Forms to explain causation, in this selection, Aristotle presents several other arguments against the Forms.

A1. Empty words
A2. Several patterns
A3. Pattern and copy

The 'empty words' argument, A1, is Aristotle's claim that one need not reify commonalities, that there need not be a one over any many.
As an alternative, Aristotle divides the world into substances and their properties.
People, animals, trees, and rocks are all substances.
We may call them natural kinds.
Tallness and beauty are not substances, but what is said of substances.
The substance and its attributes must be located together, as he insists in the following paragraph.
In essence, Aristotle is giving us the subject-predicate distinction, which remains in our grammar.
Aristotle is thus presenting an adjectival use of properties, which he will extend to mathematical objects.
Roundness (circularity) and twoness (from counting) are properties of primary substances, not substances themselves.

A2, the several patterns argument, seems just a bit of puzzle for the platonist.
It is not clear why it would be a problem for one object to participate in many forms.

A3 refers to the third man argument.
Since forms are invoked to account for similarities, we must have some explanation of why the blue sky is similar to the form of blueness.
There are two options.
We can say that the form of blueness participates in itself, in which case the same object is both pattern and copy.
Or, we can posit a higher-level form (a third man) to explain the commonality between the sky and the form of blueness.
On the latter view, we treat the form itself as a particular, and the number of forms multiplies impossibly.
There are turtles all the way down.
Even if the platonist accepts all of the higher- and higher-level forms, the infinite number of forms lack an explanation of their similarity.
So, the theory of forms fails to explain what it set out to explain.
That is, either horn of the dilemma reduces to absurdity.

Aristotle proceeds to criticize the Pythagorean-influenced view that the forms are numbers, which we will not pursue.
Aristotle's criticisms of the forms mostly focus on the multiplication of entities.

> In seeking to grasp the causes of the things around us, the introduced others equal in number to these, as if a man who wanted to count things thought he could not do it while they were few, but tried to count them when he had added to their number (*Metaphysics* I.9: 990b1-4).

As we will see, Aristotle's worries about the preponderance of forms also apply to the Platonic view of mathematical objects.
Aristotle says that Plato's accumulation of mathematical objects, like that of the forms, is absurd.