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The Explanatory Indispensability Argument

I. A Confusion of Weasels

We have been looking at the Quinean indispensability argument, and some of the most important responses to it: the dispensabilist and the weasel.
Most dispensabilists and weasels are fictionalists.
Indeed, I don't know of anyone, other than myself, who has written opposing the indispensability argument from a platonist point of view.
So, let's proceed on the assumption that all dispensabilists and weasels are fictionalists.
Further, let's not distinguish between fictionalists and nominalists.
So, we are looking at various anti-platonist responses to the indispensability argument.
We have seen three different kinds of weasels.
Let's look at them in order of increasing, say, dignity.

The most basic weasel, $weasel_1$, denies that mathematical objects exist, but mainly because of her rejection of metaphysical questions altogether.
$Weasel_1$s could be positivists, like Carnap; pragmatists, like Rorty; or others of anti-metaphysical bent.
For the $weasel_1$, the very question whether mathematical objects exist is ill-formed.
Similarly, the $weasel_1$ denies the legitimacy of the question of the existence of the external world.

A slightly more dignified weasel, $weasel_2$, accepts the legitimacy of the question of whether mathematical objects exist, and denies that they do.
$Weasel_2$s generally believe in the existence of the external world, and, unlike $weasel_1$s, will accept and assert sentences that affirm the existence of the external world.
But, they deny the existence of mathematical objects, regardless of whether they refer to them in their most serious sentences about the world.
Melia and Leng are $weasel_2$s.
They argue that mathematical objects play merely a representational or modeling role in science, so they enter our best theories in the wrong way to induce beliefs in those objects.
Despite our uses of mathematics in science, and despite our beliefs in the external world, we need not extend such beliefs to the mathematical objects used in science.

Lastly, the most dignified of the weasels, $weasel_3$, not only accepts the question whether mathematical objects exist, and denies that they do, but also explains the basis on which she determines her ontological commitments.
The most common of the $weasel_3$s is the eleatic, who denies the existence of mathematical objects because she believes that all and only existing objects are located within the causal realm.
Since mathematical objects are non-causal, they do not exist.
We did not read David Armstrong's work, but we did see Colyvan's response to some of Armstrong's eleatic view.
$Weasel_3$s recognize that it is not enough to deny beliefs in the existence of mathematical objects.
They must also explain why they deny that mathematical objects exist, and on what basis they determine what objects do exist.

If we rely on a weaseling approach to the quantifications of our best theories, then, the most dignified position will present an alternative method for determining the real commitments of our theories.
The eleatic presents one alternative.
But, others are possible.
We could say that all and only objects in space-time exist, for example.
Or, we might be idealists, or dualists, or have some other metaphysical attitude.

I have argued that the Quinean argument is most resilient to weaseling responses because of its reliance on independent arguments that we find our ontology in the domain of quantification of our best theory.
Whether we think that the weaseling response is satisfactory depends on how strong we take Quine's double-talk argument to be.
If double-talk is really inadmissible, then no weaseling will be allowable.
If our uses of mathematics are really just representational, or for modeling, as Melia and Leng argue, then weaseling may be defensible.

II. Stalemate

Over the last decade, it has started to look like the defenders of the standard indispensability argument and its detractors have argued themselves into a stalemate.
Dispensabilist projects have been produced, but they have not been as satisfying as Field initially hoped.
Colyvan and others have countered with stronger re-statements of the argument.
Weasels like Melia and Leng have refused to adopt Quine's conclusions.

But, a new version of the indispensability argument, the explanatory indispensability argument, has recently emerged.
Notice that any indispensability argument has to present some goal for which mathematics is supposed to be indispensable.
For Quine, that goal was the regimentation of our best scientific theories.
Given Quine's claim that we find our ontological commitments in the quantifications of our best theories, the indispensability of mathematics for those best theories has ontological importance.
But one of the criteria for determining whether we have a good theory is whether and how much that theory explains phenomena.
Proponents of the new, explanatory indispensability argument take explanation, rather than theory construction, to be the central goal for the argument.
They claim that we should believe in the existence of mathematical objects because they are indispensable to our best scientific explanations.

'Scientific explanation' has become something of a technical term in philosophy.
In order to assess the new argument, we should spend a few minutes considering the nature of scientific explanation.

III. Explanation

Broadly speaking, an explanation answers a why-question.
There are different kinds of why-questions.
The question why Brad Pitt is so sexy is different from the question why the earth revolves around the sun.
Some why-questions are epistemic; they solicit justifications for believing a statement.
Explanation-seeking why-questions solicit scientific explanations.

Scientific explanation became a topic of study in the wake of the logical positivists' work.
The first, and still paradigmatic theory of scientific explanation derives from the work of Carl Hempel, and is called the deductive-nomological, or D-N, model.
'Nomological' means lawlike.
D-N explanations are deductive, using covering laws and specific facts.
An explanandum, some specific phenomenon, is derived, using just logic, from the laws and initial conditions.
The covering laws are general.
They say that every time certain circumstances are realized, certain specific phenomenal will occur.
If we want to know why this swan is white, we might appeal to the general law that all swans are white, and the specific fact that this is a swan.

All swans are white.	(Covering law)
This is a swan.	(Specific fact)
So, this is white.	(Explanandum)

Both the general laws and the specific conditions mentioned in the general laws must be present in order to have a D-N explanation.
If we do not have specific conditions, we are left without application of the laws in any particular case.
If we do not have general laws, then we have an empty explanation.
For an example of a non-explanation, consider the explanation that opium puts one to sleep because it has the dormitive virtue.
The phrase 'dormitive virtue' comes from *Le Malade Imaginaire* by Molière.
We need general laws about how opium interacts with our body in order to have a real explanation.
And the best laws will be those of the highest level

We can also use the D-N model to explain lower-level laws by higher-level, covering laws.
Lower-level laws concern only a few variables.
The higher the level of the covering laws, the more variables are considered.
Boyle's Law. $P_1V_1=P_2V_2$, is a low-level law, because we keep the temperature constant while varying pressure and volume.
Similarly for Charles's Law: $V_1/T_1=V_2/T_2$.
On the other hand, the ideal law of gases is a higher-level law, combining results of the two: $PV=kT$.
Similarly, Newton's gravitational law explains Galileo's law regarding free-falling bodies.
Galileo took the acceleration of a free-falling body to be a constant.
Newton's law of gravitation shows that Galileo's law is false, but is also pretty close to a special case.
Thus, having a D-N explanation can be useful both in explaining particular phenomena and particular laws.

I will briefly discuss three of the problems which arise for the D-N model.
One problem concerns identifying the appropriate covering laws.
It is not enough merely to be a general claim.
There are many statements which look like laws, in their universality, but which are not really laws.
That a person in a room is first-born will confirm the hypothesis that all persons in a room are first-born.
That hypothesis looks, syntactically, like a law.
But, it will not increase our confidence that future people in the room will be first-born.
If younger sibling entered the room, the law (that all people in the room are first-born) would no longer hold.
Laws must be more than general statements which combine with specific cases to yield instances.
They must support counterfactuals.

A second problem with the D-N model involves relevance.
Some logical inferences of the D-N form are not explanatory because they appeal to irrelevant factors.
We can derive Kepler's laws of planetary motion from Newton's more general laws of motion.
But, we can also derive Kepler's laws from the conjunction of Newton's laws with, say, Mendel's laws of genetics.
Only the first inferences will be explanatory.

A third and last problem with the D-N model arises from its lack of asymmetry.
Explanation is asymmetric.
We can explain the length of a shadow by appealing to the height of, say, a flagpole.
But, according to the D-N model, we can just as easily explain the height of the flagpole by the length of the shadow.
Again, only the first inference would be truly explanatory.

Modifications to Hempel's D-N account attempted to fix the problems concerning laws, relevance, and symmetry.
Some alternative accounts of scientific explanation relied on pragmatic or epistemic considerations.
Bas van Fraassen developed a pragmatic account of explanation.
Others looked toward unification, rather than logical derivation, as the fundamental aspect of scientific explanation.
Philip Kitcher relies on unifying argument patterns which also answer why-questions by proffering inferences made within a serious theory.

But the most promising emendations to the D-N model and alternative accounts of explanation focused on causation as the centrally important concept.
Covering laws can be distinguished from accidental generalizations by appealing to the causal connections described by those laws.
The asymmetry of explanations can be explained by the appeals to causal laws in the explananda.
The relevance condition is again an appeal to causal connections.

We will not pursue alternative accounts of explanation any further.
On the D-N account, and related ones, the explanation of a state of affairs involves the causal laws of a serious theory combined with appropriate initial conditions.
The theories to which D-N explanations appeal are ones in which we speak most strictly.
But, there are alternative, if not very popular, accounts of explanation which do not appeal to our best theories.

IV. The Explanatory Indispensability Argument

Indispensabilists have been both emboldened by the lack of convincing success on the side of the nominalists, and eager to fortify the original argument against dispensabilist criticisms.
According to the new explanatory indispensability argument, we should believe in mathematical objects because of their indispensable roles in our scientific explanations.
Baker defends the explanatory argument.
We can see versions of it in Colyvan's more recent work, too.
Baker does not state his version of the argument explicitly, but Mancosu does.

EI
EI1. There are genuinely mathematical explanations of empirical phenomena.
EI2. We ought to be committed to the theoretical posits postulated by such explanations.
EIC. We ought to be committed to the entities postulated by the mathematics in question (Mancosu 2008: §3.2).

The provenance of EI is a matter for dispute.
Baker and Mancosu both misleadingly credit Field.

> Hartry Field, one of the more influential recent nominalists, writes that the key issue in the platonism-nominalism debate is 'one special kind of indispensability argument: one involving indispensability for explanations' (Field 1989, p. 14) (Baker 225).

A careful reading of the selection cited by Baker shows no such argument by Field.
Specifically, Field makes no claim that there is a heavier burden on the dispensabilist than recasting standard scientific theories to remove quantification over mathematical entities.
Field's interest in explanation depends exclusively on how explanatory merit factors into evaluations of our theories.
His concern is with the traditional indispensability argument, i.e. QI, where one factor in determining whether a theory is our best theory is whether it has explanatory force.
Other factors include breadth and simplicity.
For Field, once we have settled on a best theory, the central question for the indispensabilist is whether that theory can be recast to avoid quantification over mathematical objects.

In the section cited, Field explicitly refers to his own work rewriting NGT, and he moves directly from talk about explanation to talk about theories.

> What we must do is make a bet on how best to achieve a satisfactory overall view of the place of mathematics in the world... My tentative bet is that we would do better to try to show that the explanatory role of mathematical entities is not what is superficially appears to be; and the most convincing way to do that would be to show that there are some fairly general strategies that can be employed to purge theories of all reference to mathematical entities (Field 1989: 18; see also fn 15 on p 20).

Field is clearly thinking of explanation on a traditional covering-law D-N account.
In fact, Field says that an explanation is, "A relatively simple non-*ad hoc* body of principles from which [the phenomena] follow" (Field 1989: 15).

In contrast, the key feature of the explanatory argument is that it puts aside the question of whether theories can be recast in order to eliminate mathematical entities.
Instead, the proponents of the explanatory argument wonder whether non-mathematical explanations of physical phenomena are available.
Indeed, recent work on EI grants the availability of nominalist reformulations of standard scientific theories and continues to urge that mathematical explanations of empirical phenomena support belief in mathematical objects.

These options allow us to raise a question about presenting EI as an alternative to other indispensability arguments.
We can see the argument as an additional demand on the platonist, and thus an additional option for the nominalist.
Sorin Bangu and Joseph Melia say that even if dispensabilist constructions do not work, we should withhold commitments to mathematical objects since there are no genuinely mathematical explanations.
On the Bangu/Melia view, the platonist has to show mathematics indispensable from both theories and explanations; the nominalist needs to show that mathematics is eliminable only from explanations or theories.

In contrast, we can see the argument as an additional option for the platonist, and thus an additional demand on the dispensabilist.
Baker argues that even if the dispensabilist constructions do work, we should grant commitments to mathematical objects as long as there are genuinely mathematical explanations of physical phenomena.
On Baker's view, the platonist needs to show that mathematics is indispensable only from explanations, and the nominalist must show how we can eliminate mathematics from both theories and explanations.

It will not really matter here whether EI is taken as an additional burden on the nominalist or on the platonist.
The more important aspects of the argument are its departures from Quine's version.
The abandonment of Quine's method for determining ontological commitments entails that, like the weasel, the proponent of the explanatory argument should defend a new method for determining ontological commitments.
Baker substitutes an inference to the best explanation, or IBE.
He believes that an IBE is essential to the indispensability argument.

> The indispensability debate only gets off the ground if both sides take IBE seriously, which suggests that *explanation* is of key importance in this debate (Baker 225).

As I have argued at length recently, the key element of Quine's argument is his criterion for determining the ontological commitments of our theories, not an inference to the best explanation.

Also against Baker's interpretation of the argument, without Quine's holism, it is difficult to see how the evidence for the empirical elements of our best scientific explanations extends to the mathematical objects used in those explanations.
Without some measure of holism, it is impossible to make the case that evidence extends from science to mathematics.
Still, we can proceed to understanding the argument.
We will return to worries about its strength, next week.

V. Baker on Colyvan

While the difference between theories and explanations is important for evaluating the success of the explanatory indispensability argument, the two goals are closely related.
Indeed, while I took Colyvan's three examples of non-causal explanations to be relevant to Quine's argument, Baker believes that they might, in principle, be used in an explanatory argument.
The examples function similarly in each domain.
For QI, they are attempts to show that there are non-causal entities in our best theories.
The defender of the explanatory argument is looking for genuine mathematical explanations of physical phenomena.
For the explanatory argument, Colyvan's examples could be attempts to show that there are serious references to mathematical entities in our explanations.

Unfortunately, Baker believes that Colyvan's examples are, in practice, insufficient for EI.
Colyvan's first example, of the antipodes, is a predication, not an explanation.
We are unlikely to find that there are two antipodes with precisely the same temperature and pressure in two spots in the Earth's atmosphere.
The account which uses the Borsuck-Ulam topological theorem predicts that we could find two such spots.
But, prediction is not explanation, and the order of an explanation is important.
Recall the symmetry criticism of the D-N model of explanation.

Colyvan's second and third examples, according to Baker, are contentious because of their reliance on geometry.

> Individual geometrical terms such as 'triangle' may refer either to mathematical or to physical objects, and the historical trajectory of Euclidean geometry, from descriptor of physical space to free-standing formal system, shows a similar bridging of the mathematics/physics boundary at the level of geometrical theories. This is one reason why nominalists often object that geometrical explanations are not genuinely mathematical. And it suggests that we should look elsewhere than geometry for a convincing case of mathematical explanation in science (Baker 228).

Baker's criticism of Colyvan is similar to my claim that Colyvan begs the question about non-causal, non-mathematical objects.
Baker also cites Melia's criticism of Colyvan that the mathematics in each case is used merely to represent, or index, real physical properties.
In other words, Colyvan's cases seem excessively liable to weaseling responses, especially those based on an eleatic principle.

From his consideration of Colyvan's cases, and others, Baker presents three criteria which any example of a mathematical explanation of a physical phenomenon would have to meet in order to be considered genuine.

> The first condition is that the application be external to mathematics...
> The second condition is that the phenomenon in question must be in need of explanation...
> A third condition...is that the phenomenon must have been identified independently of the putative explanation (otherwise it is more like a prediction) (Baker 233-4)

VI. Cicadas, Honeycombs, and More

Baker provides an example which meets all three of his conditions.
Three species of cicadas of the genus Magicicada share a life cycle of either thirteen or seventeen years, depending on the environment.
The phenomenon of having prime-numbered life-cycles intrigued biologists, who sought an explanation.
Baker claims that the phenomenon is explained thus:

CP

CP1. Having a life-cycle period which minimizes intersection with other (nearby/lower) periods is evolutionarily advantageous.
CP2. Prime periods minimize intersection.
CP3. Hence organisms with periodic life-cycles are likely to evolve periods that are prime.
CP4. Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years.
CP5. Hence, cicadas in ecosystem-type, E, are likely to evolve 17-year periods (Baker 233).

Baker argues that the mathematical explanans, at CP2, supports the empirical explanandum, at CP3.
In fact, as Baker notes, CP3 is a "'mixed' biological/mathematical law."
He proceeds to use this law to explain the further empirical claim CP5.
In other words, that prime-numbered life-cycles minimize the intersection of cicada life-cycles with those of both predators and other species of cicadas explains why three species of cicadas of the genus Magicicada share a life cycle of either thirteen or seventeen years, depending on the environment.

Baker urges that the cicada example meets all three criteria.
There is a biological phenomenon in question, one which was noticed before any mathematical explanation was presented.
That phenomenon puzzled scientists, who looked for an explanation.

Mancosu presents three further examples.

> Why do hive-bee honeycombs have a hexagonal structure?...Part of the explanation depends on evolutionary facts. Bees that use less wax and thus spend less energy have a better chance at being selected. The explanation is completed by pointing out that "any partition of the plane into regions of equal area has perimeter at least that of the regular hexagonal honeycomb tiling". Thus, the hexagonal tiling is optimal with respect to dividing the plane into equal areas and minimizing the perimeter (Mancosu §1).

The honeycomb conjecture which states that a regular hexagonal grid or honeycomb represents the best way to divide a surface into regions of equal area with the least total perimeter is a purely geometric result.
Again, we have a biological phenomenon, independent of any mathematical prediction, that needed explanation.
The mathematical theorem is part of the explanation.

Mancosu's second and third further examples have a greater relation to physics.
Hold a tennis racket horizontally, and toss the racket attempting to make it rotate about the intermediate principal axis (which is perpendicular to the handle in the face of the racket passing through its center of mass).
After one rotation catch the racket by its handle.
The opposite face will almost always be up.
The racket makes a near-half twist around its handle.
The explanation refers to a mathematical result which has become known as the twisting tennis racquet theorem.

Mancosu's last further example, due to Peter Lipton, involves tossing a bundle of sticks.
A snapshot of the sticks in the air will show that significantly more of the sticks are close to the horizontal than to the vertical.
The explanation of this fact is mathematical, rather than physical.
It doesn't have to do with gravity, or air resistance, or the force of the throw.
There are just more ways to be horizontal than vertical.

> Such explanations...seem to be counterexamples to the claim that all explanations in the natural science [sic] must be causal (Mancosu §1).

VII. Two Questions

Recall EI.

EI
- EI1. There are genuinely mathematical explanations of empirical phenomena.
- EI2. We ought to be committed to the theoretical posits postulated by such explanations.
- EIC. We ought to be committed to the entities postulated by the mathematics in question.

To evaluate the argument, we have to evaluate each of the premises.
For EI1, is the mathematics in these Colyvan/Mancosu cases really explanatory?
For EI2, does it matter, as far as our ontological commitments are concerned?

We might, for the time being, accept an affirmative answer to the question regarding EI1.
We will pursue two responses to the question about EI2 next week.