Philosophy 405: Knowledge, Truth and Mathematics | Hamilton College
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Mathematical Recreation

I. Naturalism and Epistemology

Leng defends a weaseling version of mathematical fictionalism against the indispensability argument.
The first part of Leng's paper concerns a tension in Quine's view, between his epistemology and the indispensability argument.
Leng believes that she has found a way to reject the conclusion of the indispensability argument, while maintaining a broadly Quinean (i.e. empiricist) framework, by exploiting this tension.
This tension was explored earlier, by Penelope Maddy, who describes the problem as rooted in different understandings of Quine's naturalism.
It will be worth a moment to try to be clearer about the 'naturalist' label, and Quine's use of it.

While Quine, in places, calls himself an empiricist, and the indispensability argument is clearly an empiricist's argument, Quine prefers to use the term 'naturalism' to describe his approach to philosophy.
He characterizes naturalism as the rejection of the view of philosophy as independent of, and as a way of evaluating, science.

> Naturalism [is the] abandonment of the goal of a first philosophy. It sees natural science as an inquiry into reality, fallible and corrigible but not answerable to any supra-scientific tribunal, and not in need of any justification beyond observation and the hypothetico-deductive method (Quine, "Five Milestones of Empiricism" 72).

As with any distillation of a variety of complex views to a simple term, there are different interpretations of 'naturalism'.
One consequence of Quine's naturalism, at its most radical, is the assimilation of epistemology to empirical psychology.
If there is no supra-scientific tribunal, then there is no first philosophy (metaphysics) and no epistemology, no way to establish what exists independently of our best scientific theories.
Quine's holism ensures that claims in philosophy, and physics, and biology, and economics, and mathematics, are all inter-related.
No claim is prior to, or independent of, any other.
Epistemology, our theory of knowledge, is continuous with neuroscience, our theories of the brain, and cognitive psychology, our theories of our thought, and biology, our theories of sensation, and information processing.

> Naturalism does not repudiate epistemology, but assimilates it to empirical psychology. Science itself tells us that our information about the world is limited to irritations of our surfaces, and then the epistemological question is in turn a question within science: the question how we human animals can have managed to arrive at science from such limited information (Quine, "Five Milestones of Empiricism" 72).

Concomitantly, Quine views philosophy as a handmaiden to the sciences.
Philosophers can make progress in mathematics, say, as Quine did in set theory, or in empirical science, by contributing to the understanding of empirical research.
But, there is no special, independent discipline called philosophy.

If we want to do metaphysics, we just construct the best scientific theories, and interpret them.
We look to scientists for their work.

As Leng reviews in the first part of her paper, Maddy argues that two interpretations of Quine's naturalism are in tension with one another.
On the one interpretation, Quine privileges empirical science, understood holistically, as the locus of all our ontological commitments.
On the other, he defers all questions of what exists to the scientists.
Scientists, Maddy argues, isolate mathematics from the rest of their theories.
They do not act as if all the objects over which they quantify have the same status.
Scientists make a distinction between the real elements of their theories and the instrumental elements.
In particular, they seem to rely on something closer to an eleatic principle, when evaluating the commitments of their theories.
In practice, they reject Quine's holism.
So, the naturalist seems to have to decide between accepting Quine's holism, or rejecting it in favor of the instrumentalism that scientists actually use.

Quine, or a Quinean, would respond that instrumentalism, which we saw in the work of Carnap and Melia, employs double-talk about ontological commitment.
But, according to Leng and Maddy, such a response privileges philosophy over scientific practice.
The holistic response elevates philosophical reflection over scientific practice, and thus conflicts with a proper understanding of naturalism.
A consistent naturalism, Maddy and Leng argue, would not reject instrumentalism if that view is a part of scientific practice.
And, Maddy argues, instrumentalism is an accepted principle of scientific practice.
As Sober argues, scientists never actually test mathematical hypotheses.
Instead, they hold them fixed in the background.
Experiments only confirm the empirical portions of our hypotheses.
Counterexamples only hold against the empirical portions, too.
Remember the foxes and chickens!

Maddy thus concludes, and Leng follows her, that the indispensability argument fails because the ontological commitments of a theory are not to be found exclusively in the quantifications of the theory.
Instead, naturalism entails that epistemology is to be assimilated to empirical psychology, and that metaphysics is to be assimilated to empirical science.
The ontological commitments of a theory should be discovered by looking at the actual practice of scientists.

Leng's fictionalism is the result of privileging the naturalism which respects the practice of scientists over Quinean holism and the double-talk argument.
Let's look at her naturalistic evaluation of the relation between mathematics and science.

II. Mathematics and Science

Relying on the work of Maddy and Sober describing the relationship of scientific practice and attitudes toward mathematics, Leng presents three characteristics of mathematics with regard to science, which I will call insularity, Euclidean rescue, and bridging.

> [Insularity] 1. Mathematics is insulated from scientific discoveries, in the sense that the falsification of a scientific theory that uses some mathematics never counts as falsification of that mathematics (beyond simple cases of calculation error).
>
> [Euclidean Rescue] 2. In particular, a scientific observation that conflicts with some scientific theory may suggest a move to a different background mathematics, but does not suggest that mathematicians should abandon that mathematics...The success of a scientific theory does not confirm the mathematics used in that theory.
>
> [Bridging] 3. What does seem to be disconfirmed by the failure of a scientific theory that relies strongly on a background mathematics is the claim that this mathematics is applicable to the scientific phenomena that it has been used to describe (Leng 411).

Quine argues that when we are faced with a contradiction in our theory, as when an hypothesis is contradicted by an experiment, we have the choice to revise an empirical hypothesis in our grand, holistic theory, or a mathematical hypothesis, or a logical one.
Since the entire theory is interconnected in a single, holistic web, we can restore consistency in lots of distinct ways.
As a logical matter, Quine's confirmation holism is indisputable.
Sober's argument against the Quinean view derives from the observation that in practice we never choose to give up the logical or mathematical principles.
Again, this practical fact is not contentious.
Quine explains this fact as the consequence of a methodological principle of theory choice: the maxim of minimum mutilation.
In accordance with the maxim of minimum mutilation, we hold logical and mathematical principles fixed in order to do as little damage as possible to other portions of our theory.
Sober and Leng take the fact that we never give up our mathematics in light of recalcitrant data to indicate that mathematics is insulated from science, at the most basic level.

The term 'Euclidean rescue' comes from the work of Michael Resnik.
Before the nineteenth century, people generally thought that there were only one geometry.
When it became clear that there were consistent, non-Euclidean spaces, people tended to think of them as somehow lesser geometries.
Some people thought of non-Euclidean geometries as consistent, but false, or uninterpreted.
When it became clear that physical space was non-Euclidean, some people inferred that hyperbolic geometry was true, and Euclidean geometry was false, or uninterpreted.
We perform a Euclidean rescue when we accept all three geometries that result from the three different parallel postulates as equally true.
In this paradigmatic case, the Euclidean rescue entails that the discovery of which geometry is actually applied in our space is irrelevant to the truth of the mathematical theory.

> It is...difficult to maintain that the empirical discoveries confirmed the truth of non-Euclidean geometry and showed the falsity of Euclidean geometry in any sense other than that one was a correct model of the physical world and the other was not. But this is not *mathematical* truth: the applicability of non-Euclidean geometry did not falsify any mathematical theorems in Euclidean geometry - the Pythagorean theorem still holds for Euclidean triangles - it merely confirmed the assumption of Gauss and others that the scope of the theorems of Euclidean geometry only covers systems that assume the parallel axiom (Leng 402).

Euclidean rescues are not limited to this one case.
They are available in all cases in which a mathematical theory is shown inapplicable in science.
Leng cites the case of catastrophe theory.
Initially, the mathematical theory called catastrophe theory was thought to have profound implications for physical science.
Later, it was seen not to apply as broadly as was initially thought.
Still, the mathematics was not impugned.
Euclidean rescues are related to insularity in that if one thinks that mathematics is insular, then one is predisposed to perform Euclidean rescues.
One might perform Euclidean rescues because one thinks that mathematics is and should be held insular.

Similarly, Euclidean rescues are supported by Bridging.
For a mathematical theory to be used in a physical theory, there must be bridge principles which map some mathematical claims into some physical claims.
When one finds an inconsistency in one's physical theory, one can always restore consistency without falsifying one's mathematical claims by denying just the bridge principles, and not the actual mathematical theorems.
By denying only the bridge principles, we perform a Euclidean rescue, holding the mathematics to be insulated from any falsification of the empirical theory.
In Leng's case of catastrophe theory, only the bridge principles, the claims of its applicability to physics, were denied.

> Catastrophe Theory became a much less popular area of research, but no one would claim that the mathematics of Catastrophe theory had been *falsified* by its magnificent scientific failures (Leng 407).

Leng's fictionalist proposal rests both on her exploitation of the tension in Quinean naturalism and these three characteristics of mathematical practice and its relation to science.
It also relies on Quine's attitude toward the mathematics which is not used in empirical science.

III. Mathematical Recreation

We saw Quine's view concerning the un-applied portions of mathematics when we first looked at the indispensability argument.

> My view of pure mathematics is oriented strictly to application in empirical science...Pure mathematics extravagantly exceeds the needs of application...but I see these excesses as a simplistic matter of rounding out...I recognize indenumerable infinites only because they are forced on me by the simplest known systematizations of more welcome matters. Magnitudes in excess of such demands, e.g., $\beth_\omega$ or inaccessible numbers, I look upon only as mathematical recreation and without ontological rights (Quine on Mathematical Recreation).

Strictly speaking, the indispensabilist has no justification for beliefs in mathematics that are not used in physical theory.
There is some room for extending our beliefs in the disjoint pieces of mathematics that are applied in science to full mathematical theories.
There is no room for beliefs in portions of mathematics which have no relation to our empirical science.

Quine and the fictionalist agree that such sub-theories of mathematics are false or merely vacuously true.
Colyvan follows Quine in appealing to a fictionalist interpretation of the un-applied portions of mathematics.
This is what Leng calls the doctrine of mathematical recreation: we should be fictionalists about the portions of mathematics that are not used in science.

Before the break, we had asked in class whether our lack of access to mathematical objects is the same kind of lack as our lack of access to unicorns.
There seem to be two different kinds of accounts of our knowledge of fictional objects.
Similarly, one can argue that there is a distinction among fictional objects.
The way in which mathematical recreation is fiction and the way in which *Moby Dick* is fiction are different.
Moby Dick and other fictional stories are not constructed in the way that non-fictional works are created.
But, mathematical methodology is the same for portions which are used in science and portions which are not used in science.
Recreational results are continuous with applied mathematical results.
They are reducible to the same foundational theories, like set theory.
They are written in the same, formal language.
There are no mathematical differences among the two groups of results.

Leng's proposal, then, is simply to extend the fictionalist attitude toward recreational or un-applied portions of mathematics to all of mathematics.
From the characteristics of Insularity, Euclidean Rescue, and Bridging, Leng concludes that mathematics plays merely a modeling role in science.
Mathematical theories lack any serious ontological rights because they are used merely as models, without any presumption that they are true or refer to real objects.

> When we use mathematics to model physical situations in this way, we never refer to mathematical objects or assume the (mathematical) truth of their relations. Rather, we interpret our mathematical stories physically and assume that our model is good enough in the relevant respects that the theorems derived in our mathematical recreations, when transcribed into physical language, will give us truths about the physical phenomena we are considering...If Colyvan is right (and I think he is) that mathematics that is not assumed by science to be true should be seen as recreational (and given some important status as such), then it follows from the modeling picture of the relationship between mathematics and science that *all* mathematics is recreational (Leng 411-412).

In later work, Leng discusses the modeling aspect of mathematical objects in terms of their ability to represent physical objects.

> We are not committed to belief in the existence of objects posited by our scientific theories *if their role in those theories is merely to represent configurations of physical objects*. Fictional objects can represent just as well as real objects can (Leng 2005: 179).

From Quine's naturalism, Leng concludes that there is no mathematical reason to distinguish between applied and un-applied results in mathematics.
She also infers that the work of mathematicians (and scientists) need not entail our beliefs in the truth of mathematical claims.

Where Quine claimed that un-applied portions of mathematics could be considered recreational, Leng argues that all of mathematics should have that status.

IV. A Real Platonist Option

For those impressed with the argument concerning the continuity of applied and un-applied results, there is a non-fictionalist option.
I agree with Maddy and Leng that Quine's naturalism is in tension with itself.
The indispensabilist's view of mathematics is in tension with the actual practice of mathematics.

The continuum hypothesis provides a clear example of how mathematical practice conflicts with the indispensabilist's philosophy of mathematics.
According to the indispensabilist, mathematical questions are to be answered by examining our best (naturalist) scientific theory.
Our current mathematical axioms do not settle the question of the size of the continuum, and provably so.
Gödel suggested that we adopt new axioms to settle the question.
The question arises about how to decide on which axioms to adopt.
Do we look to physics?
Or, do we look directly to mathematics?
The indispensabilist insists that the answers have to be found in the needs of our best scientific theory.
The mathematician looks to purely mathematical criteria.
Thus, the methods of the indispensabilist conflict with the methods of the mathematician.

If we are looking to the practice of science as having consequences for what we should believe exists, we should look more broadly.
We can reject Quine's view that there is no supra-scientific tribunal.
We can, and need to, develop criteria for determining which so-called scientific practices are legitimate.
Mathematics itself is good science.
The naturalist should be willing to accept the practice of mathematicians.
Instead of calling all of mathematics recreational, we can call it all true!

The lesson we take from the continuity claim is either to accept all of mathematics or to reject it all.
Leng argues that a good Quinean, with her preference for desert landscapes, should reject it all.
I believe that a good Quinean naturalist should accept all of mathematics, on the basis of the practice of mathematicians, and the legitimacy of mathematics as a science.