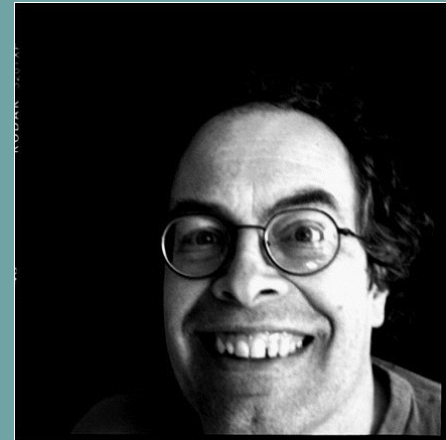# Knowledge, Truth, and Mathematics

Philosophy 405
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Hamilton College, Fall 2010
November 15
Class 23: Finishing Field
Melia's Weasel

Field's Project

- "If one *just* advocates fictionalism about a portion of mathematics, without showing how that part of mathematics is dispensable in applications, then one is engaging in intellectual doublethink: one is merely taking back in one's philosophical moments what one asserts in doing science..." (Field 2).

- First, he develops a nominalist counterpart to a standard scientific theory
  - ‣ Choose some arbitrary regions of space-time to serve as the bases for measurement in place of the real numbers.
  - ‣ Impose a structure, including measurement (greater and less) and order (betweenness), on space-time regions.
  - ‣ Proving representation theorems that show that any uses of the real numbers can be replaced by the structure he describes for space-time points.

- Second, he tries to show that mathematics applies conservatively to nominalist theories, to assure us that nominalist counterparts are adequate substitutes.

Fictionalism and Mathematical Knowledge

Field accounts for what is ordinarily seen as mathematical knowledge in three ways.

1. Much of what we take to be mathematical knowledge is really empirical knowledge.
    - Of space-time
    - Of which axioms are generally accepted by the mathematical community

2. For limited results, there is no knowledge to be had.
    - no physical facts

3. We have logical knowledge of which theorems follow from which axioms.
    - "[Y]ou don't need to make mathematics actually be about anything for it to be possible to objectively assess the logical relations between mathematical premises and mathematical conclusions" (Field 1998a: 317).

Worries About Field's Project I: Space-Time Points

- There are two ways in which mathematics might be said to remain in Field's nominalist theory.

- First, we might claim that space-time points are really mathematical objects.
  - ‣ Hartry Field has claimed...that one can do physics without reference to abstract entities. But his construction requires that we accept *absolute space-time points* and arbitrary sets of space-time points as 'concrete'; most philosophers (including myself) would regard this as 'cheating' (Putnam 1981: 175).
  - ‣ Owing to the richness of Field's physical ontology, philosophers in Quine's tradition might object that Field has just hidden his mathematical objects in physical disguises (Resnik 1985b: 192).
  - ‣ [In space-time, we have] everything essential to the real numbers (Maddy 1990b: 201).

Against Worry I

- We use space-time points in field formulations of physical theories.

- They are empirical posits, as opposed to *a priori* ones.
  - ▸ "Perhaps it is a bit odd to use the phrase 'physical entity' to apply to space-time points. But however this may be, space-time points are not abstract entities in any normal sense. After all, from a typical platonist perspective, our knowledge of mathematical structures of abstract entities (e.g. the mathematical structure of real numbers) is *a priori*; but the structure of physical space is an empirical matter" (Field 31).

- Field even argues that we have sensory access to space-time points.
  - ▸ "For there are quite unproblematic physical relations, viz., spatial relations, between ourselves and space-time regions, and this gives us epistemological access to space-time regions. For instance, because of their spatial relations to us, certain space-time regions can fall within our field of vision" (Field 1982a: 68).
  - ▸ Sensory access to empty regions is implausible.
  - ▸ Worries about access to space-time points are moot.

Worries About Field's Project II: Ideology

- A second about what remains in the reformulated theory: the ideology that goes with both mathematics and Field's nominalist theory is what really matters to mathematics.

- Both Field's space-time ontology and the standard mathematical ontology contain continuous objects.

- Both kinds of objects satisfy the mathematical properties required by science.

- But:
  - "Postulating physical space isn't like postulating real numbers...the ideology that goes with the postulate of points of space is less rich than that which goes with the postulate of the real numbers" (Field 32).

Worries About Field's Project III:
Are Reformulations Available?

- Classical Hamiltonian mechanics, a phase-space theory

- Quantum mechanics

- General relativity
  - There is no available suitable version of the curved space-time geometry.

- Statistical inference
  - Psychology and economics
  - Physics may be easier to nominalize, just because it is so mathematized.

- Speculation
  - We do not know the nature of our ideal physical theory.
  - And we do not know what kinds of dispensabilist techniques may be developed.
  - "As a consequence of nominalism's being mainly a philosopher's concern, this open research problem is...one that has so far been investigated only by amateurs - philosophers and logicians -not professionals - geometers and physicists; and the failure of amateurs to surmount the obstacles is no strong grounds for pessimism about what could be achieved by professionals" (Burgess and Rosen 1997: 118).

Ground Rules 1-2

- GR.1: *Adequacy*: A reformulation must not omit empirical results of the standard theory.

- GR.2: *Logical Neutrality*: A reformulation must not reduce ontology merely by extending logic, or ideology.
  - First-order logic makes no commitments
  - Second-order logic is "set theory in sheep's clothing" (Quine).
  - Modal logics make commitments to possible worlds.
  - "Avoidance of modalities is as strong a reason for an abstract ontology as I can well imagine" (Quine, "Reply to Charles Parsons" 397).
  - "[I]t can be seen that there is something dubious about the practice of just helping oneself to whatever logical apparatus one pleases for purposes of nominalistic reconstruction while ignoring any customary definitions that would make the apparatus nominalistically unpalatable: for by doing so, one can make the task of nominalistic reconstruction absolutely trivial – and so absolutely uninteresting" (Burgess and Rosen, *A Subject with No Object* 175).

Ground Rules 3-4

- **GR.3:** *Conservativeness*: The addition of mathematics to the reformulated theory should license no additional nominalist conclusions.
  - ▸ The top-down argument alleges that set theory is conservative over any nominalistically acceptable body of assertions.
  - ▸ Field's bottom-up arguments for conservativeness rely on his representation theorems.

- **GR.4:** *Attractiveness*
  - ▸ The dispensabilist must show, "[T]hat one can always reaxiomatize scientific theories so that there is no reference to or quantification over mathematical entities in the reaxiomatization (*and one can do this in such a way that the resulting axiomatization is fairly simple and attractive*)." (Field viii, emphasis added)
  - ▸ Subjective?
  - ▸ Few axioms?
  - ▸ Elegant proofs?
  - ▸ "If no attractiveness requirement is imposed, nominalization is trivial... Obviously, such ways of obtaining nominalistic theories are of no interest" (Field 41).

The Trivial Strategy

- Adopt only the nominalistically-acceptable theorems of a scientific theory.

- Field: such theories are unattractive.

- Melia: even conservative mathematical theories can be insufficient.
  - Mereological theories
  - "There can be more to the nominalist consequences of a theory than the set of sentences entailed by that theory in the nominalist vocabulary. If the nominalist simply takes his theory to be the set of nominalistically acceptable sentences entailed by some platonist theory, he has no guarantee that his theory actually has the same nominalist content as the platonist theory" (Melia 461).

- Let's grant the failure of the trivial strategy.

- If weaseling is acceptable, we don't need the trivial strategy anyway.

Melia and Carnap

- Melia defends nominalism against the indispensabilist without a Field-style rewriting of scientific theory.
  - "A physicist who is suspicious of abstract entities may perhaps try to declare a certain part of the language of physics as uninterpreted and uninterpretable, that part which refers to real numbers as space-time coordinates or as values of physical magnitudes, to functions, limits, etc. More probably he will just speak about all these things like anybody else but with an uneasy conscience, like a man who in his everyday life does with qualms many things which are not in accord with the high moral principles he professes on Sundays" (Carnap, "Empiricism, Semantics, and Ontology" 205).

- Quine accused Carnap of doubletalk, or doublethink.
  - Field followed Quine.
  - Quine's holism supports his denigration of doubletalk: there is no external perspective from which to stand to evaluate linguistic frameworks.

- Melia attempts to reclaim Carnap's attitude toward the uses of mathematics in science without Carnap's view concerning linguistic frameworks.
  - "It is quite common for both scientists and mathematicians to think that their everyday, working theories are only partially true" (Melia 2000: 457).

Weaseling

- Melia calls the practice of taking back a portion of one's claims weaseling.
- The weasel accepts that mathematics is ineliminable from scientific theory but maintains that we need not believe that mathematical objects exist.
- The weasel can accept, say, that vectors in Hilbert space are indispensable to the practice of quantum mechanics.
- The weasel just adds that we can, when speaking most seriously and parsimoniously, deny that our best theory really posits them.
- Thus, Melia denies Quine's claim that our ontological commitments are (all of) those objects over which we first-order quantify in our best theories.
  - ▸ "The mathematics is the necessary scaffolding upon which the bridge must be built. But once the bridge has been built, the scaffolding can be removed" (Melia 2000: 469).

The Weasel and Scientist

- Joe is a hypothetical nominalist
  - He prefers standard theories, like T*, because of their inferential strength.
  - He rejects the mathematics used in those standard theories.
  - "Joe does *not* simultaneously hold contradictory beliefs. Just because, in the process of telling us his beliefs about the world, Joe asserts all the sentences of T*, it does not follow that Joe believes all the sentences of T*. Indeed, since Joe believes there are no abstract objects, he will explicitly say that T* is false" (Melia 467).

- Compare Joe to his intellectual ancestor.
  - "Some contemporary nominalists label the admission of variables of abstract types as "Platonism". This is, to say the least, an extremely misleading terminology. It leads to the absurd consequence, that the position of everybody who accepts the language of physics with its real number variables (as a language of communication, not merely as a calculus) would be called Platonistic, even if he is a strict empiricist who rejects Platonic metaphysics" (Carnap, "Empiricism, Semantics, and Ontology" 215).

The Geometry Analogy

r = 8 m
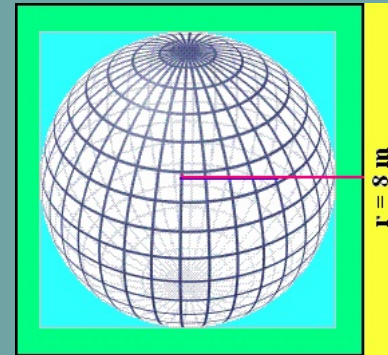
- Consider the two-dimensional surface of a sphere.

- From a three-dimensional perspective, it is easy to describe the surface as the locus of all points equidistant from the center of the sphere.

- In order to describe the spherical surface, we appeal to the center point of the sphere and its three-dimensional properties.

- But, the center is not part of the two-dimensional surface.

- From the point of view of the surface of the sphere, we can appeal to the center while not really taking it as part of the world.

- "We do successfully and unproblematically describe a particular non-Euclidean world by taking back some of the implications of what we earlier said" (Melia 468).

Angels and Stars
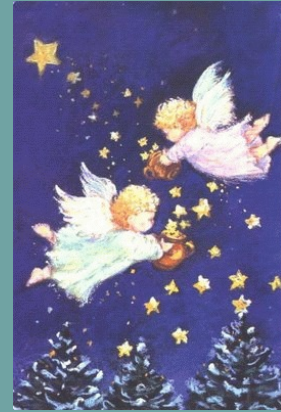
- "In charge of each star is an angel, no two angels are in charge of the same star, and at the precise moment that each star is created the corresponding angel is also created. Moreover, the angels in charge of stars a and b were created at the very same time" (Melia 470).

- We can infer that two stars are created at the same time.

- Joe, being a nominalist about angels, retracts all the consequences about them.

- But, he holds the inference about the stars.

- He could have said that there were two stars created at one time directly.

- But in stating that claim in terms of angels, and then taking back the parts of the claim that refer to angels, Joe is not speaking incoherently or contradictory.

We Need the Weasel

- Melia: our ability to weasel is fortunate, since nominalistic reformulations of our best theories are not always available.

- "Sometimes, we just cannot say what we want to say first time round. Sometimes, in order to communicate our picture of the world, we *have* to take back or modify part of what we said before" (Melia 468-9).

- Instrumentalism
  - ‣ Penelope Maddy alleges that scientists often view certain commitments, e.g. to infinitely-deep water waves, instrumentally.
  - ‣ Jody Azzouni points out that no one believes that the center of mass of a physical system must be a real thing.

- If scientists view some of the posits of their theory instrumentally, they could avoid commitments to mathematical objects, as well.

A False Inference

- We use aesthetic and pragmatic considerations in choosing among theories.
- Thus, we have a choice whether to believe in the existence of all the objects over which we first-order quantify.
- Melia uses this argument against Quine.
- It is more appropriate against Putnam.

Putnam and the Weasel

- Melia accurately represents Putnam's version of the indispensability argument as lacking an explicit statement of the method one is to use to determine the objects to which a theory commits.
  - ▸ "Our best scientific theories entail the existence of numbers, sets and functions...Since such claims entail the existence of *abstracta*, we cannot consistently assert or believe in our scientific theories whilst denying the existence of *abstracta*"(Melia 455).

- Quine provides an explicit method for determining the ontological commitments of a theory while Putnam leaves that question open.

- Quine's argument is resistant to alternative interpretations of the language of science, and thus resists weaseling.
  - ▸ We can not, for Quine, take back some of what we allege.

- Given Putnam's argument, for which we do not have explicit rules for interpretation of scientific discourse, a weaseling strategy might succeed.

- Melia has to provide an alternative method of determining one's ontological commitments.
  - ▸ Eleatic Principle?

Competing Theories, Competing Ontologies

- We are often faced with competing theories.

- Sometimes these theories are empirically equivalent: no evidence sways us to one over the other.

- If no empirical factors sway us toward one theory over another, we may suppose ourselves to be like Buridan's ass between the two theories, unable to choose.

- The Immanent Virtues: Simplicity (for example)
  - ▸ Some theories are simpler ontologically.
  - ▸ Some theories are simpler in formulation.
  - ▸ These factors of simplicity are inversely proportional.

- Melia claims that we should prefer ontological simplicity over formulational simplicity.
  - ▸ "I accept that considerations of simplicity play an important role in theory choice. But I prefer the hypothesis that makes *the world* a simpler place. For sure, all else being equal, I prefer the simpler ontology. For sure, all else being equal, I prefer the theory that postulates the least number of fundamental properties and relations. But the simplicity I value attaches to the kind of world postulated by the theory - not to the *formulation* of the theory itself" (Melia 473).

Always Ontological Simplicity?

- Melia compares two theories.
  - ‣ T_1 has no numbers but lots of numerical predicates.
    - – All distance relations are different properties.
  - ‣ T_2 accomplishes the same tasks with few predicates and an arithmetic ontology.
    - – The same predicate may be used for any distance.

- Melia: there is no sense in which T_2 is simpler.

- Really?

- We can reduce ontology at the expense of ideology.

- Thus, one must be very careful to choose a satisfying ideology.
  - ‣ Remember GR2: Logical Neutrality
  - ‣ "The quest of a simplest, clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate categories, a limning of the most general traits of reality" (Quine, *Word and Object* 161).

- One could interpret the connection of metaphysics and choice of language as a flaw in Quine's project, as Melia does.

- Or, one could reconsider Quine's arguments for holism and first-order logic, and engage them directly, as Melia does not.

- This latter route is the only charitable and intellectually responsible one.