Knowledge, Truth, and Mathematics

Philosophy 405
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Class 21: Finishing the Indispensability Argument

Quine's Metaphysics

- There are differences between meaning, naming, and ontologically committing.
- The most effective way of formulating a theory is to put it in the language of first-order logic.
- "We can very easily involve ourselves in ontological commitments by saying, for example, that *there is something* (bound variable) which red houses and sunsets have in common; or that *there is something* which is a prime number larger than a million. But this is, essentially, the *only* way we can involve ourselves in ontological commitments: by our use of bound variables" ("On What There Is" 12).
- "To be assumed as an entity is, purely and simply, to be reckoned as the value of a variable" ("On What There Is" 13)
- Existence questions become questions about how best to write one's best theory.
- The question of whether numbers exist becomes a question about whether we quantify over them when our language is made most precise, and formalized into first-order logic.

Constructing a Theory

- We adopt, at least insofar as we are reasonable, the simplest conceptual scheme into which the disordered fragments of raw experience can be fitted and arranged. Our ontology is determined once we have fixed upon the over-all conceptual scheme which is to accomodate science in the broadest sense... ("On What There Is" 16-17).
- We construct a theory of our sense experience.
- Then, we look at the theory, and decide what values it takes for its bound variables.

Posits and Myths

- The values of the bound variables are what a theory presupposes.
- These are the posits, the postulated entities, of the theory.
- Quine, in early work, calls them myths.
  - They are the result of our choice of a theory.
- This methodology is not intended to denigrate the objects posited.
  - “To call a posit a posit is not to patronize it” (*Word and Object* 22).

Empirically Equivalent Theories

- Sense experience serves as the evidence for our theory.
  - boundary conditions
- Sense experience under-determines any theory.
- We choose among competing, empirically equivalent theories according to their formal characteristics, their immanent virtues.
  - simplicity
  - elegance
  - utility
  - explanatory strength

Holism and Posits

- The statements of any theory are interconnected.
  - Quine’s rejection of reductionism
- When we tinker with our theory, in response to new experiences, we can adjust any theory in various ways.
  - All evidence is evidence for the theory as a whole, not for individual statements.
- The posits come out all together, as values for the variables.
  - “The considerations which determine a reasonable construction of any part of that conceptual scheme, for example, the biological or the physical part, are not different in kind from the considerations which determine a reasonable construction of the whole” (“On What There Is” 17).

Quine and Carnap and Double-Talk

- Quine agrees with Carnap’s claim that metaphysical claims, from an internal perspective, are trivial.
  - “One’s ontology is basic to the conceptual scheme by which he interprets all experiences, even the most commonplace ones. Judged within some particular conceptual scheme - and how else is judgment possible? - an ontological statement goes without saying, standing in need of no separate justification at all” (“On What There Is” 10).
- Quine agrees with Carnap that we can choose among various theories, or conceptual schemes.
- Quine disagrees with Carnap’s characterizations of the choices among conceptual schemes.
  - If, with Carnap, we say that numbers exist (internally) while denying, at the same time, that “numbers exist” is meaningful, we are contradicting ourselves.
  - Carnap’s physicist
- “For us common men who believe in bodies and prime numbers, the statements ‘There is a rabbit in the yard’ and ‘There are prime numbers between 10 and 20' are free from double-talk. Quantification does them justice” (“Existence and Quantification” 99).

Quine's Argument

QI.1: We should believe the theory which best accounts for our sense experience.

QI.2: If we believe a theory, we must believe in its ontological commitments.

QI.3: The ontological commitments of any theory are the objects over which that theory first-order quantifies.

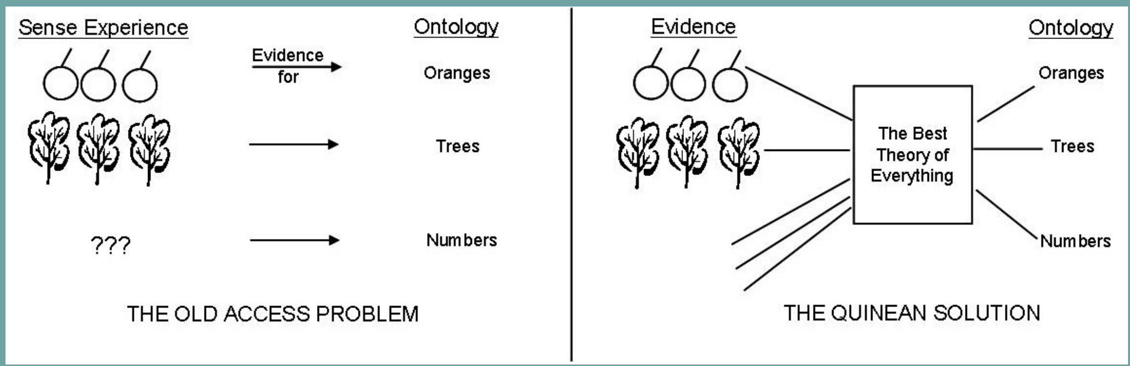
QI.4: The theory which best accounts for our sense experience first-order quantifies over mathematical objects.

QI.C: We should believe that mathematical objects exist.

Indispensability and Access

- Quine, like his empiricist predecessors, sought the best theories for explaining our sense experience.
- Unlike traditional empiricists, he does not reduce all claims of existence directly to sense experiences.
- Traditional empiricists were burdened with an access problem: how can we justify beliefs in objects unavailable to our senses?
- The access problem is the source of the epistemological branch of Benacerraf's problem.
  - We have no causal (or otherwise reliable) access to abstract objects.
- Quine's method avoids the access problem by denying the possibilities of reduction.
- Our best epistemology is just figuring out how best to construct and interpret scientific theory.

Quine's Solution to the Access Problem



There is nothing preventing us from having both a standard semantics and our best epistemology.

Three Worries

1. Holism is false

- Sober's foxes and chickens
- Basic beliefs

2. Quine's indispensability argument makes the justification of mathematical beliefs subordinate to the justification of empirical scientific beliefs.

- "My view of pure mathematics is oriented strictly to application in empirical science. Parsons has remarked, against this attitude, that pure mathematics extravagantly exceeds the needs of application. It does indeed, but I see these excesses as a simplistic matter of rounding out...I recognize indenumerable infinites only because they are forced on me by the simplest known systematizations of more welcome matters. Magnitudes in excess of such demands, e.g., $\beth_\omega$ or inaccessible numbers, I look upon only as mathematical recreation and without ontological rights" (Quine on Mathematical Recreation).
- Mathematical methodology is consistent across mathematical theories, no matter what the scientists do with the mathematical results.

3. Instrumentalism

- Center of mass
- Our theory may be committed, in the formal sense, to objects to which we are not committed.
- Among those objects could be the mathematical objects.

These worries are about the first three premises of QI.

Field's Project

- Field's work is aimed at the fourth premise of QI.
- He denies QI4.
  - The theory which best accounts for our experience need not quantify over mathematical objects.
- If Field is right that QI4 is false, Quine's indispensability argument fails, and Benacerraf's dilemma re-emerges.
- At the heart of Field's project is his proposed nominalistic reformulation of Newtonian Gravitational Theory (NGT).
  - Replaces quantification over mathematical objects in NGT with quantification over space-time points or regions.

Formal Axiomatic Physical Theories

- Standard Theories
  - 1. A logical system, used for inference
  - 2. Mathematical axioms
  - 3. Scientific axioms
- Bridge functions relate the theorems of mathematics to the theorems of science.
  - measurements of quantities like mass and velocity
  - The speed of light $c = 3 \times 10^8$ cm/s^2
  - G = 8 π T, where G is the gravitational tensor and T is the stress-energy tensor
- Field produces representation theorems to show how the space-time points and regions can do the work that mathematical objects do in standard theories.

Fictionalism

- Mathematical terms are empty names.
- Mathematical sentences are either false, for existence claims, or vacuously true, for purely mathematical entailments.
- Field aims at the indispensability argument:

  "The only non-question-begging arguments that I have ever heard for the view that mathematics is a body of truths all rest ultimately on the applicability of mathematics to the physical world" (Field viii).
- An Argument

  FF1. We should take mathematical sentences at face value.

  FF2. If we take (some of them) to be non-vacuously true, then we have to explain our access to them.

  FF3. The only good account of access is the indispensability argument.

  FF4. But, the indispensability argument fails.

  FFC. So, we should take the non-vacuous ones to be false.

Reformulating Mathematical Theories

- Some nominalists try to re-interpret the mathematical axioms.
  - possible inscriptions
- Such strategies give up standard semantics for mathematical propositions.
- Field's work eliminates, rather than re-interprets, the mathematical axioms.
- He interprets the mathematical axioms standardly, and then claims that mathematical propositions are false.

Two Parts to Field's Project

- First, he develops a nominalist counterpart to a standard scientific theory
- Second, he tries to show that mathematics applies conservatively to nominalist theories, to assure us that nominalist counterparts are adequate substitutes.

Ground Rules

- GR.1: *Adequacy*: A reformulation must not omit empirical results of the standard theory.
- GR.2: *Logical Neutrality*: A reformulation must not reduce ontology merely by extending logic, or ideology.
  - First-order logic makes no commitments
  - Second-order logic is "set theory in sheep's clothing" (Quine).
  - Modal logics make commitments to possible worlds.
    - "Avoidance of modalities is as strong a reason for an abstract ontology as I can well imagine" (Quine, "Reply to Charles Parsons" 397).
  - "[I]t can be seen that there is something dubious about the practice of just helping oneself to whatever logical apparatus one pleases for purposes of nominalistic reconstruction while ignoring any customary definitions that would make the apparatus nominalistically unpalatable: for by doing so, one can make the task of nominalistic reconstruction absolutely trivial – and so absolutely uninteresting" (Burgess and Rosen, *A Subject with No Object* 175).
- GR.3: *Conservativeness*: The addition of mathematics to the reformulated theory should license no additional nominalist conclusions.
  - Deductive conservativeness: mathematics does not allow new theorems to be derived from the nominalistic theory.
  - Semantic conservativeness: no additional statements come out true in any model of the theory which includes mathematics.

The Importance of Conservativeness

- First, it serves as a check on the adequacy of the nominalist reformulation.
  - If mathematics does not apply conservatively to NGT*, then the standard theory will yield more consequences.
  - The nominalist theory will be shown to omit some theorems of the standard theory.
- Second, conservativeness provides an account of the applicability of mathematics to science.
  - Why is mathematics useful?
- Field's project is especially alluring.
  - He does not merely eliminate mathematics from scientific theory.
  - He attempts to show that our ordinary uses of mathematics are consistent with nominalist principles.

Attractiveness: A Fourth Ground Rule

- GR.4: *Attractiveness*: The dispensabilist must show, "[T]hat one can always reaxiomatize scientific theories so that there is no reference to or quantification over mathematical entities in the reaxiomatization (*and one can do this in such a way that the resulting axiomatization is fairly simple and attractive*)." (Field viii, emphasis added)
  - Subjective?
  - Few axioms
  - Elegant proofs
- A Bad Theory: All and only the nominalistic consequences of standard science
  - will not reduce diverse experiences to a few, simple axioms.
- "If no attractiveness requirement is imposed, nominalization is trivial... Obviously, such ways of obtaining nominalistic theories are of no interest" (Field 41).

Purpose of the Reformulation

- Useful theory?
  - But, the standard theory regimented in first-order logic is similarly unacceptable.
  - Conservativeness
  - GR.4, in this sense, is too strong a requirement.
- Explanatory Strength?
  - "The elimination of numbers [from science], unlike the elimination of electrons, helps us to further a plausible methodological principle: the principle that *underlying every good extrinsic explanation there is an intrinsic explanation*. If this principle is correct, then real numbers (unlike electrons) have *got* to be eliminable from physical explanations, and the only question is how precisely this is to be done" (Field 44).
  - No theory regimented into first-order logic can be explanatory, since it can not be perspicuous.
  - The original un-regimented theory is the one which does all the explanatory work.
- Reduce the laws to a neat and tidy few axioms.
- Translate axioms of standard science directly into nominalist language.