Knowledge, Truth, and Mathematics

Philosophy 405
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Class 20: The Indispensability Argument

Quine's Platonism

Ordinary interpreted scientific discourse is as irredeemably committed to abstract objects - to nations, species, numbers, functions, sets - as it is to apples and other bodies. All these things figure as values of the variables in our overall system of the world. The numbers and functions contribute just as genuinely to physical theory as do hypothetical particles ("Success and the Limits of Mathematization 149-50).

Quine: Not a Rationalist

- Descartes and Leibniz defend innate ideas.
- Gödel holds a platonism based on mathematical intuition, insight into the structure of the mathematical universe analogous to sense perception.
- Quine's indispensability argument eschews any appeal to the rationalists' innateness or intuition.
- It is intended as an empiricist's argument.
- Michael Resnik: an empiricist epistemology for a platonist ontology.

Quine's Argument

QI.1: We should believe the theory which best accounts for our sense experience.

QI.2: If we believe a theory, we must believe in its ontological commitments.

QI.3: The ontological commitments of any theory are the objects over which that theory first-order quantifies.

QI.4: The theory which best accounts for our sense experience first-order quantifies over mathematical objects.

QI.C: We should believe that mathematical objects exist.

Quine's Projects

- Negative
  - In "Two Dogmas", Quine argues against the traditional empiricist's reductive method of determining ontological commitments.
- Positive
  - In "On What There Is", Quine argues for his new method, involving the construction of a best theory of our sense experience, and the modeling (or interpretation) of that theory.
- The indispensability argument is a corollary of the argument for Quine's new method.
- It all starts with Frege.

Frege's Distinction Between Sense and Reference

Three Problems

1. Hesperus = Phosphorus
2. Bellerophon rode Pegasus.
3. Opaque contexts
   - Lois Lane believes that Superman can fly.
   - Superman is Clark Kent
   - So, Lois Lane believes that Clark Kent can fly.

McX, Wyman, and Frege on Empty Names

- McX appeals to the idea of Pegasus as the referent of my term.
  - "McX would sooner be deceived by the crudest and most flagrant counterfeit than grant the nonbeing of Pegasus" ("On What There Is" 2).
- Wyman distinguishes between existence and subsistence.
  - "Wyman...is one of those philosophers who have united in ruining the good old word 'exist'" ("On What There Is" 3).
  - What of impossible objects (e.g. the round square cupola)?
- Frege: terms have both sense (meaning, intension) and reference (extension)

Extensionalism

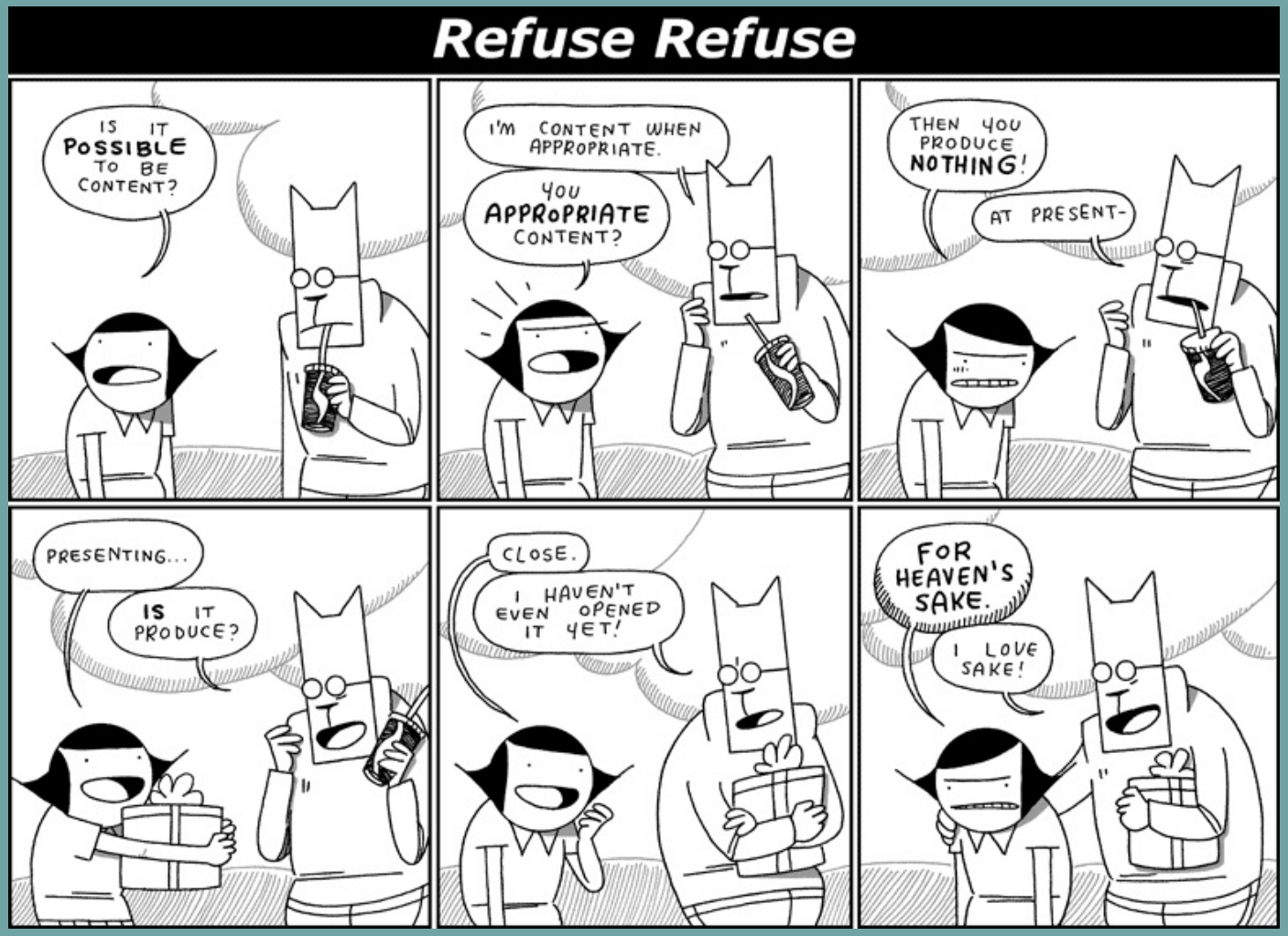
- Quine opposes intensions because of their lack of identity conditions.
  - Propositions and modalities are also intensions
  - “Wyman’s overpopulated universe is in many ways unlovely. It offends the aesthetic sense of us who have a taste for desert landscapes, but this is not the worst of it. Wyman’s slum of possibles is a breeding ground for disorderly elements. Take, for instance, the possible fat man in that doorway; and again, the possible bald man in that doorway. Are they the same possible man, or two possible men? How do we decide? How many possible men are there in that doorway? Are there more possible thin ones than fat ones? How many of them are alike? Or would their being alike make them one? Are no *two* possible things alike? Is this the same as saying that it is impossible for two things to be alike? Or, finally, is the concept of identity simply inapplicable to unactualized possibles?” (“On What There Is” 4).’
  - The myth of the museum
- Quine approves of extensions.
  - The identity of a set depends only on its members.

A Problem for Extensionalists

- ‘Creature with a heart’ and ‘Creature with a kidney’
  - same extension (extensionally equivalent)
  - different intension
- Let’s not worry about this one.

Quine on Empty Names

- There are no meanings, so Frege's route is out.
  - Meanings skepticism/nihilism
  - Meaningfulness can be expressed in terms of sentences and our beliefs (revealed by our behaviors) about them.
- There are names, but we shouldn't worry too much about them.
  - Names can have referents or not.
  - Cantor's diagonal argument
  - 'I slew a Jabberwock for Julie's sake'.


Refuse Refuse
IS IT POSSIBLE TO BE CONTENT?
I'M CONTENT WHEN APPROPRIATE.
YOU APPROPRIATE CONTENT?
THEN YOU PRODUCE NOTHING!
AT PRESENT-
PRESENTING...
IS IT PRODUCE?
CLOSE.
I HAVEN'T EVEN OPENED IT YET!
FOR HEAVEN'S SAKE.
I LOVE SAKE!

Quine on Empty Names

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  - Names can have referents or not.
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  - 'I slew a Jabberwock for Julie's sake'.
- All things being equal, we should prefer standard semantics.
- But, if we have over-riding reasons to disavow a commitment that a standard semantics would imply, we should re-write our language.
  - "We could [appeal] to the *ex hypothesi* unanalyzable, irreducible attribute of *being Pegasus*, adopting, for its expression, the verb 'is-Pegasus' or 'pegasizes'. The noun 'Pegasus' itself could then be treated as derivative, and identified after all with a description: 'the thing that is-Pegasus', 'the thing that pegasizes'" ("On What There Is" 8).
  - "Whatever we say with the help of names can be said in a language which shuns names altogether" ("On What There Is" 13).

God Exists

From First-Order Logic

1. ~(∃x)x=g	Assumption, for indirect proof
2. (∀x)x=x	Principle of identity
3. (∀x)~x=g	1, Change of quantifier rule
4. g=g	2, UI
5. ~g=g	3, UI
6. (∃x) x=g	1-5, Indirect proof

QED

Quine's Metaphysics

- There are differences between meaning, naming, and ontologically committing.
- The most effective way of formulating a theory is to put it in the language of first-order logic.
- "We can very easily involve ourselves in ontological commitments by saying, for example, that *there is something* (bound variable) which red houses and sunsets have in common; or that *there is something* which is a prime number larger than a million. But this is, essentially, the *only* way we can involve ourselves in ontological commitments: by our use of bound variables" ("On What There Is" 12).
- "To be assumed as an entity is, purely and simply, to be reckoned as the value of a variable" ("On What There Is" 13)

Quinean Metaphysics and Mathematics

- Our metaphysics reduces to a process of interpreting our first-order theory.
  - We interpret a first-order theory by specifying a domain of discourse, a set of objects over which the quantifiers range.
  - We assign values to variables in order to model the theory, or provide an interpretation which makes the sentences of the theory come out true.
  - Our metaphysics is the simple byproduct of modeling the theory.
- Existence questions become questions about how best to write one's best theory.
- The question of whether numbers exist becomes a question about whether we quantify over them when our language is made most precise, and formalized into first-order logic.

Is Quine's Criterion Semantic?

- Quine does not turn metaphysical questions into semantic ones.
- "How are we to adjudicate among rival ontologies? Certainly the answer is not provided by the semantical formula "To be is to be the value of a variable"; this formula serves rather, conversely, in testing the conformity of a given remark or doctrine to a prior ontological standard. We look to bound variables in connection with ontology not in order to know what there is, but in order to know what a given remark or doctrine, ours or someone else's, *says* there is; and this much is quite properly a problem involving language. But what there is is another question" ("On What There Is" 15-16).
- The question of whether mathematical objects exist is the question of how to specify the prior ontological standard.

Constructing a Theory

- We adopt, at least insofar as we are reasonable, the simplest conceptual scheme into which the disordered fragments of raw experience can be fitted and arranged. Our ontology is determined once we have fixed upon the over-all conceptual scheme which is to accomodate science in the broadest sense... (“On What There Is” 16-17).
- We construct a theory of our sense experience.
- Then, we look at the theory, and decide what values it takes for its bound variables.

Posits and Myths

- The values of the bound variables are what a theory presupposes.
- These are the posits, the postulated entities, of the theory.
- Quine, in early work, calls them myths.
  - They are the result of our choice of a theory.
- This methodology is not intended to denigrate the objects posited.
  - "To call a posit a posit is not to patronize it" (*Word and Object* 22).

Empirically Equivalent Theories

- Sense experience serves as the evidence for our theory.
  - boundary conditions
- Sense experience under-determines any theory.
- We choose among competing, empirically equivalent theories according to their formal characteristics, their immanent virtues.
  - simplicity
  - elegance
  - utility
  - explanatory strength

Holism and Posits

- The statements of any theory are interconnected.
  - Quine's rejection of reductionism
- When we tinker with our theory, in response to new experiences, we can adjust any theory in various ways.
  - All evidence is evidence for the theory as a whole, not for individual statements.
- The posits come out all together, as values for the variables.
  - "The considerations which determine a reasonable construction of any part of that conceptual scheme, for example, the biological or the physical part, are not different in kind from the considerations which determine a reasonable construction of the whole" ("On What There Is" 17).

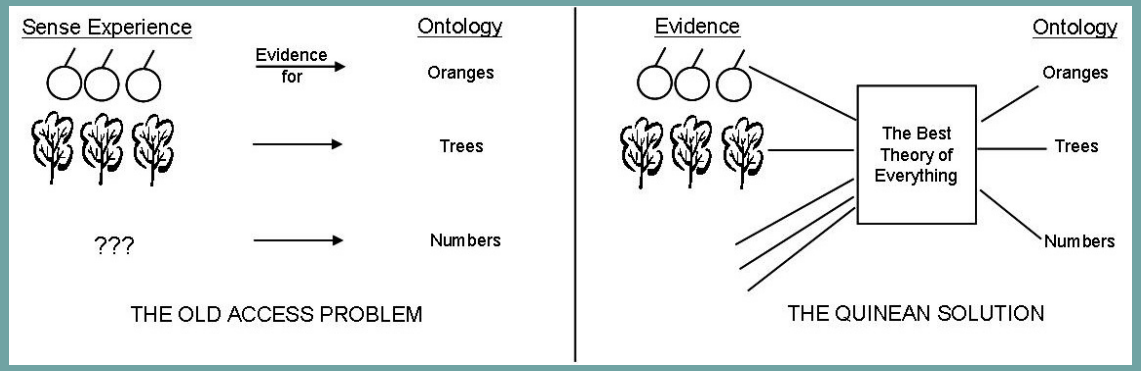
Quine and Carnap and Double-Talk

- Quine agrees with Carnap's claim that metaphysical claims, from an internal perspective, are trivial.
  - "One's ontology is basic to the conceptual scheme by which he interprets all experiences, even the most commonplace ones. Judged within some particular conceptual scheme - and how else is judgment possible? - an ontological statement goes without saying, standing in need of no separate justification at all" ("On What There Is" 10).
- Quine agrees with Carnap that we can choose among various theories, or conceptual schemes.
- Quine disagrees with Carnap's characterizations of the choices among conceptual schemes.
  - If, with Carnap, we say that numbers exist (internally) while denying, at the same time, that "numbers exist" is meaningful, we are contradicting ourselves.
  - Carnap's physicist
- "For us common men who believe in bodies and prime numbers, the statements 'There is a rabbit in the yard' and 'There are prime numbers between 10 and 20' are free from double-talk. Quantification does them justice" ("Existence and Quantification" 99).

Indispensability and Access

- Quine, like his empiricist predecessors, sought the best theories for explaining our sense experience.
- Unlike traditional empiricists, he does not reduce all claims of existence directly to sense experiences.
- Traditional empiricists were burdened with an access problem: how can we justify beliefs in objects unavailable to our senses?
- The access problem is the source of the epistemological branch of Benacerraf's problem.
  - We have no causal (or otherwise reliable) access to abstract objects.
- Quine's method avoids the access problem by denying the possibilities of reduction.
- Our best epistemology is just figuring out how best to construct and interpret scientific theory.

Quine's Solution to the Access Problem



There is nothing preventing us from having both a standard semantics and our best epistemology.

Three Worries

1. Holism is false
- Sober's foxes and chickens
- Basic beliefs

2. Quine's indispensability argument makes the justification of mathematical beliefs subordinate to the justification of empirical scientific beliefs.
- "My view of pure mathematics is oriented strictly to application in empirical science. Parsons has remarked, against this attitude, that pure mathematics extravagantly exceeds the needs of application. It does indeed, but I see these excesses as a simplistic matter of rounding out...I recognize indenumerable infinites only because they are forced on me by the simplest known systematizations of more welcome matters. Magnitudes in excess of such demands, e.g., $\beth_\omega$ or inaccessible numbers, I look upon only as mathematical recreation and without ontological rights" (Quine on Mathematical Recreation).
- Mathematical methodology is consistent across mathematical theories, no matter what the scientists do with the mathematical results.

3. Instrumentalism
- Center of mass
- Our theory may be committed, in the formal sense, to objects to which we are not committed.
- Among those objects could be the mathematical objects.

For Next Week

- Field's response, to QI.4 (the minor premise)
- Some of the Field reading is technical
  - Appendix to Chapter 1
  - Skim