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Pythagoras and the Pythagoreans

I. Sign up for seminar papers.

II. Metaphysics of Mathematics

Let's characterize the three most dominant positions on the metaphysics of mathematics.
Realism is the claim that numbers exist, objectively, and that mathematical claims can be non-vacuously true.
Plato, Descartes, Frege, Gödel, and Quine were all realists.
We can divide realists into object and sentence realists.
Object realists believe that there are mathematical objects.
Sentence realists believe that mathematical claims can be non-vacuously true.
There are sentence realists who are not object realists.
But, on standard semantics for language, for a claim like 'the square root of two is irrational' to be true, there have to be irrational numbers, like two.
Sentence realists are often motivated by the supposed spookiness of abstract mathematical objects.
Mathematical objects are supposed to be real objects, but not ones that we can see, or otherwise sense.

Nominalism is the claim that numbers do not exist, that they are merely empty names.
The nominalist denies that there are any types corresponding to number tokens, inscriptions.
Nominalists are often motivated by empiricist premises that knowledge comes, ultimately, from the senses.
Since mathematical objects are not the kinds of things that are available to the senses, we have no good reason to believe that they exist.
The most prominent contemporary nominalist position is called fictionalism.
The fictionalist claims that sentences like 'the square root of two is irrational' are, strictly speaking, false, since there are no mathematical objects.
Hartry Field is the most prominent fictionalist.

Idealism is the position that numbers are mental constructs.
Idealists are compelled by the objectivity of the realist interpretation of mathematical claims.
They agree that mathematical sentences can be true or false.
But, they also agree with the nominalist that there are no non-empirical objects, like the purportedly spooky abstract objects of mathematics.
The idealist grounds the objectivity of mathematical claims in our thoughts about mathematics.
Mathematics is, for the idealist, about mental objects.
Kant and Brouwer are idealists.

One achievement of the Greeks in mathematics, as opposed to earlier or contemporaneous civilizations, like the Babylonians or the Egyptians, was to recognize that realism about mathematics entails believing in an unseen world.
The Pythagoreans seem to be among the first to recognize that the mathematical realm goes beyond the senses.
Furthermore, they demanded proofs of mathematical theorems, rather than mere practical utility.

III. Chaos, Order, and Mathematics

Kline identifies the Pythagorean discovery of the transcendence of mathematics with psychological empowerment over an irrational world.

> The civilizations that preceded the Greek or were contemporaneous with it regarded nature as chaotic, mysterious, capricious, and terrifying (Kline, 146).

The world, as we perceive it, is orderly and predictable.
The odd surprise (a tsunami, a terrorist attack) is always explicable, in hindsight.
We complain about our weather forecasters and political scientists.
But the alien-robot-from-the-future weather person/sociologist would have known.
In contrast, if Kline is right, this order may be unnatural.
Kline's claim is that the Greeks, through a variety of methods, tamed the chaos.
The Ionians, like Thales, argued that all diversity is the result of different combinations of a few familiar substances, or even a single substance.
Thales gave order and organization to the chaos by positing a single element, water.
Democritus favored unseen atoms.
Empedocles, and other Greeks following him, posited four elements: earth, air, fire, and water.
In contrast, the Pythagoreans posited numbers as the fundamental constituent of all that we experience: all things are numbers.
Numbers, as Kline says, were to the Pythagoreans as atoms are to us.
That is, the Pythagoreans did not distinguish between mathematics and science.

Prima facie, the Pythagorean claim is absurd.
The Pythagoreans were a secret cult.
They had some odd beliefs, independently of their metaphysics, including transmigration of the soul (see Shakespeare's *Twelfth Night* IV.2).
Here are some rules of the Pythagorean order:

1. To abstain from beans.
2. Not to pick up what has fallen.
3. Not to touch a white cock.
4. Not to break bread.
5. Not to step over a crossbar.
6. Not to stir the fire with iron.
7. Not to eat from a whole loaf.
8. Not to pluck a garland.
9. Not to sit on a quart measure.
10. Not to eat the heart.
11. Not to walk on highways.
12. Not to let swallows share one's roof.
13. When the pot is taken off the fire, not to leave the mark of it in the ashes, but to stir them together.
14. Do not look in a mirror beside a light.
15. When you rise from the bedclothes, roll them together and smooth out the impress of the body (Bertrand Russell, *A History of Western Philosophy* 31-2).

Looking at the Pythagorean superstitions, one can see them as a bridge between the irrational world view and a rational one.
One problem with examining the Pythagoreans is that there is very little extant writing.
We know about them only on the basis of what others, like Aristotle, wrote.
Many of their beliefs were likely not the beliefs of Pythagoras, anyway, but of his later followers.

Kline's claim is that in seeing the world as mathematical, the Pythagoreans were able to turn a chaotic world into a tame one.
I'm not sure what would it mean to regard the world as chaotic, or mysterious, capricious, and terrifying.
Consider *Koyaanisqatsi*, or *Guernica*.

IV. The Child-Development Analogy

One way to make sense of Kline's claim is to draw an analogy between the development of civilization and the development of the individual.
We can extend the evolutionary claim that ontogeny recapitulates phylogeny.
The individual begins life by perceiving a chaotic world, and learns to order and organize that world around him/her.
This was Kant's contention, as well: the noumenal world, if we can say anything about it, is completely unordered.
We impose the order from our concepts.

The basis for the analogy seems acceptable.
We don't really know what the world is like for the baby, in immediate apprehension.
Perhaps, the baby is given a chaotic world, which would explain its screaming and crying.
So, child, as it grows, progresses through Piagetian stages, and learns to make order.

The Four Piagetian Stages

1. Sensorimotor stage (birth - age 2): The child builds concepts about the external world and how it works, correlating sense experiences with external objects. The child lacks, and learns, object permanence.
2. Pre-operational stage (ages 2 - 7): The child is not able to think abstractly. The child lacks and learns conservation of quantity.
3. Concrete operations (ages 7 - 11): The child starts to reason logically about concrete events. Some limited abstract problem-solving is possible, but only applied to concrete phenomena.
4. Formal operations (ages 11 - 15): The child's develops abstract reasoning.

The argument on the table is a metaphor.
Early civilizations are like babies, seeing the world as chaotic.
The development of civilization, like the development of a child, progresses through stages.
The Greek advance, then, is analogous to humanity entering the formal operations stage.

Can we really expect that the civilizations prior to the Greeks were all like babies?
Maybe, this is plausible in earlier stages of evolutionary development.
But, once we have something like mythology, we would need to think that the people had enough leisure,

and freedom from fear and chaos, to develop the myths.
The child development analogy is uncharitable, as well as unhelpful.

It may not be helpful at all to see the pre-Greek cultures as childlike.
Kline argues that the difference between the Greeks and earlier cultures is that the Greeks provided a rational view of nature.
He points out that the mythology of earlier cultures makes life and death the whims of the gods.
But, the gods are presumably rational, too.
Their reasons are just potentially hidden from our view.
The Greeks provided scientific reasons, as opposed to mythological reasons.

V. Explanations

Examining the differences between science and mythology helps explain the achievement of the Greeks.
We can understand Kline's claim as that the Greeks allowed us to see reasons in nature.
That is, what the Greeks were doing was not giving order to chaos, but providing natural explanations where no explanations were available.

Explanations are useful, but they do not solve the underlying mystery of the universe.
If we want to know why A fell off of a cliff, it is useful to know that B pushed her.
It begs the question of why B pushed her.
We might find out that C pushed B.
And that D pushed C.
Et cetera.
The ultimate causes get pushed back, but are not disappeared.
We do get an order to some portion of the universe, which may be Kline's point.

In any case, we do have some explanations of causes.
And, the explanations replace a visible world with a less-visible world.

The Pythagorean claim was that the less-visible world is mathematical in nature.
Is there any sense that we can make of the claim that the world is mathematical?

VI. Later Pythagoreanism

One way in which the world is mathematical is in our uses of mathematics to describe the world.

> Philosophy is written in this grand book of the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and to read the alphabet in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one wanders in a dark labyrinth (Galileo, *The Assayer*, 1623).

Consider, for example, Coulomb's law: $F = k\ |q_1q_2|/ r^2$.
It refers to a real number, k, the Coulomb's Law constant.

It includes two functions, absolute value and squaring, as well as multiplication and division.
Furthermore, the function maps real numbers, measurements of charge and distance, to other real numbers, which measure force.
We presume that the particles are not mathematical.
But, their representations surely are.

The inference from the uses of mathematics to describe the world to the existence of mathematical objects is central in the work we will study in the second half of the course.
The contemporary Pythagorean view arises out of Quine's work on the indispensability argument.
We start thinking of bodies as physical objects, but these have vague boundaries, and puzzling identity conditions over time.
Am I the same person I was when I was younger, or the same person I will be later?
Is the cragged old tree the same as the small sapling?
We avoid some of the problems by taking bodies to be composed of smaller particles.
We can think of the world as composed of four-dimensional aggregates of these atomic elements.

But, atomism has its problems, too.
Electrons, for example, do not seem to have great identity conditions.
It is arbitrary, at times, to say whether two point events are moments in the career of one electron, or two different ones.

Another option is a field theory of distributions of states over space-time.
There are electromagnetic fields, and gravitational fields, and others.
If we take these overlapping field theories to describe the universe, we think of the world as composed of space-time points and their states.
The objects are the space-time regions themselves, and their properties.
But, we can identify space-time regions with Cartesian coordinates, making an arbitrary choice of coordinate axes.
So, the world is essentially numerical.

> Predicates that formerly attributed states to points or regions will now apply rather to quadruples of numbers, or to sets of quadruples... I seem to have ended up with this as my ontology: pure sets (Quine, "Whither Physical Objects", 501-2).