Philosophy 405: Knowledge, Truth and Mathematics Fall 2010

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Class #19: November 1 Two Dogmas of Empiricism

I. "Two Dogmas," Mathematics, and Indispensability

Our interest in studying Quine has mainly to do with the indispensability argument, which will occupy us for the remainder of the course.

In a sense, we have reached a turning point in our class.

Our studies of the history of the philosophy of mathematics are done.

Now, we are engaged in a live, contemporary debate.

There are a variety of different versions of the indispensability argument, but Quine's version is the most important.

It may be seen as a way out of Benacerraf's dilemma.

Benacerraf argued that standard readings of mathematical claims entail the existence of mathematical objects.

But, our best epistemic theories seem to debar any knowledge of mathematical objects. Thus, the philosopher of mathematics faces a dilemma: either abandon standard readings of mathematical claims or give up our best epistemic theories. Neither option is attractive.

The indispensability argument in the philosophy of mathematics is an attempt to avoid Benacerraf's dilemma by showing that our best epistemology is consistent with standard readings

of mathematical claims. Broadly speaking, it is an attempt to justify knowledge of an abstract mathematical ontology

using only a strictly empiricist epistemology.

The <u>indispensability argument in the philosophy of mathematics</u>, in its most general form, consists of two premises.

The major premise states that we should believe that mathematical objects exist if we need them in our best scientific theory.

The minor premise claims that we do in fact require mathematical objects in our scientific theory. The argument concludes that we should believe in the abstract objects of mathematics.

Quine does not present a canonical version of the argument, so it has to be reconstructed from comments throughout his work.

In "Two Dogmas," allusion to the argument appears at the end.

As an empiricist I continue to think of the conceptual scheme of science as a tool, ultimately, for predicting future experience in the light of past experience. Physical objects are conceptually imported into the situation as convenient intermediaries - not by definition in terms of experience, but simply as posits... Physical objects, small and large, are not the only posits. Forces are another example... Moreover, the abstract entities of mathematics - ultimately classes and classes of classes and so on up - are another posit in the same spirit. Epistemologically these

are myths on the same footing with physical objects and gods, neither better nor worse except for differences in the degree to which they expedite our dealings with sense experiences ("Two Dogmas" 167-8)

The core claim of Quine's indispensability argument is that all of our ontology, what we believe exists, is determined by looking at the posits of our best theory.

Quine makes a radical methodological break from his empiricist predecessors, from Locke through the positivists.

For earlier empiricists, our ontology was determined by examining our sense experience, our ideas or our sense data.

For Quine, the question of ontology is deferred to empirical science, and the methods of constructing scientific theories.

We construct a theory which best explains our sense experience, and then look to that theory to determine what it says exists.

Thus, the first step in Quine's argument is to argue that the classic empiricist reductionist methodology is flawed.

That's the goal of "Two Dogmas."

II. The Dogmas

Quine's paper is an attempt to cleanse empiricism from reliance on two presuppositions:

- D1. There is an analytic/synthetic distinction; and
- D2. Reductionism: statements can be translated to terms which refer only to immediate experience.

Quine concludes that the two dogmas are essentially the same.

Reductionism entails that statements are confirmed (or disconfirmed) individually, by the experiences that justify them.

It is the claim we find in Locke and Hume, that all ideas must derive from initial impressions, and in Carnap and Ayer, that the meaning of a statement is the method we use to verify that statement. Radical reductionism says that every meaningful statement is reducible to a report about immediate experience.

The early reductionists (e.g. Locke and Hume) intolerably focused on reducing terms, rather than statements.

Bentham appealed to sentences as the basic units of communication.

The positivists continued to seek reductions of sentences.

But, the positivists failed to explain how, precisely, we were to build statements out of sense data. The most valiant attempt actually to carry out the in-principle reductions was Carnap's *Der Logische Aufbau der Welt*, or *The Logical Structure of the World*.

The *Aufbau* included the whole language of pure mathematics in addition to sense data.

Carnap treated spatio-temporal points as quadruples of real numbers, to which qualities then applied. So the ultimate constituents of the world were taken to be sensory events plus classes, or sets.

Quine notes that there are problems getting from sense data to the appropriate ascriptions. Further, Carnap leaves the 'is at' relation as a primitive.

The most basic spatio-temporal property is unexplained.

More importantly, even Carnap's more conservative reductionism relies on the claim that statements are confirmed individually, by the experiences we take to justify them or give them meaning. This isolation of sentences remains with positivism, whatever the status of Carnap's *Aufbau* project.

The notion lingers that to each statement, or each synthetic statement, there is associated a unique range of possible sensory events such that the occurrence of any of them would add to the likelihood of truth of the statement... (2D, 165-6)

And the isolation of sentences implicit in reductionism is inextricably linked to the belief in an analytic/ synthetic distinction.

As long as it is taken to be significant in general to speak of the confirmation and infirmation of a statement, it seems significant to speak also of a limiting kind of statement which is vacuously confirmed, *ipso facto*, come what may; and such a statement is analytic (2D, 166).

Ignoring any connections to positivism, there seems to be a distinction between sentences like:

1. Bachelors are unmarried.

and

2. Bachelors are unhappy.

Quine argues that since there is no way to draw a line between analytic and synthetic statements, such statements differ only in degree of empirical confirmation, not in kind.

It is obvious that truth in general depends on both language and extralinguistic fact. The statement "Brutus killed Caesar" would be false if the world had been different in certain ways, but it would also be false if the word "killed" happened rather to have the sense of "begat." Hence, the temptation to suppose in general that the truth of a statement is somehow analyzable into a linguistic component and a factual component. Given this supposition, it next seems reasonable that in some statements the factual component should be null; and these are the analytic statements. But, for all its a priori reasonableness, a boundary between analytic and synthetic statements simply has not been drawn. That there is such a distinction to be drawn at all is an unempirical dogma of empiricists, a metaphysical article of faith (2D, 163-4).

The bulk of "Two Dogmas" consists in showing how the analytic/synthetic distinction, while intuitive, is unsupported by evidence.

III. Characterizing Analyticity

Quine discusses five different ways to characterize analyticity:

- A1. Truth in all possible worlds
- A2. Can not be false/true
- A3. Denial as self-contradiction
- A4. Conceptual containment
- A5. Truth in virtue of meaning

Quine traces A1 and A2 to Leibniz, who distinguished truths of reason and truths of fact. The notion of possible worlds is metaphorical, and will not suffice as a foundation for analyticity. Quine interprets A1 as A2, which we can see in Hume's distinction between relations of ideas and matters of fact.

We also see A2 in Kant.

Note that truth, which is central to A2, is a metaphysical, rather than semantic, notion.

A1 and A2 are not useful as explanations of analyticity at all.

They concern whether statements are necessary or contingent.

To understand why Quine would make the serious error of blurring semantic questions with metaphysical ones, one has to recognize that Quine is responding directly to the positivists.

The positivists followed Frege in rejecting Kant's claim that arithmetic is synthetic a priori.

By adopting Frege's plant-in-the-seed notion of analyticity, they returned to the Humean view that the denial of a necessary proposition had to lead to a contradiction.

For a statement to be necessary, it would have to be analytic, since, as Hume argued, there are no necessary connections of matters of fact.

The analytic statements are precisely the same as the necessary ones.

Conversely, for the positivists, all analytic claims would have to be necessarily true, since experience cannot disconfirm an analytic truth.

The truth of an analytic statement depends only on the meaning of its symbols.

In contrast to the positivists's identification of necessity and analyticity, and putting Kant aside, there are other examples of propositions that some people think are necessary, but not analytic.

Kripke argued that theoretical identifications (e.g. water is H_2O) are known *a posteriori*, but are necessary.

Finding examples of analytic statements that are only contingently true is more difficult. Here is an example from Hilary Putnam that might work.

Imagine that all cats turn out to be cleverly disguised robots, sent to Earth from Mars, for some nefarious purposes.

Now, consider:

3. Cats are animals.

You might argue that 3 is analytically true, but false.

It is part of the concept of a cat that it is an animal, but in the actual world, it turns out that 3 is false.

There are (at least) two distinct ways we could describe this situation.

First, we could say that it turns out that cats are robots.

Second, we could say that it turns out that there are no cats.

On the first description, 3 is false, but it is not analytic. On the second description, we hold that cats are a natural kind, essentially animals. Thus, we re-gain the analyticity of 3, but we lose its falsity.

In any case, Quine is not concerned with the cleavage of analyticity and necessity.

He is, following the positivists, identifying the two.

This identification explains why he thinks that A1 and A2 could characterize analyticity, even though they are metaphysical notions.

He is casting a wide net for characterizations of analyticity, so that he can show that no attempt succeeds.

A3, on which analytic statements are those whose denials are self-contradictory, characterizes Hume's relations of ideas, as well as Frege's version of analyticity. Quine notes that A3 is in need of explanation, just as A1 and A2 were. It is fairly easy to characterize the sentences whose denials are false. But, self-contradiction is trickier.

This definition has small explanatory value; for the notion of self-contradictoriness, in the quite broad sense needed for this definition of analyticity, stands in exactly the same need of clarification as does the notion of analyticity itself. The two notions are the two sides of a single, dubious coin (2D 155).

We do have a logical notion of self-contradiction. The sentence:

4. P • ~P

is clearly a contradiction, given the meanings of the logical terms and that the token 'P' refers to the same thing in both instances in 4.

But:

5. Fred is a married bachelor.

is not of the form of 4.

We need a broader notion of self-contradiction, which is just another way to state the original problem of characterizing analyticity.

So, A3 will not suffice, either.

A4 is of course ambiguous between Frege's plant-in-the-seeds analyticity and Kant's beams-in-the-house version.

Quine presents two problems with containment.

One problem is the presumption that all statements are best understood in subject-predicate form. Russell, in recasting sentences that contain definite descriptions as existential sentences already questioned the identification of grammatical form and logical form. Consider:

6. All swans are white.7. 7 > 5.

6 and other general claims, including most scientific laws, are best understood as universally quantified sentences.

Even the specific sentence 7 may best be understood as a relation between two particular objects, and not as a property ascribed to an individual object.

We can cast sentences such as 6 and 7 in subject-predicate form, but this interpretation is forced.

Another problem with containment is the vagueness of the metaphor. Quine understands Kant to mean that an analytic statement is true in virtue of meanings, and independently of fact.

Thus, Quine takes analyticity in the form A5 as his quarry. A5 which is broader than the traditional A4. If Quine can argue that there are no statements which are analytic A5, it would cover A3 and A4, as well.

A5 coheres neatly with the positivists' claim that analytic sentences are true independent of facts. It clearly evokes the picture of a linguistic component and a factual component of a statement. Quine agrees that truth in general depends on language and extralinguistic fact. But, he denies that the factual portion ever disappears.

IV. Logical Truth

We can distinguish two types of analytically true statements. Compare

- 8. No unmarried man is married.
- 9. No bachelor is unmarried.

8 is a logical truth.

It displays its analyticity on its surface. It is clear, by rules of logic, that the predicate of is contained in the subject.

9 is merely analytic.Its truth depends on the meaning of 'bachelor'.The difference between 8 and 9 may be clearer in their logical regimentations:

8'. $(x)(\sim \sim Mx \supset Mx)$ 9'. $(x)(Bx \supset \sim Mx)$

8' is true under any interpretation of the predicates.

9' has false interpretations.

Quine has no complaint about logical truth.

But, the analytic/synthetic distinction can not rest on logical truth, since that concept is too narrow.

9 can be turned into a logical truth by substitution of synonyms (i.e. putting 'unmarried man' for 'bachelor').

But, to make such a substitution, we require an explanation of synonymy. The bulk of "Two Dogmas," then, is an attempt to discredit synonymy.

V. Synonymy

The argument against the analytic/synthetic distinction rests, in the first half of the paper, on Quine's argument against synonymy.

Quine tries three possibilities to characterize synonymy:

- S1. Logic (meaning postulates)
- S2. Dictionary definition
- S3. Interchangeability (substitutivity) salva veritate

S1 fails because it presumes the notion to be explained.

Carnap tries to use <u>meaning postulates</u> to characterize synonymy.

If we want to say that 'Fx' and 'Gx' are synonymous predicates, within the theory, we add an axiom, or semantic rule:

10. (x)(Fx \equiv Gx)

To model a theory, we consider state-descriptions.

State-descriptions ascribe truth values to atomic sentences of a theory.

We can build complex sentences using logical particles and the constraints of the semantic rules.

Carnap's analytic sentences are those that come out true on every state-description.

The semantic rules assure us that all substitutions of synonymous expressions will maintain analyticity. That is, the synonymy rules preserve substitutivity salva analyticity.

Meaning postulates might characterize logical truth.

They do not help to explain analyticity, which was Carnap's intent.

Carnap's procedure involves presenting a list of semantic rules of synonymy, and their consequences. There are two types of semantic rules.

The first type specify, recursively or otherwise, which sentences are analytic.

The second type of semantic rule just says that certain statements, those entailed by the rules specified, are among the truths.

Then we can define analyticity as 'true according to the semantic rules'.

In both cases, we would like an explanation of the semantic rules themselves. We want to know why certain semantic rules are picked out as special.

Semantical rules determining the analytic statements of an artificial language are of interest only in so far as we already understand the notion of analyticity; they are of no help in gaining this understanding (2D, 163).

Further, lists of semantic rules would provide a definition of analyticity for a given language, but not a general explanation of analyticity for all languages.

Quine's challenge is not merely to characterize analyticity by listing the analytic sentences. He wants to define it, to express its essence.

Quine's criticism of Carnap, here, is reminiscent of many of Plato's dialogues. Socrates repeatedly (e.g. the beginnings of *Euthyphro* and *Republic*) chides interlocutors for listing examples of some form, rather than characterizing its essence.

Saying that analytic statements are true 'by definition', S2, also presupposes, rather than explains, synonymy.

The lexicographer is a sociologist, who reports synonymy, and so can not ground it.

Terms are found to be synonymous, not made so by fiat.

Explication, which adds clarifying information to the definition, relies on other, preexisting synonymies. Only when a term is created explicitly to act as a synonym (e.g. an abbreviation) do we have a definition that does not rely on synonymy.

For example, scientists might name a planet, or molecule.

But these exceptions are rare.

The characterization of synonymy as substitutivity, S3, comes from linguistics.

Quine presents a few silly examples of failure of substitutivity.

While you can substitute 'unmarried man' for 'bachelor' in 9, you can not substitute (*salva veritate* or meaning) the one for the other in 11.

11. When I graduated, I received a bachelor of arts diploma.

Or in 12.

12. 'Bachelor' has eight letters.

His example of not being able to substitute 'unadorned' for 'plain' in 13 is similarly silly.

13. Burlington is on the shores of Lake Champlain.

We can easily appeal to the notion of a word or term to block the above objections. We should only expect substitution of terms for terms.

'Bachelor of arts' is a complete term, and the 'bachelor' within it is just part of a term.

Similarly, 'plain' in 'Lake Champlain' is not a whole term.

The scare quotes provide a context of indirect reference in 12.

We should not expect interchangeability in any of the examples 11-13.

Still, we have not seen whether substitutivity *salva veritate* actually explains synonymy. Quine pretends to attempt to defend substitutivity *salva veritate* by appealing to modal contexts. (This is the "hocus-pocus" argument on 160.)

The idea is that we can isolate synonymous expressions as those whose identities are necessary. For example:

- 14. Necessarily, bachelors are unmarried men.
- 15. Necessarily, anything plain is unadorned.

Claims 14 and 15, Quine argues, are useless as explanations of synonymy.

They are circular, in that they explain one intensional idiom (synonymy/analyticity) in terms of another (modality).

What Quine wants is to explain synonymy without appealing to any other intensional contexts. He wants a reduction of the intensional.

His preference would be an explanation of analyticity in extensional terms.

In a purely extensional language, we would get interchangeability for all coextensive terms. But'creature with a heart' and 'creature with kidneys' would come out as synonymous.

Also, 'square circle' and 'short tall hairy bald man' would be synonymous, since they both have null extensions.

Interchangeability in an extensional language is different from the version we need to explain cognitive synonymy.

We need a cognitive synonymy that doesn't merely get us the truth of 'no bachelor is married', but its analyticity, as well.

We need something stronger than extensional equivalence.

It may not be possible to reductively analyze meaning at all.

But, without a reduction, the explanations (like those of 14 and 15) seem question-begging. 'Necessity' presupposes analyticity, so we get a kind of circle.

Our argument is not flatly circular, but something like it. It has the form, figuratively speaking, of a closed curve in space (2D, 160).

One response to Quine's accusation of circularity is to distinguish between virtuous and vicious circles. If we had a set of inter-theoretically linked intensional terms, we could justify the whole group by appealing to their systematic virtues for the intensional idioms themselves.

VI. The Argument Against D1, Redux

From his arguments against S1-S3, Quine concludes that there is no such thing as analyticity as apart from logical truth.

Synonymy must be explicated either in terms of definition or in terms of interchangeability *salva veritate*.

(I omit dictionary definition, S2, which was a non-starter.)

Since both of these options fail, there must be no analytic/synthetic distinction.

- QD QD1. If there is an analytic/synthetic distinction, there must be a good explanation of synonymy.
 - QD2. The only ways to explain synonymy are by interchangeability *salva veritate* or definition.
 - QD3. But interchangeability can not explain synonymy.
 - QD4. And definition (meaning postulates) can not explain synonymy.
 - QD5. Thus, there is no good explanation of synonymy.
 - QDC. And thus there is no analytic/synthetic distinction.

I'll mention one objection to QD, and a Quinean response and a half, before proceeding to examine the ramifications of Quine's conclusion.

The strongest response, I think, to QD is to challenge the weak QD2.

Quine does not consider all possible ways to explain synonymy, only two.

Perhaps a better theory of meaning could generate a good definition of synonymy, one from which analyticity would follow.

Quine's response would concede that QD2 is not fully defended.

But, he thinks that he has shown enough for us to see that no other ways of explaining synonymy will succeed.

So, Quine would urge us to take QD as enthymemic, an incomplete argument that could easily be finished.

Another way to look at QD is to take it not as an air-tight argument, but as painting a picture. Quine is describing a world without an analytic/synthetic distinction.

The defense of adopting that world-view would be not a small, tight argument, but, rather, how satisfying the picture is on the whole.

Thus, in order to understand the details, we have to look both at other work of Quine's, and at the ramifications of the failure of the two dogmas.

We now proceed to the latter.

VII. Meanings Skepticism

Quine's arguments against analyticity are motivated by Ockhamist concerns about spooky entities. His argument is twofold.

First, if we are to posit an object, it must have clear identity conditions.

We must be able to determine when two meanings are identical.

Elsewhere, Quine urges that we should admit "no entity without identity".

Quine uses the need for identity conditions, and the related need to be able to individuate objects, in order to deride alethic modal operators: possibility and necessity.

Quine asks, rhetorically, how many possible persons stand in a doorway.

Thus, Quine's response to Frege's problem of substitutivity in opaque contexts is to deny the legitimacy of (some of) those contexts.

The desire for identity conditions for objects in one's ontology is defensible.

A specific demand for identity conditions on meanings is even more defensible.

Meanings, if there are any, play a central role in communication.

A theory of meaning that entails that people can share their (identical) meanings seems a requirement if one wants to explain communication.

Meanings, e.g. Frege's propositions, inhabit a third realm.

So do mathematical objects, like sets.

Quine does not argue that we can avoid all abstract objects.

In fact, he defends the belief in sets, as long as they are construed extensionally.

But, he believes that the empiricist can avoid intensional objects.

The difference between intensional and extensional objects is that we have clear identity conditions for extensional objects.

We know when we have two and when we have one.

Sets are identified with their members; two sets with exactly the same members are the same set. But, we don't know when two meanings are the same.

There seems little hope of erecting a fruitful science about them. It is not even clear, granted meanings, when we have two and when we have one; it is not clear when linguistic forms should be regarded as *synonymous*, or alike in meaning, and when they should not ("Two Dogmas" original version, 22).

The second part of Quine's Ockhamist argument is that even if we could construct identity conditions for meanings, they would be otiose.

What we need from meanings is an explanation of synonymy and analyticity. If we can get these without meanings, then we don't need them.

If a standard of synonymy should be arrived at, we may reasonably expect that the appeal to meanings as entities will not have played a very useful part in the enterprise. A felt need for meant entities may derive from an earlier failure to appreciate that meaning and reference are distinct. Once the theory of meaning is sharply separated from the theory of reference, it is a short step to recognizing as the business of the theory of meaning simply the synonymy of linguistic forms and the analyticity of statements; meanings themselves, as obscure intermediary entities, may well be abandoned (ibid; see also 156 in our version).

The spookiness of meanings is related to the spookiness of essences.

Meaning is what essence becomes when it is divorced from the object of reference and wedded to the word (2D, 156).

For Aristotle, objects had essential characteristics, and accidental ones.

Persons had essential characteristics (e.g. rationality) and accidental ones (e.g. two-leggedness).

This difference now supposedly goes into the meaning of a term.

The meaning of 'biped' includes being two-legged but the meaning of 'man' may not.

Kripke's rehabilitation of essences, and necessity, were of course unwelcome to Quine and Quineans.

In "Ontological Relativity," Quine calls meanings realism (like that of Frege) the myth of the museum.

VIII. Confirmation Holism and Semantic Holism

The final section of "Two Dogmas" is called "Empiricism without the Dogmas."

We have been looking at Quine's skepticism about intensional objects: there are no meanings.

Quine does not, though, deny that sentences (and terms, derivatively) have meaning.

He defends meaningfulness without meanings.

Further, he believes that meaning is properly the property of a much larger unit than the word (as Locke and Hume held), or even the sentence (as Frege and the positivists held).

The unit of significance, for Quine, is one's entire theory, the whole of science.

(See Quine's "Five Milestone's of Empiricism" for a brief, helpful discussion of these differences.)

Ayer characterized analytic statements as those onto which we hold come what may.

In the absence of an analytic/synthetic distinction, one might think that Quine would argue that there are no sentences that are immune from revision or abandonment.

This is correct, but there is another consequence.

If we see science as a total field of information, then we can hold on to any statements we want, as long as we adjust our body of knowledge and the logical framework along with it.

The claim that any sentence may be held immune from revision because no sentence is absolutely immune, is called confirmation holism.

Confirmation holism follows from the failure of the analytic/synthetic distinction, and the identities of necessity with analyticity and contingency with syntheticity.

It is the claim that all sentences are confirmed or disconfirmed as a whole body.

One can also argue for confirmation holism from weaker premises.

It is just a point of the logic of theory construction.

Quine's stronger argument for confirmation holism also yields the more contentious semantic holism:

16. Semantic holism: the unit of meaning is the entire language.

Most people's beliefs, their entire language, are too loose to serve as a good semantic theory. Instead, to find meaning, Quine looks at our best theory, the theory which best accounts for all our sense experience.

Quine compares our scientific beliefs to a giant web.

Experience forms the boundary conditions on the web of belief.

Peripheral statements are those most closely tied to sensory experience.

Central statements are those about logic, mathematics, and the self, the guiding principles of science, highly theoretical statements.

Quine also compares the web to a field of force.

Experience forces us to adjust and readjust the whole field, not one sentence at a time, but altogether. The under-determination of the field by boundary conditions gives us play among the statements.

We can hold to the truth of any statement, come what may, by merely adjusting other statements. Some may require large scale adjustments, others minor ones.

In the face of an odd sense experience, we can, drastically, claim hallucination, or revise logic.

Different persons growing up in the same language are like different bushes trimmed and trained to take the shape of identical elephants. The anatomical details of twigs and branches will fulfill the elephantine form differently from bush to bush, but the overall outward results are alike" (*Word and Object*, 8).

IX. Ontology and Posits

Along with the loss of reductive justifications of particular sentences, we lose a straightforward method for determining our ontology.

Locke and Hume and the positivists all demanded reductions of claims about the world to our sense experience of the world, in some way.

They demanded direct lines from physical objects to sense data to singular terms.

On Quine's holistic view, we have to determine our ontology by appeal to the whole of science.

Science is a tool, for predicting future experience in the light of past experience.

Physical objects are just convenient posits of a scientific theory.

Ontological questions, under this view, are on a par with questions of natural science (2D 168).

We already accept an ontology of posits for distant objects and very small objects, like electrons. Quine argues that all our ontology is of that form.

Quine's method is not intended to denigrate physical objects.

"To call a posit a posit is not to patronize it" (Word and Object, 22).

The method of positing is just a result of the failure of reductionism and the turn towards holism. The difference between questions of the existence of sets, say, or quarks, and questions of the existence of houses is only one of degree, not of type.

Posits are accepted or rejected according to pragmatic considerations of theory construction, as well as their coherence and consistency with our broader theory, the web of belief.

Quine's procedure for determining our ontic commitments (QP) is as follows.

- QP.1: Select a best scientific theory, one which balances simplicity, strength, and fit with sense experience.
- QP.2: Regiment that theory in first-order logic with identity.
- QP.3: Model the resulting formal theory.
- QP.4: Examine the domain of quantification of the theory to see what objects the theory needs to come out as true.

One consequence of QP is that Quine is able to reconcile mathematical ontology with empiricist epistemology.

Traditionally, empiricists had difficulty explaining how we could have knowledge of the abstract objects of mathematics.

By turning all ontology into scientific posits, Quine opens the door for any objects which facilitate, in serious ways, the construction and regimentation of scientific theory.

Quine's epistemology is still empiricist, broadly, as was Benacerraf's.

But, Quine has no problem explaining access to mathematical objects, since his method does not require access to objects for them to be in our ontology.

The only requirement is that they be elements in the domain of quantification of the best interpretation of our best scientific theory.

So, contra Benacerraf's claim, Quine can have both standard semantics and his preferred epistemology.

Quine's justification of our mathematical beliefs is called the indispensability argument, for its claim that mathematics is indispensable to science.

That's our topic for next time.