Knowledge, Truth, and Mathematics

Philosophy 405
Russell Marcus
Hamilton College, Fall 2010
Class #1: Mathematics and Philosophy

Why I Like James Robert Brown

If we taught philosophy today in a way that reflected its history, the current curriculum would be overwhelmed with the philosophy of mathematics. Think of these great philosophers and how important mathematics is to their thought: Plato, Descartes, Leibniz, Kant, Frege, Russell, Wittgenstein, Quine, Putnam, and so many others. And interest in the nature of mathematics is not confined to the so-called analytic stream of philosophy; it also looms large in the work of Husserl and Lonergin, central figures in, respectively, the continental and Thomistic philosophical traditions. Anyone sincerely interested in philosophy must be interested in the nature of mathematics... As for those who persist in thinking otherwise - let them burn in hell (Brown, xi-xii).

Philosophers Doing Mathematics

- Plato's Academy: "Let no one enter who is ignorant of geometry."
- Aristotle: "Mathematics has come to be the whole of philosophy for modern thinkers" (*Metaphysics* I.9: 992a32).
- Descartes founded analytic geometry.
- Leibniz developed the calculus.
- Frege and Russell made advances in the foundations of mathematics proper.
- Quine, Kripke, Field and many others contribute to set theory and the foundations of mathematics.

Mathematicians Doing Philosophy

- Euclid's method
- Cantor's work on transfinite numbers
- Kripke and the mathematical treatment of modality
- Hilbert, Gödel, von Neumann, and Tarski

The Effects of Mathematics on Metaphysics

- Plato used the abstractness of mathematics to motivate the reality of the forms.
- Descartes cleaved thought from sensation by considering how mathematical beliefs were not ultimately sensory.
- Kant's transcendental idealism begins with the question of what the structure of our reasoning must be in order to yield mathematical certainty.
- Wittgenstein's *Remarks on the Foundations of Mathematics* contain core elements of his philosophical positions, specifically his skepticism about rule-following.

Effects of Metaphysics on Mathematics?

- Berkeley tried to debunk the calculus.
- But.
- "Philosophy may in no way interfere with the actual use of language; it can in the end only describe it. For it cannot give it any foundation either. It leaves everything as it is. It also leaves mathematics as it is, and no mathematical discovery can advance it" (Wittgenstein, *Philosophical Investigations*, §124).
- "There is no mathematical substitute for philosophy" (Kripke, "Is There a Problem About Substitutional Quantification").

This Course

- Historical and contemporary approaches to the philosophy of mathematics
- First half: a broad survey of historical approaches to the philosophy of mathematics, from the Pre-Socratic philosophers through the early twentieth century.
- Second half: the indispensability argument
- The course will culminate in some new work that I am preparing.

The Syllabus and Website

- **readings**
- **reading prècises**
- **seminar papers**
- **term paper**
  - abstract on **Wednesday, October 13**
  - full draft on **Monday, November 15**
  - final draft on **Monday, December 6**
  - course bibliography
- **final exam**
  - **Wednesday, December 15**, from 9am to noon
  - Preparatory questions will be posted on the course website.

On Proof

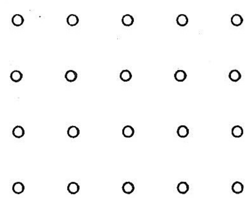
- Greek mathematics was essentially geometric.
- The Pythagoreans were fascinated by figurate numbers: triangular numbers, square numbers, pentagonal numbers
- The triangular numbers, were especially interesting to the Pythagoreans:
  1, 3, 6, 10, 15, 21, 28, 36...
- The formula for the calculating the nth triangular number is: (n/2)(n+1)
- The sum of two consecutive triangular numbers is a square number.
- This is easily shown algebraically:

$$(n/2)(n+1) + ((n+1)/2)((n+1)+1) =$$
$$(n^2 + n)/2 + (n+1)(n+2)/2 =$$
$$(n^2 + n)/2 + (n^2 + 3n + 2)/2 =$$
$$(2n^2 +4n + 2)/2 =$$
$$n^2 + 2n + 1 =$$
$$(n+1)^2$$

- Kline: "That the Pythagoreans could prove this general conclusion, however, is doubtful" (30).
- Is it?

Wittgenstein on Commutativity

17. The mere picture



regarded now as four rows of five dots, now as five columns of four dots, might convince someone of the commutative law. And he might thereupon carry out multiplications, now in the one direction, now in the other.

17. The mere picture

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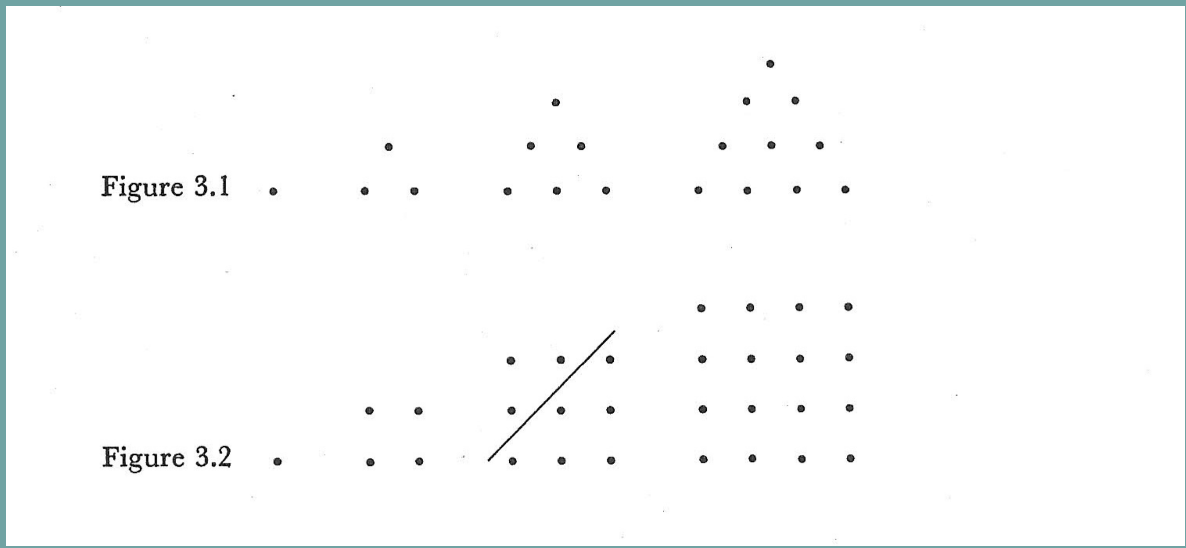
The Pythagoreans Had a Picture



Limitations of Pictures

- It is difficult to draw intuitively useful pictures of odd spaces.
- Some pictures are misleading.
- Compare the sums of two infinite series:
  - 1, 1/4, 1/9, 1/16...
  - 1, ½, 1/3, 1/4...

Graphs

1, 1/4, 1/9, 1/16...

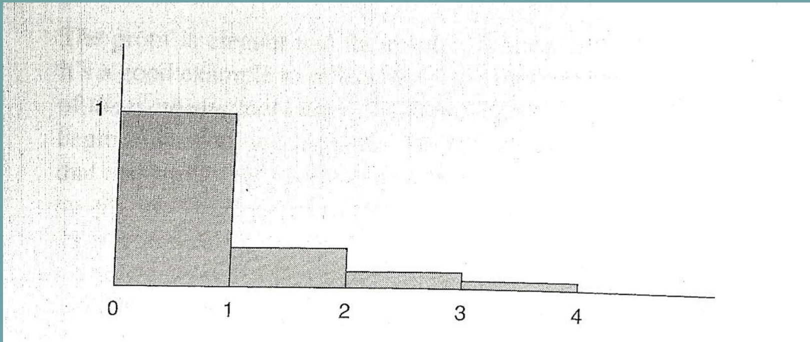


Figure 1.2 Shaded blocks correspond to terms in the series

1, ½, 1/3, 1/4...

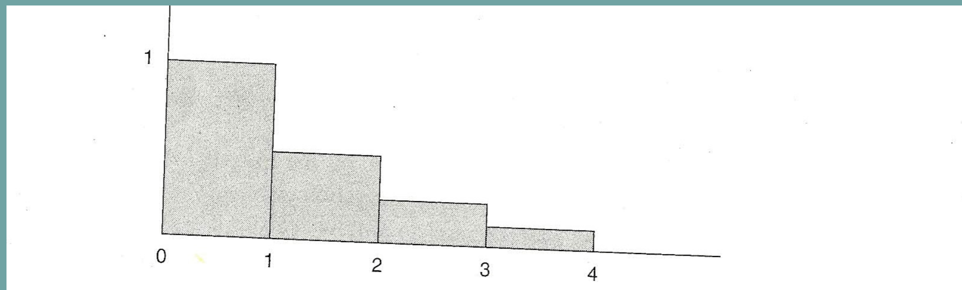


Figure 1.3

But

- The first series (1, 1/4, 1/9, 1/16...) sums to a finite number, $\pi^2/6 \approx 1.64$
- The sum of the second series (1, ½, 1/3, 1/4...) is infinite
- So much for pictures?
- Still, do we overvalue the algebraic proof?
- What about the ‘aha’ moment that we can get from pictures?

Brown’s “Mathematical Image”

1. Mathematical results are certain.
2. Mathematics is objective.
3. Proofs are essential.
4. Diagrams are psychologically useful, but prove nothing.
5. Diagrams can even be misleading.
6. Mathematics is wedded to classical logic.
7. Mathematics is independent of sense experience.
8. The history of mathematics is cumulative.
9. Computer proofs are merely long and complicated regular proofs.
10. Some mathematical problems are unsolvable in principle.

a priori knowledge

- The question of whether we have *a priori* knowledge is widely debated.
- A proposition is known *a priori* if the knowledge is not based on any "experience of the specific course of events of the actual world'"(Blackburn, in Shapiro, 22).
- The debates over the *a priori* are subtle and complex.
- But, the question of whether there is *a priori* knowledge seems easily answered in mathematics.
- We could never discover that the square root of two is irrational by experience.
  - The rationals are dense.
  - We can always find a rational which will fulfill our measurement needs.

That $\sqrt{2}$ is irrational

- Suppose that $\sqrt{2}$ is rational.
- Then, it's expressible as a/b, where a and b are integers.
- We can suppose a/b to be in lowest terms, which means that a and b have no common divisors.
- $a^2 = 2b^2$
- So, a^2 is even.
- Thus **a is even**, since only even numbers have even squares.
- So, a = 2c, for some c.
- $a^2 = 4c^2 = 2b^2$
- So, $b^2 = 2c^2$.
- Which means that **b is also even**.
- So a and b have been shown even, which contradicts our assumption that a/b is in lowest terms.
- *Tilt*

Hippasus

Aside on *reductio ad absurdum*

- Assume the opposite of what one wants to demonstrate, and show that it leads to a contradiction.
- Reductios assume bivalence.
- Some philosophers reject bivalence, and its object-language correlate called the law of the excluded middle:
  - Law of the excluded middle: $P \vee {\sim}P$
  - Intuitionists demand constructive proofs.

Apriority and Necessity

- Long confounded
  - Old view: anything believed *a priori* must be true
  - not
- Consider Kant's claim that Euclidean space is the result of the a priori application of our concepts on the noumenal world.
  - Space is non-Euclidean
  - What seemed *a priori* turns out to be false.
  - On the old view, if a statement turns out false, it must never have been believed *a priori*.
  - Kant's entire metaphysical system depended on the application of *a priori* concepts to the noumenal world.
  - When space turned out to be non-Euclidean, Kant's system seemed to fall apart.
- Shapiro still calls apriority and necessity "twin notions" (23).

The Fallibilist *a priori*

- Apriority is about the acquisition and justification of our beliefs.
- Necessity is about their modal status.
- We can be wrong about a proposition, even if we hold it *a priori*.
- We can believe a proposition independently of experience, and still be wrong about that belief.
  - Cantor and Frege and the axiom of comprehension: every property determines a set
  - The set of all things that aren't woodchucks is too big.
- Had Kant held a fallibilistic *a priori*, he might have been able to salvage some of his work.
- The fallibilist can hold that statements believed on the basis of a priori reasoning are necessarily true, *if true*.
  - And if they are false, they are necessarily false.

Analyticity

- Thought by many to be an explanation of apriority
- ‘Bachelors are unmarried’
- ‘We walk with those with whom we stroll’
- Analyticity is a semantic notion, about meanings of terms.
- Apriority is an epistemic notion, about belief and knowledge.
- Necessity is a metaphysical notion, about the nature of the universe, broadly conceived.
- Certainty is an epistemic notion, masquerading as a metaphysical notion.
  - I can be certain about something non-necessary, like that I am here now.
  - I can be uncertain about something necessary, like whether Goldbach’s conjecture is true.
  - Even Brown makes mistakes: 1. Mathematical results are certain.

Wednesday:

Pythagoreans