

To Prepare for the Final Exam

For the final exam, I will give you ten of the following questions, and ask you to write responses to eight of them.

1. How, according to the Pythagoreans, were numbers the ultimate components of real things?
2. How did Aristotle's view of mathematics differ from that of Plato and the Pythagoreans?
3. How does Plato argue that knowledge is not perception?
4. How does Plato divide the visible realm? Describe the two parts. How does Plato divide the intelligible realm according to the methods used to investigate the two parts? What are the different elements of the two parts of the intelligible realm?
5. For Aristotle, why can't mathematical objects exist in sensible things? Why can't mathematical objects exist separate from sensible things? Given that mathematical objects do not exist in substances or separate from substances, does it follow that they do not exist at all? Explain.
6. "What I believe must be considered above all here is the fact that I find within me countless ideas of certain things that, even if perhaps they do not exist anywhere outside me, still cannot be said to be nothing" (Descartes). Explain. How does Descartes argue that mathematical objects, like the triangle, exist objectively (immutably)?
7. How does Descartes argue that mathematical knowledge does not arise from sense experience?
8. How does Leibniz criticize Descartes's criterion of clear and distinct ideas?
9. What does Locke take to be the most important argument for innate ideas? How, according to Locke, is that argument invalid? How is it unsound? Describe Locke's account of our knowledge of general ideas.
10. Distinguish, following Leibniz, the temporal order of ideas from their order of justification.
11. According to Locke, how do words become general? How do ideas become general?
12. For Locke, are mathematical claims certain? What are the objects of these claims?
13. How, for Berkeley, does the notion of an abstract, general idea lead to a contradiction?
14. Distinguish Hume's relations of ideas from matters of fact. How do we learn propositions that are solely concerned with relations of ideas?
15. How does Kant argue that mathematical judgments are *a priori*? How does Kant argue that arithmetic is synthetic? How does Kant argue that geometry is synthetic?
16. "Number is therefore simply the unity of the synthesis of the manifold of a homogeneous intuition in general, a unity due to my generating time itself in the apprehension of the intuition" (Kant). Explain.
17. For Mill, in what way are the theorems of geometry false?
18. "There are no such things as numbers in the abstract" (Mill). Explain. What are the subjects of the theorems of arithmetic?
19. What is Cantor's continuum hypothesis?
20. For Frege, are arithmetical theorems analytic or synthetic? Explain. Contrast Frege's definition of analyticity (plant in the seeds) with Kant's definition (beams in the house).
21. How does Hilbert adopt a finitary view of mathematics? What are the four requirements for a Hilbert-style system?
22. What are the three possibilities regarding the continuum hypothesis? Which does Gödel think is the most promising?
23. How was non-Euclidean geometry a problem for Kant's intuitionism? How does Brouwer's neo-intuitionism respond to this problem?

24. How do intuitionists differ from classical logicians concerning the nature of mathematical existence?  
How do formalists and intuitionists differ in their approaches to mathematics?
25. "Some contemporary nominalists label the admission of variables of abstract types as "Platonism". This is, to say the least, an extremely misleading terminology. It leads to the absurd consequence, that the position of everybody who accepts the language of physics with its real number variables (as a language of communication, not merely as a calculus) would be called Platonistic, even if he is a strict empiricist who rejects Platonic metaphysics" (Carnap). Explain. How does the internal/external distinction help solve the problem?
26. How, according to Ayer, do we respond to purported counter-examples to mathematical or logical claims?
27. How are concerns about semantics and epistemology central to discussions of mathematical truth? How are they in tension?
28. "It is obvious that truth in general depends on both language and extralinguistic fact...Thus one is tempted to suppose in general that the truth of a statement is somehow analyzable into a linguistic component and a factual component. Given this supposition, it next seems reasonable that in some statements the factual component should be null; and these are the analytic statements" (163-4). According to Quine, what is wrong with this argument?
29. How, for Quine, do we commit ourselves to an ontology?
30. "Ordinary interpreted scientific discourse is irredeemably committed to abstract objects - to nations, species, numbers, functions, sets - as it is to apples and other bodies...The numbers and functions contribute just as genuinely to physical theory as do hypothetical particles" (149-50). Explain.
31. What, for Quine, determines whether a mathematical result is legitimate? What is mathematical recreation?
32. Describe the Quinean doublethink objection to fictionalism. How does Field avoid that criticism?
33. What is a representation theorem? How does Hilbert's representation theorem facilitate inference within Euclidean geometry? How does Hilbert adapt Hilbert's representation theorems for his project?
34. Describe Melia's taking-back (weasel) strategy. How does it apply to the mathematical case?
35. What is the eleatic principle? How does it oppose Quine's method for determining ontological commitment?
36. How is the holism which supports the indispensability argument in conflict with mathematical practice?
37. Contrast Leng's fictionalism with Field's version. Why, according to Leng, do we not need to reformulate scientific theories to be fictionalists?
38. What is the explanatory indispensability argument? Are there genuine mathematical explanations of physical phenomena? Explain.
39. How, according to Bangu, is Baker's explanatory indispensability argument circular? How does it beg the question against the nominalist?