Philosophy 405: Knowledge, Truth and Mathematics | Hamilton College
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Reading Guide #7: Mill

John Stuart Mill, *A System of Logic*, Book II, Chapters V-VI
Gottlob Frege, *The Foundations of Arithmetic*, §§7-17

Mill

1. In what way are the theorems of geometry false?
2. Do the theorems of geometry refer to possible objects?
3. Are geometric objects mental objects? Explain.
4. "I much question if any one who fancies that he can conceive what is called a mathematical line, thinks so from the evidence of his consciousness: I suspect it is rather because he supposes that unless such a conception were possible, mathematics could not exist as a science" (169). Explain.
5. How do we correct our mathematical reasoning after applying our results? Explain the analogy from physical science.
6. How is geometry built on hypotheses?
7. What role does exaggeration play in the development of geometry?
8. Are there any propositions in mathematics which are precisely true?
9. What evidence supports our beliefs in mathematical axioms?
10. "Experimental proof crowds in upon us in such endless profusion, and without one instance in which there can be even a suspicion of an exception to the rule, that we should soon have stronger ground for believing the axiom, even as an experimental truth, than we have for almost any of the general truths which we confessedly learn from the evidence on our senses" (173). Explain.
11. Explain the argument against taking mathematical theorems to be inductive generalizations that appears in the footnote on pp 173-4. How does Mill respond?
12. How does Whewell characterize intuition? How does Mill argue that such intuition is inductive?
13. How does Mill interpret Whewell's various claims of the necessity of mathematical axioms? How is it an "affair of accident"?
14. "The most practiced intellect is not exempt from the universal laws of our conceptive faculty" (178). Explain.
15. Is Newton's first law of motion intuitively obvious? According to Mill, what do historical differences regarding people's beliefs in this law show?
16. How does Mill account for philosophical claims about necessity? Why do we call mathematical theorems necessary?
17. Why do we believe that the weight of the whole is found in the weight of its elements?
18. In what sense does Mill accept that the theorems of mathematics are necessary?
19. How might the theorems of arithmetic be merely verbal?
20. "There are no such things as numbers in the abstract" (189). Explain.
21. What are the subjects of the theorems of arithmetic?
22. Are theorems of arithmetic mere identities or definitions? Explain.
23. What kind of certainty can we find in arithmetic? In geometry?

Frege

1. “What a mercy, then, that not everything in the world is nailed down” (§7). Explain both Mill’s account of number and Frege’s criticism here.
2. What problem arises for Mill’s definition of number by considering zero and one?
3. How are considerations of sensations and methods of solving an equation problematic for Mill’s definition of number?
4. How are considerations of large numbers problematic for Mill’s definition of number?
5. Can we accept Mill’s claim, that the definitions of numbers include an observed fact, for merely small numbers?
6. “If we call a proposition empirical on the ground that we must have made observations in order to have become conscious of its content, then we are not using the word ‘empirical’ in the sense in which it is opposed to ‘a priori’” (§8). Explain.
7. How does Frege distinguish the meaning from the application of arithmetic theorems? How does he use this distinction against Mill?
8. How does induction rest on arithmetic (via probability theory)?
9. How does Frege argue against taking arithmetic to be synthetic?
10. How does Frege argue that geometry is synthetic?
11. “The truths of arithmetic [are] related to those of logic in much the same way as the theorems of geometry to the axioms” (§17). Explain. (This is a big one.)