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Truth tables for propositions, §6.3

I. **What is a truth table?**

A truth table is a method for characterizing any logically complex proposition on the basis of the truth conditions of its component propositions.
We can also use them to separate valid from invalid arguments.

We can construct truth tables for any proposition, by using the basic truth tables.
The basic truth tables are printed on the inside of the front cover of the text.
Truth tables show us the distributions of all possible truth values of component propositions.

II. **Constructing truth tables for propositions**

The Method:

1) How many rows do we need?
1 variable: 2 rows
2 variables: 4 rows
3 variables: 8 rows
4 variables: 16 rows

2) Assign truth values to the component variables.
We start truth tables always in the same ways.
See below for examples.

3) Work inside out, placing the column for each letter or connective directly beneath the letter or connective.

Consider: ‘(P ∨ ~Q) · (Q ⊃ P)’

Step 1: We have two variables, so we need four rows.
Step 2: Assign truth values to component variables:

(P	∨	~	Q)	·	(Q	⊃	P)
T			T		T		T
T			F		F		T
F			T		T		F
F			F		F		F

Note that the same values we assign to P in the first column, we also use for P in the last column, and similarly for Q.
Also, all four row truth tables begin with this set of assignments.

Step 3, in stages:
First do the negation:

(P	∨	~	Q)	·	(Q	⊃	P)
T		**F**	T		T		T
T		**T**	F		F		T
F		**F**	T		T		F
F		**T**	F		F		F

Then the disjunction and conditional:

(P	∨	~	Q)	·	(Q	⊃	P)
T	**T**	F	T		T	**T**	T
T	**T**	T	F		F	**T**	T
F	**F**	F	T		T	**F**	F
F	**T**	T	F		F	**T**	F

And finally, the main connective, the conjunction, using the columns for the disjunction and the conditional:

(P	∨	~	Q)	·	(Q	⊃	P)
T	**T**	F	T	**T**	T	**T**	T
T	**T**	T	F	**T**	F	**T**	T
F	**F**	F	T	**F**	T	**F**	F
F	**T**	T	F	**T**	F	**T**	F

Thus, this proposition is false when P is false and Q is true, and true otherwise.
Note that you only have to write out the truth table once, like the last one in this demonstration.

III. **Exercises A**. Construct truth tables for each of the following propositions.
1) ~P ⊃ Q
2) (P ≡ P) ⊃ P
3) ~Q ∨ (P ⊃ Q)

IV. **Classifying propositions using truth tables**
We may use truth tables to make important distinctions among tautologies, contingencies, and contradictions,
Consider the truth table for 'P ⊃ P'

P	⊃	P
T	**T**	T
F	**T**	F

This is a *tautology*: statement that is always true.
Another tautology: 'Either the Mets win the World Series this year, or they don't.'

Here's a longer tautology: '[(P ⊃ Q) · (Q ⊃ R)] ⊃ (P ⊃ R)'

[(P	⊃	Q)	·	(Q	⊃	R)]	⊃	(P	⊃	R)
T	T	T	T	T	T	T	**T**	T	T	T
T	T	T	F	T	F	F	**T**	T	F	F
T	F	F	F	F	T	T	**T**	T	T	T
T	F	F	F	F	T	F	**T**	T	F	F
F	T	T	T	T	T	T	**T**	F	T	T
F	T	T	F	T	F	F	**T**	F	T	F
F	T	F	T	F	T	T	**T**	F	T	T
F	T	F	T	F	T	F	**T**	F	T	F

Note the method for constructing an 8-row truth table. (Look at the columns under P, Q, and R.)
Tautologies aren't really true, themselves, but, rather, true on all substitutions of propositions for the variables.

Consider the truth table for 'P ∨ ~Q'

P	∨	~	Q
T	**T**	F	T
T	**T**	T	F
F	**F**	F	T
F	**T**	T	F

This is a *contingency*: statement that may or may not be true.
It's true in at least one row of the truth table.
It's false in at least one row.
The truth of the complex proposition is contingent (depends) on the values of the component premises.

Consider the truth table for 'P · ~P'

P	·	~	P
T	F	F	T
F	F	T	F

This is a *self-contradiction*: statement that is never true.
Here's another: '(~P ⊃ Q) ≡ ~(Q ∨ P)'

(~	P	⊃	Q)	≡	~	(Q	∨	P)
F	T	T	T	**F**	F	T	T	T
F	T	T	F	**F**	F	F	T	T
T	F	T	T	**F**	F	T	T	F
T	F	F	F	**F**	T	F	F	F

V. **Exercises B**. Classify each proposition as tautologous, contingent, or self-contradictory.
1) ~A ⊃ ~A
2) B · (B ∨ F)
3) (~D · E) · (E ⊃ D)

VI. **Classifying pairs of sentences using truth tables**

Consider '(A ∨ B) ≡ (~B ⊃ A)'.

(A	∨	B)	≡	(~	B	⊃	A)
T	T	T	**T**	F	T	T	T
T	T	F	**T**	T	F	T	T
F	T	T	**T**	F	T	T	F
F	F	F	**T**	T	F	F	F

It's a tautology.

Eliminate the biconditional, and consider the two remaining halves as separate statements:

A	∨	B		~	B	⊃	A
T	**T**	T		F	T	**T**	T
T	**T**	F		T	F	**T**	T
F	**T**	T		F	T	**T**	F
F	**F**	F		T	F	**F**	F

These are *logically equivalent*: Two or more statements with identical truth values in every row of the truth table. This concept will help us understand some left-over issues about translation. (See below, §VIII.)

Consider 'A ∨ ~B' and 'B · ~A'.

A	∨	~	B		B	·	~	A
T	**T**	F	T		T	**F**	F	T
T	**T**	T	F		F	**F**	F	T
F	**F**	F	T		T	**T**	T	F
F	**T**	T	F		F	**F**	T	F

These statements form a *contradiction*: Two statements with opposite truth values in all rows of the truth table.
Note that the biconditional connecting the two statements of a contradiction is self-contradictory.
'P · ~P' is a simple contradiction, with common use.
In English: "It's raining. It's not raining."
A person who makes both statements together has to be wrong about at least one of them.

Consider ‘E ⊃ D’ and ‘~E · D’.

E	⊃	D		~	E	·	D
T	**T**	T		F	T	**F**	T
T	**F**	F		F	T	**F**	F
F	**T**	T		T	F	**T**	T
F	**F**	F		T	F	**F**	F

These are neither contradictory (see rows 2, 3, and 4) nor logically equivalent (see row 1).
But a person who makes both statements can be making true statements. (See row 3).
It depends on what the substitutions are (for E and D).

If two statements are neither logically equivalent nor contradictory, they may be consistent or inconsistent.
Consistent: Can be true together, for at least one valuation (one row of the table).
Inconsistent: Not consistent. I.e. there is no row of the truth table in which both statements are true.
An inconsistent pair: ‘E · F’ and ‘~(E ⊃ F)’

E	·	F		~	(E	⊃	F)
T	**T**	T		**F**	T	T	T
T	**F**	F		**T**	T	F	F
F	**F**	T		**F**	F	T	T
F	**F**	F		**F**	F	T	F

Note that the conjunction of two inconsistent statements is a self-contradiction.
When comparing two propositions, first look for the stronger conditions: logical equivalence and contradiction.
Then, if these fail, look for the weaker conditions: consistency and inconsistency.

VII. **Exercises C**. Are the statements logically equivalent or contradictory? If neither, are they consistent or inconsistent?

1) A ⊃ ~B ~(B · A)
2) A · ~B B · ~A
3) B · A A ⊃ ~B
4) A ≡ B ~(A ∨ B)
5) A ∨ (B · D) ~A · ~(B ∨ ~D)

VIII. **Clearing up some translation issues**

Unless and Exclusive Disjunction:

Previously, we translated ‘unless; with a ‘v’.
Consider the complex proposition: ‘A car will not run unless there’s gas in the tank.’

The car does not run	The car runs	**unless**	The car has gas
F	T	**T**	T
F	T	**F**	F
T	F	**T**	T
T	F	**T**	F

In the first row, the car runs and has gas, so the complex proposition should be true.
In the second row, the car runs, but does not have gas, and so the complex proposition should be false.
In the third row, the car does not run, but has gas.
This does not falsify the complex proposition, which does not indicate what else the car needs in order to run.
The complex proposition indicates a necessary condition (having gas) but not sufficient conditions for a car to run.
Thus the statement should be considered true.
In the fourth row, the car does not run, but does not have gas, and so the proposition should be true.
The following truth table thus appropriately represents the complex proposition, translating 'unless' as '∨', since it is logically equivalent to the one we want.

~	R	∨	G
F	T	T	T
F	T	F	F
T	F	T	T
T	F	T	F

But now, consider: 'Carol will attend school full time unless she gets a job'

Carol attends school	unless	Carol gets a job
T	?	T
T	T	F
F	T	T
F	F	F

In the second row, she attends school but doesn't get a job, and so the proposition should be true.
In the third row, she gets the job, and doesn't go to school, and so the proposition should be true.
In the last row, she doesn't get the job but doesn't go to school, and so the proposition should be false.
What about the first row?
Here she gets a job but attends school anyway.
Is the proposition true or false?
If we say it's false, we arrive at a truth table for exclusive disjunction.
Unless is as ambiguous as 'or', and in the same way: there's an inclusive and exclusive 'unless'.
The following translation will suffice for exclusive disjunction and exclusive unless: ~A ≡ J, since it is logically equivalent to the one we want.

~	A	≡	J
F	T	**F**	T
F	T	**T**	F
T	F	**T**	T
T	F	**F**	F

We thus think of the exclusive unless as a biconditional: Carol will not attend school if, and only if, she gets a job.
So, when translating unless, use the wedge for inclusive senses, and as the default translation.
Use the biconditional (with one element negated) for exclusive senses.

Given two variables, there are 16 possible distributions of truth values.
We have labels for four.
We can define the other 12, using combinations of the five connectives.
(This is kind of a fun exercise. You might try it.)
As long as we can define all possibilities, it doesn't matter which we take to be basic.
We just have to be careful to translate correctly.

Here's another logically equivalent formulation for exclusive disjunction: '(P ∨ Q) · ~(P · Q)'.

(P	∨	Q)	·	~	(P	·	Q)
T	T	T	**F**	F	T	T	T
T	T	F	**T**	T	T	F	F
F	T	T	**T**	T	F	F	T
F	F	F	**F**	T	F	F	F

The Biconditional
The biconditional is superfluous, since it's logically equivalent to a statement which uses only other connectives: '(P ⊃ Q) · (Q ⊃ P)'

P	≡	Q		(P	⊃	Q)	·	(Q	⊃	P)
T	**T**	T		T	T	T	**T**	T	T	T
T	**F**	F		T	F	F	**F**	F	T	T
F	**F**	T		F	T	T	**F**	T	F	F
F	**T**	F		F	T	F	**T**	F	T	F

Other connectives can be shown to be superfluous, in similar ways.

IX. **Solutions**
Answers to Examples A:
1)

~	P	⊃	Q
F	T	**T**	T
F	T	**T**	F
T	F	**T**	T
T	F	**F**	F

2)

(P	≡	P)	⊃	P
T	T	T	**T**	T
F	F	F	**F**	F

3)

~	Q	∨	(P	⊃	Q)
F	T	**T**	T	T	T
F	T	**T**	F	T	T
T	F	**T**	T	F	F
T	F	**T**	F	T	F

Answers to Exercises B
1) Tautologous
2) Contingent
3) Contradictory

Answers to Exercises C.
1) Logically equivalent
2) Inconsistent
3) Contradictory
4) Consistent
5) Inconsistent