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Truth tables for arguments, §6.4

I. Valid and Invalid Arguments

Compare the following arguments:

A)

1. If God exists then every effect has a cause.

2. God exists.

 \therefore Every effect has a cause.

B)

1. If God exists then every effect has a cause.

2. Every effect has a cause.

 \therefore God exists.

A) is valid, and has the following form: $P \supset Q$ P / Q

This form is known as Modus Ponens.

Note that we write the premises on sequential lines, and the conclusion on the same line as the final premise, following a single slash.

B) is invalid, and has the following form: P \supset Q Q / P

This is known as the Fallacy of Affirming the Consequent.

Recall: In a valid argument, if the premises are true then the conclusion must be true.

Note, it says nothing about what happens if the premises are false.

An invalid argument is one in which it is possible for true premises to yield a false conclusion.

By focusing on valid arguments, we can make sure that if all our premises are true, so must our conclusions be.

II. A method for determining if an argument is valid

Step 1: Line up premises and conclusion horizontally, separating premises with a single slash and separating the premises from the conclusion with a double slash.

Step 2: Construct truth tables for each premise and the conclusion, using consistent assignments to component variables. Step 3: Look for a counter-example: a row in which all premises are true and the conclusion is false.

If there is a counter-example, the argument is invalid.

If there is no counterexample, the argument is valid.

A valid argument:

Р	n	Q	/	Р	//	Q
Т	Т	Т		Т		Т
Т	F	F		Т		F
F	Т	Т		F		Т
F	Т	F		F		F

Note: On no line are the premises true and the conclusion false. There is no counter-example.

An invalid argument:

Р	n	Q	/	Q	//	Р
Т	Т	Т		Т		Т
Т	F	F		F		Т
F	Т	Т		Т		F
F	Т	F		F		F

This third row is the *counterexample*: The argument is invalid when P is false and Q is true.

Consider: Is this a valid argument? 1. $P \supset (Q \supset P)$ 2. $\sim P / Q$

Р		(Q		P)	/	~ P	//	Q
Т	Т	Т	Т	Т		F		Т
Т	Т	F	Т	Т		F		F
F	Т	Т	F	F		Т		Т
F	Т	F	Т	F		Т		F

Row 4 is a counter-example.

The argument is shown invalid when P is false and Q is false.

III. Exercises. Determine whether each argument is valid. If invalid, specify the counter-example.

$\begin{array}{l} 1)\\ A \supset B\\ \sim A \end{array}$	/~B
2) C ∨ D ~D	/ C
3) E = F	/~E \/ F
4) G • H H ⊃ I	/~(G·I)
5) J ⊃ K K ⊃ ~J ~J ⊃ K	/ J ∨ ~K

IV. Solutions

- 1. Invalid, when A is false, and B is true
- 2. Valid

3. Valid

- 4. Invalid, when G, H, and I are all true
- 5. Invalid, when J is false and K is true