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Translation To Predicate Logic, §8.1

I. **Introduction**

Consider the following argument:
All philosophers are happy.
Emily is a philosopher.
So, Emily is happy.

If we translate into propositional logic, the conclusion does not follow:
P
Q / R

The conclusion follows logically, but not with propositional logic.
Propositional Logic is insufficient to derive all logical consequences.
We need a stronger logic, one that explores the logic inside the sentences.
This is called predicate logic.

In Propositional Logic, we have the following elements:
Terms for statements , simple letters
Five connectives
Punctuation (brackets)

In Predicate Logic, we have the following elements:
Complex Terms for statements, made of objects and predicates
Quantifiers
The same five connectives as in propositional logic.
The same punctuation as in propositional logic.

II. **Objects and Predicates**

We represent objects using lower case letters.
'a, b, c,...w' stand for specific objects, and are called constants.
'x, y, z' are used as variables.

We represent predicates using capital letters.
These stand for properties of the objects, and are placed in front of the object letters.
Pa: means object a has property P, and is said "P of a"
Pe: Emily is a philosopher
He: Emily is happy

III. **Exercises A**. Translate each sentence into predicate logic.

1. Alice is clever.
2. Bobby works hard.
3. Chuck plays tennis regularly.
4. Dan will see Erika on Tuesday at noon in the gym.

IV. **Quantifiers**

Consider: All philosophers are happy.
The subject of this sentence is not a specific philosopher, no specific object.
Similarly for “Something is made in the USA”
There is no a specific thing to which the sentence refers.
For sentences like these, we use quantifiers.

There are two quantifiers:
1) The existential quantifier: $(\exists x)$. It is used with any of the following expressions:
There exists an x, such that
For some x
There is an x
For at least one x
Something

2) The universal quantifier: (x). It is used with:
For all x
Everything

Some terms, like ‘anything’, can indicate either quantifier, depending on the context:
In ‘If anything is missing, you’ll be sorry’, we use an existential quantifier.
In ‘Anything goes’, we use a universal quantifier.

Examples of simple translations using quantifiers:

Something is made in the USA:	$(\exists x)Ux$		
Everything is made in the USA:	$(x)Ux$		
Nothing is made in the USA:	$(x){\sim}Ux$	or	${\sim}(\exists x)Ux$

Note that all statements with quantifiers and negations can be translated in at least two different ways.

Most sentences get translated as (at least) two predicates:
One is used for the subject of the sentence.
One is used for the attribute of the sentence.
Universals tend to use conditionals (as main connective) to separate the subject from the attribute.
Existentials usually use conjunctions between the subject predicate and the attribute predicate.

More sample translations:

All persons are mortal:	$(x)(Px \supset Mx)$		
Some actors are vain:	$(\exists x)(Ax \cdot Vx)$		
Some gods aren’t mortal:	$(\exists x)(Gx \cdot {\sim}Mx)$		
No frogs are people:	$(x)(Fx \supset {\sim}Px)$	or	${\sim}(\exists x)(Fx \cdot Px)$

V. **Exercises B**. Translate each sentence into predicate logic.
1) All roads lead to Rome. (Rx, Lx)
2) Beasts eat their young. (Bx, Ex)
3) Everything worthwhile requires effort. (Wx, Rx)
4) Some jellybeans are black. (Jx, Bx)
5) Some jellybeans are not black.

VI. **Scope and Binding**

Compare:

(x)(Px ⊃ Qx)	and	(x)Px ⊃ Qx
Every P is Q		If everything is P, then x is Q

The scope of the quantifier is whatever term follows immediately.
If it's a bracket, then whatever is in the bracket is in the scope of the quantifier.
If it's a negation, then everything negated is in the scope of the quantifier.
Quantifiers bind every instance of their variable in its scope.
So, in the first, the 'x' in 'Qx' is bound, but in the second, it is not.

Examples of unbound terms

(x)Px ∨ Qx	'Qx' is not in the scope of the quantifier, so is unbound.
(∃x)(Px ∨ Qy)	'Qy' is in the scope of the quantifier, but 'y' is not the quantifier variable, so is unbound.

VII. **Many propositions contain more than two predicates**

Examples with more than one predicate in the subject part:

Some wooden desks are uncomfortable:	(∃x)[(Wx · Dx) · Ux]
All wooden desks are uncomfortable:	(x)[(Wx · Dx) ⊃ Ux]

Examples with more than one predicate in the attribute part:

Many applicants are untrained or inexperienced:	(∃x)[Ax · (Ux ∨ Ix)]
All applicants are untrained or inexperienced:	(x)[Ax ⊃ (~Tx ∨ ~Ix)]

Examples using 'only':

Only men have been presidents: (x)(Px ⊃ Mx)

That is, if it has been a president, it must have been a man, since all presidents have been men.
Note that 'only' inverts the antecedent and consequent.

Only famous men have been presidents:	(x)[(Px · Mx) ⊃ Fx]	or	(x)[(Px ⊃ (Mx · Fx)]
Only intelligent students understand Kant:	(x)[(Sx · Kx) ⊃ Ix]		

VIII. **Some sentences contain more than one quantifier**

If anything is damaged, then everyone in the house complains:	(∃x)Dx ⊃ (x)[(Ix · Px) ⊃ Cx]
Either all the gears are broken, or a cylinder is missing:	(x)(Gx ⊃ Bx) ∨ (∃x)(Cx · Mx)

IX. **Exercises C**. Translate each sentence into predicate logic.

1) Some jellybeans are tasty. (Jx, Tx)
2) Some black jellybeans are tasty. (Jx, Bx, Tx)
3) No green jellybeans are tasty. (Gx, Jx, Tx)
4) Some politicians are wealthy and educated. (Px, Wx, Ex)
5) All wealthy politicians are electable. (Wx, Px, Ex)
6) If all jellybeans are black then no jellybeans are red. (Jx, Bx, Rx)
7) If everything is physical then there are no ghosts. (Px, Gx)
8) Some one walked the dog, but no one washed the dishes. (Px, Wx, Dx)
9) Everyone can go home only if all the work is done. (Px, Gx, Wx, Dx)

X. **Exercises D**. Translate each sentence into predicate logic.
1. All mice are purple. (Mx, Px)
2. No mice are purple.
3. Some mice are purple.
4. Some mice are not purple.
5. Snakes are reptiles. (Sx, Rx)
6. Snakes are not all poisonous. (Sx, Px)
7. Children are present. (Cx, Px)
8. Executives all have secretaries. (Ex, Sx)
9. Only executives have secretaries.
10. All that glitters is not gold. (Gx, Ax)
11. Nothing in the house escaped destruction. (Hx, Ex)
12. Blessed is he that considers the poor. (Bx, Cx)
13. Some students are intelligent and hard working. (Sx, Ix, Hx)
14. He that hates dissembles with his lips, and lays up deceit within him. (Hx, Dx, Lx)
15. Everything enjoyable is either illegal, immoral, or fattening. (Ex, Lx, Mx, Fx)
16. Some medicines are dangerous if taken in excessive amounts. (Mx, Dx, Tx)
17. Some medicines are dangerous only if taken in excessive amounts.
18. Victorian houses are attractive (Vx, Hx, Ax)
19. Slow children are at play. (Sx, Cx, Px)
20. Any horse that is gentle has been well-trained. (Hx, Gx, Wx)
21. Only well-trained horses are gentle.
22. Only gentle horses have been well-trained.
23. A knowledgeable, inexpensive mechanic is hard to find. (Kx, Ex, Mx, Hx)
24. Dogs and cats chase birds and squirrels. (Dx, Cx, Bx, Sx)
25. If all survivors are women, then some women are fortunate. (Sx, Wx, Fx)
26. Some, but not all, of us got away. (Ux, Gx)
27. If all ripe bananas are yellow, then some yellow things are ripe. (Rx, Bx, Yx)
28. If any employees are lazy and some positions have no future, then some employees will not be successful. (Ex, Lx, Px, Fx, Sx)
29. No coat is waterproof unless it has been specially treated. (Cx, Wx, Sx)
30. A professor is a good lecturer if and only if he is both well-informed and entertaining. (Px, Gx, Wx, Ex)

XI. **Solutions**

Answers to Exercises A:
1. Ca
2. Wb
3. Pc
4. Sd

Answers to Exercises B:
1) $(x)(Rx \supset Lx)$
2) $(x)(Bx \supset Ex)$
3) $(x)(Wx \supset Rx)$
4) $(\exists x)(Jx \cdot Bx)$
5) $(\exists x)(Jx \cdot \sim Bx)$

Answers to Exercises C:

1) (∃x)(Jx · Tx)
2) (∃x)[(Bx · Jx) · Tx)]
3) (x)[(Gx · Jx) ⊃ ~Tx] or ~(∃x)[(Gx · Jx) · Tx]
4) (∃x)[Px · (Wx · Ex)]
5) (x)[(Wx · Px) ⊃ Ex]
6) (x)(Jx ⊃ Bx) ⊃ (x)(Jx ⊃ ~Rx) or (x)(Jx ⊃ Bx) ⊃ ~(∃x)(Jx · Rx)
7) (x)Px ⊃ ~(∃x)Gx
8) (∃x)(Px · Wx) · ~(∃x)(Px · Dx)
9) (x)(Px ⊃ Gx) ⊃ (x)(Wx ⊃ Dx)

Answers to Exercises D:

1. (x)(Mx ⊃ Px)
2. (x)(Mx ⊃ ~Px)
3. (∃x)(Mx · Px)
4. (∃x)(Mx · ~Px)
5. (x)(Sx ⊃ Rx)
6. ~(x)(Sx ⊃ Px) or (∃x)(Sx · ~Px)
7. (∃x)(Cx · Px)
8. (x)(Ex ⊃ Sx)
9. (x)(Sx ⊃ Ex)
10. ~(x)(Gx ⊃ Ax)
11. (x)(Hx ⊃ ~Ex)
12. (x)(Cx ⊃ Bx)
13. (∃x)[Sx · (Ix · Hx)]
14. (x)[Hx ⊃ (Dx · Lx)]
15. (x){Ex ⊃ [(~Lx ∨ ~Mx) ∨ Fx]
16. (∃x)[Mx · (Tx ⊃ Dx)]
17. (∃x)[Mx · (Dx ⊃ Tx)]
18. (x)[(Hx · Vx) ⊃ Ax]
19. (∃x)[(Cx · Sx) · Px]
20. (x)[(Hx · Gx) ⊃ Wx]
21. (x)[(Hx · Gx) ⊃ Wx]
22. (x)[(Hx · Wx) ⊃ Gx]
23. (x){[(Kx · ~Ex) · Mx] ⊃ Hx}
24. (x)[(Dx ∨ Cx) ⊃ (Bx · Sx)]
25. (x)(Sx ⊃ Wx) ⊃(∃x)(Wx · Fx)
26. (∃x)(Ux · Gx) · ~(x)(Ux ⊃ Gx)
27. (x)[(Bx · Rx) ⊃ Yx] ⊃ (∃x)(Yx · Rx)
28. [(∃x)(Ex · Lx) · (∃x)(Px · ~Fx)] ⊃ (∃x)(Ex ·~Sx)
29. (x)[Cx ⊃ (~Wx ∨ Sx)] or (x)[Cx ⊃ (~Sx ⊃ ~Wx)] or (x)[(Cx · Wx) ⊃ Sx] or ~(∃x)(Cx · Wx · ~Sx)
30. (x){Px ⊃ [Gx ≡ (Wx · Ex)]}