Philosophy 109, Modern Logic, Queens College
Russell Marcus, Instructor
email: philosophy@thatmarcusfamily.org
website: http://philosophy.thatmarcusfamily.org
Office phone: (718) 997-5287

Conditional and Indirect proof in Predicate Logic, §8.4

I. **A problem arising from using CP and IP in Predicate Logic**

With unrestricted CP we can construct the following derivation:

1. (x)Rx ⊃ (x)Bx	Premise
|2. Rx	ACP
|3. (x)Rx	2, UG
|4. (x)Bx	1, 3, MP
|5. Bx	4, UI
6. Rx ⊃ Bx	2-5, CP
7. (x)(Rx ⊃ Bx)	6, UG

This would mean that we could prove that everything red is blue (the conclusion) from 'If everything is red, then everything is blue' (the premise).
But that premise can be true while the conclusion is false.
So, the derivation should be invalid.

Moral of the story: we must restrict conditional proof.
The problem is in step 3.
We may not generalize on x within the assumption.
The assumption just means that a random thing is R, not that everything is R.
We may generalize after we've discharged, as in line 7.

The **Restriction on (CP) and (IP)**:
Never UG within an assumption on a variable that's free in the first line of the assumption.

II. **Examples of CP and IP in Predicate Logic**

One of two typical uses of (CP)

1. (x)[Ax ⊃ (Bx ∨ Dx)]		
2. (x)~Bx	/(x)(Ax ⊃ Dx)	
|3. Ay	ACP	Pick a random object that has property A.
|4. Ay ⊃ (By ∨ Dy)	1, UI	
|5. By ∨ Dy	4, 3, MP	
|6. ~By	2, UI	
|7. Dy	5, 6, DS	
8. Ay ⊃ Dy	3-7, CP	Given any object, if it has A, it provably has D.
9. (x)(Ax ⊃ Dx)	8, UG	Since we are no longer within the scope of the assumption, we may UG.
QED		

So, to prove statements of the form (x)(Px ⊃ Qx):
Assume Px.
Derive Qx.
Discharge (Px ⊃ Qx).
Then (UG).

Another typical use of CP:

1. (x)[Px ⊃ (Qx · Rx)]		
2. (x)(Rx ⊃ Sx)	/ (∃x)Px ⊃ (∃x)Sx	
| 3. (∃x)Px	ACP	Pick a random object that has property A.
| 4. Pa	3, EI	
| 5. Pa ⊃ (Qa · Ra)	1, UI	
| 6. Qa · Ra	5, 4, MP	
| 7. Ra	6, Com, Simp	
| 8. Ra ⊃ Sa	2, UI	
| 9. Sa	8, 7, MP	
| 10. (∃x) Sx	9, EG	
11. (∃x)Px ⊃ (∃x)Sx	3-10, CP	

QED

Indirect Proof works basically in the same way as in propositional logic.
But the same restriction on CP holds for IP, too.
The restriction holds any time one makes an assumption.

Typical use of (IP):

1. (x)[(Ax ∨ Bx) ⊃ Ex]		
2. (x)[(Ex ∨ Dx) ⊃ ~Ax]	/(x)~Ax	
| 3. ~(x)~Ax	AIP	Remember, you're looking for a contradiction.
| 4. (∃x)Ax	3, CQ	
| 5. Aa	4, EI	
| 6. (Ea ∨ Da) ⊃ ~Aa	2, UI	
| 7. ~(Ea ∨ Da)	6, 5, DN, MT	
| 8. ~Ea · ~Da	7, DM	
| 9. ~Ea	8, Simp	
| 10. (Aa ∨ Ba) ⊃ Ea	1, UI	
| 11. ~(Aa ∨ Ba)	10, 9, MT	
| 12. ~Aa · ~Ba	11, DM	
| 13. ~Aa	12, Simp	
| 14. Aa · ~Aa	5, 13, Conj	
15. (x)~Ax	3-13, IP, DN	

QED

Note that with CP, sometimes you only assume part of a line, then generalize outside the assumption, but with IP, you almost always assume the negation of the whole conclusion.

III. **Exercises**. Derive the conclusions of the following arguments:

1)
1. (x)(Fx ⊃ Gx)
2. (x)(Fx ⊃ Hx) / (x)[Fx ⊃ (Gx · Hx)]

2)
1. (x)(Jx ⊃ ~Kx) / ~(∃x)(Jx · Kx)

3)
1. (x)(Rx ⊃ Bx) / (x)Rx ⊃ (x)Bx

4)
1. (x)(Lx ⊃ Mx)
2. ~(∃x)Lx ⊃ (∃x)Mx / ~(x)~Mx

Solutions may vary.