Philosophy 109, Introduction to Modern Logic, Queens College Russell Marcus, Instructor email: <u>philosophy@thatmarcusfamily.org</u> website: <u>http://philosophy.thatmarcusfamily.org</u> Office phone: (718) 997-5287

Relational predicates and overlapping quantifiers Handout

I. Prove:

1. Bob is taller than Charles.

2. Andrew is taller than Bob.

3. For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z. Therefore, Andrew is taller than Charles.

II. Translate each:1. John loves Mary. (Lxy: x loves y)

2. Tokyo isn't smaller than New York. (Sxy: x is smaller than y)

3. Marco was introduced to Erika by Paco. (Ixyz: x introduced y to z)

4. America took California from Mexico. (Txyz: x was taken by y from z)

III. Introducing quantifiers.

1. Joe is bigger than something. (Bxy: x is bigger than y)

2. Something is bigger than Joe.

3. Joe is bigger than everything.

4. Everything is bigger than Joe.

5. Everything loves something.

6. Something loves everything.

V. More complex examples.

1. Something taught Plato. (Txy: x taught y)

2. Someone taught Plato.

3. Plato taught everyone.

4. Everyone knows something. (Kxy: x knows y)

5. Everyone is wiser than someone. (Wxy: x is wiser than y)

6. Someone is wiser than everyone.

7. Some financier is richer than everyone. (Fx, Rxy: x is richer than y)

8. No deity is weaker than some human. (Dx, Hx, Wxy: x is weaker than y)

9. Honest candidates are always defeated by dishonest candidates. (Hx, Cx, Dxy: x defeats y)

10. No mouse is mightier than himself. (Mx, Mxy: x is mightier than y)

11. Everyone buys something from some store. (Px, Sx, Bxyz: x buys y from z)

12. There is a store from which everyone buys something.

13. No store has everyone for a customer.

V. On the order of quantifiers: 1. Everyone loves everyone: $(x)[Px \supset (y)(Py \supset Lxy)]$ $(x)(y)[(Px \cdot Py) \supset Lxy]$ $(y)(x)[(Px \cdot Py) \supset Lxy]$

2. Someone loves someone $(\exists x)[Px \cdot (\exists y)(Py \cdot Lxy)]$ $(\exists x)(\exists y)[(Px \cdot Py) \cdot Lxy]$ $(\exists y)(\exists x)[(Px \cdot Py) \cdot Lxy]$

3. $(x)(\exists y)[(Px \cdot Py) \cdot Lxy]$ Everyone loves someone

4. $(\exists y)(x)[(Px \cdot Py) \supset Lxy]$ Someone is loved by everyone 4a. $(x)[Px \supset (\exists y)(Py \cdot Lxy)]$ 4b. $(\exists y)[Py \cdot (x)(Px \supset Lxy)]$

5. $(\exists x)(y)[(Px \cdot Py) \supset Lxy]$ Someone loves everyone

6. $(y)(\exists x)[(Px \cdot Py) \cdot Lxy]$ Everyone is loved by someone

VI. Translate:1. Everyone loves something. (Px, Lxy)

2. No one knows everything. (Px, Kxy)

- 3. No one knows everyone.
- 4. Every woman is stronger than some man. (Wx, Mx, Sxy: x is stronger than y)
- 5. No cat is smarter than any horse. (Cx, Hx, Sxy: x is smarter than y)
- 6. Dead men tell no tales. (Dx, Mx, Tx, Txy: x tells y)
- 7. There is a city between New York and Washington. (Cx, Bxyz: y is between x and z)
- 8. Everyone gives something to someone. (Px, Gxyz: y gives x to z)
- 9. A dead lion is more dangerous than a live dog. (Ax: x is alive, Lx, Dx, Dxy: x is more dangerous than y)
- 10. A lawyer who pleads his own case has a fool for a client. (Lx, Fx, Pxy: x pleads y's case; Cxy: y is a client of
 - x)

VII. Deduce:

1)	1. (x)(Cax \supset Dxb)	
	2. $(\exists x)Dxb \supset (\exists y)Dby$	$/(\exists x)Cax \supset (\exists y)Dby$

- 2) 1. $(x)[Ex \supset (y)(Fy \supset Gxy)]$ 2. $(\exists x)[Ex \cdot (\exists y) \sim Gxy]$ / $(\exists x) \sim Fx$
- 3) 1. $(\exists x)Ax \supset (\exists x)Bx$ /($\exists y$)(x)($Ax \supset By$)
- 4) 1. $(x)[Mx \supset (y)(Ny \supset Oxy)]$ 2. $(x)[Px \supset (y)(Oxy \supset Qy)]$ /($\exists x)(Mx \cdot Px) \supset (y)(Ny \supset Qy)$