17. ~(x)(Wx ⊃ ~Px)	16, CQ
18. (∃x)(Sx • Px)	2, 17, Com, DS
19. Sn • Pn	18, EI
20. Sn ⊃ ~Pn	10, UI
21. Sn	19, Simp
22. ~Pn	20, 21, MP
23. Pn	19, Com, Simp
24. Pn • ~Pn	22, 23, Conj
25. ~(∃x)[(Rx • Px) • Wx]	3-24, IP

(10)

1. (∃x)(Gx • Px) ∨ (∃x)(Ax • Px)	
2. (∃x)Px ⊃ (∃x)[Ax • (Cx • Dx)]	/ (∃x)(Dx • Cx)
3. ~(∃x)(Dx • Cx)	AIP
4. (x)~(Dx • Cx)	3, CQ
5. ~(Dx • Cx)	4, UI
6. ~(Cx • Dx)	5, Com
7. ~(Cx • Dx) ∨ ~Ax	6, Add
8. ~Ax ∨ ~(Cx • Dx)	7, Com
9. ~[Ax • (Cx • Dx)]	8, DM
10. (x)~[Ax • (Cx • Dx)]	9, UG
11. ~(∃x)[(Ax • (Cx • Dx)]	10, CQ
12. ~(∃x)Px	2, 11, MT
13. (x)~Px	12, CQ
14. ~Px	13, UI
15. ~Px ∨ ~Gx	14, Add
16. ~Gx ∨ ~Px	15, Com
17. ~(Gx • Px)	16, DM
18. (x)~(Gx • Px)	17, UG
19. ~(∃x)(Gx • Px)	18, CQ
20. (∃x)(Ax • Px)	1, 19, DS
21. Am • Pm	20, EI
22. Pm	21, Com, Simp
23. ~Pm	13, UI
24. Pm • ~Pm	22, 23, Conj
25. ~~(∃x)(Dx • Cx)	3-24, IP
26. (∃x)(Dx • Cx)	25, DN

(An alternate method appears in the back of the text.)

Exercise 8.5
Part I

1. All cats are animals.
   No cats are dogs.
   Therefore, no dogs are animals.

Exercise 8.5

2. Some animals are fish.
   All cats are animals.
   Therefore, some cats are fish.

3. All women are humans.
   Chuck Norris is a human.
   Therefore, Chuck Norris is a woman.

4. Some mammals are dogs.
   Some mammals write books.
   Therefore, some mammals are dogs that write books.

5. Everything is either an animal, vegetable or mineral.
   Therefore, either everything is an animal, or everything is a vegetable or everything is a mineral

6. All dogs are either animals or sharks.
   All All animals that are sharks are fish.
   Therefore, all dogs are fish.

7. There are flowers.
   There are dogs.
   No flowers are animals.
   Therefore. some dogs are not animals.

8. Anything that is either a dog or a cat is an animal.
   Any animal that has a trunk is an elephant.
   Therefore, all dogs are elephants.

9. All large cats are mammals.
   All large dogs are animals.
   Therefore, all large animals are mammals.

10. Some mammals are felines.
    Some animals are not felines.
    All mammals are animals.
    Therefore, some feline animals are not mammals.

Part II

(1) 1. $(x)(Ax \supset Bx)$
    2. $(x)(Ax \supset Cx)$ $\quad$ / $(x)(Bx \supset Cx)$

For a universe consisting of one member, we have,

Aa ⊃ Ba	/	Aa ⊃ Ca	//	Ba ⊃ Ca
F T T		F T F		T F F

(2) 1. $(x)(Ax \vee Bx)$
2. $\sim An$ $\quad$ / $(x)Bx$

For a universe consisting of two members, we have,

$(An \vee Bn) \bullet (Aa \vee Ba)$ / $\sim An$ // $Bn \bullet Ba$
F T T T T T F $\quad$ T F $\quad$ T F F

(3) 1. $(\exists x)Ax \vee (\exists x)Bx$
2. $(\exists x)Ax$ $\quad$ / $(\exists x)Bx$

For a universe consisting of one member, we have,

$Aa \vee Ba$ / Aa // Ba
T T F $\quad$ T $\quad$ F

(4) 1. $(x)(Ax \supset Bx)$
2. $(\exists x)Ax$ $\quad$ / $(x)Bx$

For a universe consisting of two members, we have,

$(Aa \supset Ba) \bullet (Ab \supset Bb)$ / $Aa \vee Ab$ // $Ba \bullet Bb$
T T T T F T F $\quad$ T T F $\quad$ T F F

(5) 1. $(x)[Ax \supset (Bx \vee Cx)]$
2. $(\exists x)Ax$ $\quad$ /$(\exists x)Bx$

For a universe consisting of one member, we have,

$Aa \supset (Ba \vee Ca)$ / Aa // Ba
T T F T T $\quad$ T $\quad$ F

(6) 1. $(\exists x)Ax$
2. $(\exists x)Bx$ $\quad$ / $(\exists x)(Ax \bullet Bx)$

For a universe consisting of two members, we have,

$Aa \vee Ab$ / $Ba \vee Bb$ // $(Aa \bullet Ba) \vee (Ab \bullet Bb)$
T T F $\quad$ F T T $\quad$ T F F F F F T

(7) 1. $(x)(Ax \supset Bx)$
2. $(\exists x)Bx \supset (\exists x)Cx$ $\quad$ / $(x)(Ax \supset Cx)$

Exercise 8.5

For a universe consisting of two members, we have,

$$
\begin{array}{l}
(Aa \supset Ba) \cdot (Ab \supset Bb) \;/\; (Ba \vee Bb) \supset (Ca \vee Cb) \;//\; (Aa \supset Ca) \cdot (Ab \supset Cb) \\
\text{T T T T T T T} \quad \text{T T T T T T F} \quad \text{T T T F T F F}
\end{array}
$$

(8) 1. $(\exists x)(Ax \cdot Bx) \equiv (\exists x)Cx$
2. $(x)(Ax \supset Bx)$ $\quad$ / $(x)Ax \equiv (\exists x)Cx$

For a universe consisting of two members, we have,

$$
\begin{array}{l}
[(Aa \cdot Ba) \vee (Ab \cdot Bb)] \equiv (Ca \vee Cb) \;/\; (Aa \supset Ba) \cdot (Ab \supset Bb) \\
\text{T T T T F F F T T T} \quad \text{T T T T F T F}
\end{array}
$$

$$
\begin{array}{l}
//\; (Aa \cdot Ab) \equiv (Ca \vee Cb) \\
\quad \text{T F F F T T}
\end{array}
$$

(9) 1. $(\exists x)(Ax \cdot \sim Bx)$
2. $(\exists x)(Bx \cdot \sim Ax)$ $\quad$ / $(x)(Ax \vee Bx)$

For a universe consisting of three members, we have,

$$
\begin{array}{l}
[(Aa \cdot \sim Ba) \vee (Ab \cdot \sim Bb)] \vee (Ac \cdot \sim Bc) \\
\text{T T T F T F F F T T F F T F}
\end{array}
$$

$$
\begin{array}{l}
/\; [(Ba \cdot \sim Aa) \vee (Bb \cdot \sim Ab)] \vee (Bc \cdot \sim Ac) \\
\quad \text{F F F T T T T T F T F F T F}
\end{array}
$$

$$
\begin{array}{l}
//\; [(Aa \vee Ba) \cdot (Ab \vee Bb)] \cdot (Ac \vee Bc) \\
\quad \text{T T F T F T T F F F F}
\end{array}
$$

(10) 1. $(\exists x)(Ax \cdot Bx)$
2. $(\exists x)(\sim Ax \cdot \sim Bx)$ $\quad$ / $(x)(Ax \equiv Bx)$

For a universe consisting of three members, we have,

$$
\begin{array}{l}
(Aa \cdot Ba) \vee [(Ab \cdot Bb) \vee (Ac \cdot Bc)] \\
\text{T T T T T F F T F F F}
\end{array}
$$

$$
\begin{array}{l}
/\; (\sim Aa \cdot \sim Ba) \vee [(\sim Ab \cdot \sim Bb) \vee (\sim Ac \cdot \sim Bc)] \\
\quad \text{F T F F T T F T F T F T T F T T F}
\end{array}
$$

$$
\begin{array}{l}
//\; (Aa \equiv Ba) \cdot [(Ab \equiv Bb) \cdot (Ac \equiv Bc)] \\
\quad \text{T T T F T F F F F T F}
\end{array}
$$

Part III

(1) 1. (x)[Vx • Px) ⊃ (Ax • Mx)
2. (∃x)(Vx • Ox) / (∃x)(Mx • Ax)

For a universe consisting of one member, we have,

(Va • Pa) ⊃ (Aa • Ma) / Va • Oa // Ma • Aa
T F F T F F F T T T F F F

(2) 1. (x)[(Px ∨ Hx) ⊃ Mx]
2. Pa / (x)Mx

For a universe consisting of two members, we have,

[(Pa ∨ Ha) ⊃ Ma] • [(Pc ∨ Hc) ⊃ Mc] / Pa // Ma • Mc
T T T T T F F F T F T T F F

(3) 1. (∃x)Ox ⊃ (∃x)Bx
2. (∃x)Cx ⊃ (∃x)Fx
3. Oa • Ca / (∃x)(Bx • Fx)

For a universe consisting of two members, we have,

(Oa ∨ Ob) ⊃ (Ba ∨ Bb) / (Ca ∨ Cb) ⊃ (Fa ∨ Fb)
T T T T T F T T T F T T

/ Oa • Ca // (Ba • Fa) ∨ (Bb • Fb)
T T T T F F F F F T

(4) 1. (x)(Tx ⊃ Hx)
2. (∃x)(Tx • Hx) ⊃ (∃x)(Px • Ox) / (x)(Tx ⊃ Ox)

For a universe consisting of two members, we have,

(Ta ⊃ Ha) • (Tb ⊃ Hb) / [(Ta • Ha) ∨ (Tb • Hb)] ⊃ [(Pa • Oa) ∨ (Pb • Ob)]
T T T T T T T T T T T T T T T F F F T T T T

// (Ta ⊃ Oa) • (Tb ⊃ Ob)
T F F F T T T

(5) 1. (x)[(Cx ∨ Vx) ⊃ Mx]
2. (∃x)(Vx • ~Cx)
3. (∃x)(Cx • ~Vx) / (x)Mx

Exercise 8.6

For a universe consisting of three members, we have,

$\{[(Ca \vee Va) \supset Ma] \bullet [(Cb \vee Vb) \supset Mb]\} \bullet [(Cc \vee Vc) \supset Mc]$
T T F T T T F T T T T T F F F T F

$/ [(Va \bullet {\sim} Ca) \vee (Vb \bullet {\sim} Cb)] \vee (Vc \bullet {\sim} Cc)$
F F F T T T T T F T F F T F

$/ [(Ca \bullet {\sim} Va) \vee (Cb \bullet {\sim} Vb)] \vee (Cc \bullet {\sim} Vc)$
T T T F T F F F T T F F T F

$// (Ma \bullet Mb) \bullet Mc$
T T T F F

Exercise 8.6
Part I

1. Rcp

2. $(x)(Rxp \supset Ex)$

3. $Fje \vee Fjc$

4. $(\exists x)Fxj \supset Fmj$

5. $(x)(Tjx \supset Gx)$

6. $(\exists x)(Mx \bullet Tnx)$

7. $(x)[Px \supset (\exists y)Sxy]$

8. $(\exists x)[Px \bullet (y){\sim}Sxy]$

9. $(x)[Px \supset (\exists y){\sim}Sxy]$

10. $(\exists x)[Px \bullet (y)Sxy]$

11. $(x)[(Dx \bullet Srx) \supset Gx]$ (assuming the hotel also serves food)
 $(x)[Srx \supset (Gx \bullet Dx)]$ (assuming the hotel serves no food)

12. $(x)(Pcx \supset Acx)$

13. $(\exists x)[(Cx \bullet Lx) \bullet Dpx]$

14. $(x)[(Cx \bullet Lx) \supset Djx]$

15. $(x)(Isx \supset Fxs)$

16. $(\exists x)(Fxc \cdot Icx)$

17. $(\exists x)[Px \cdot (y)(Ixy \supset Bxy)]$

18. $(\exists x)\{Px \cdot (y)[(Py \cdot Syx) \supset Sxy]\}$

19. $(x)\{Px \supset (\exists y)[Py \cdot (Mxy \supset Axy)]\}$

20. $(\exists x)\{Px \cdot (y)[(Py \cdot Mxy) \supset Axy]\}$

21. $(\exists x)[Px \cdot (y)(Axy \supset Ty)]$

22. $(\exists x)\{Px \cdot (y)[(Ty \cdot Sxy) \supset Axy]\}$

23. $(\exists x)Cx \supset (\exists x)(Cx \cdot Px)$

24. $(x)\{Cx \supset [(y)(Ry \supset Vy) \supset Px]\}$

25. $(x)\{Lx \supset (y)[(Wy \cdot Cy) \supset Rxy]\}$

26. $(\exists x)\{Lx \cdot (y)[(Py \cdot \sim Ryy) \supset Rxy]\}$

27. $(\exists x)\{(Cx \cdot Tx) \cdot (y)[(By \cdot Ly) \supset Rxy]\}$

28. $(x)\{(Cx \cdot Fx) \supset (y)[(By \cdot Ly) \supset Rxy]\}$

29. $(x)\{(Sx \cdot Dx) \supset [(y)(Ty \supset \sim By) \supset Hx]\}$

30. $(\exists x)(Sx \cdot Dx) \supset (\exists x)[(Sx \cdot Dx) \cdot Hx]$

Part II

(1)

1.	$(x)[Ax \supset (y)Bxy]$	
2.	Am	/ $(y)Bmy$
3.	$Am \supset (y)Bmy$	1, UI
4.	$(y)Bmy$	2, 3, MP

(2)

1.	$(x)[Ax \supset (y)(By \supset Cxy)]$	
2.	$Am \cdot Bn$	/ Cmn
3.	$Am \supset (y)(By \supset Cmy)$	1, UI
4.	Am	2, Simp
5.	$(y)(By \supset Cmy)$	3, 4, MP
6.	$Bn \supset Cmn$	5, UI
7.	Bn	2, Com, Simp
8.	Cmn	6, 7, MP

(7)	1. (∃x)[Ax • (y)(Ay ⊃ Bxy)]	/ (∃x)Bxx
	2. Am • (y)(Ay ⊃ Bmy)	1, EI
	3. Am	2, Simp
	4. (y)(Ay ⊃ Bmy)	2, Com, Simp
	5. Am ⊃ Bmm	4, UI
	6. Bmm	3, 5, MP
	7. (∃x)Bxx	6, EG

(8)	1. (∃x)[Ax • (y)(By ⊃ Cxy)]	
	2. (x)(∃y)(Ax ⊃ By)	/ (∃x)(∃y)Cxy
	3. Am • (y)(By ⊃ Cmy)	1, EI
	4. Am	3, Simp
	5. (y)(By ⊃ Cmy)	3, Com, Simp
	6. (∃y)(Am ⊃ By)	2, UI
	7. Am ⊃ Bn	6, EI
	8. Bn	4, 7, MP
	9. Bn ⊃ Cmn	5, UI
	10. Cmn	8, 9, MP
	11. (∃y)Cmy	10, EG
	12. (∃x)(∃y)Cxy	11, EG

(9)	1. (∃x)(y)(Axy ⊃ Bxy)	
	2. (x)(∃y)~Bxy	/ ~(x)(y)Axy
	3. (y)(Amy ⊃ Bmy)	1, EI
	4. (∃y)~Bmy	2, UI
	5. ~Bmn	4, EI
	6. Amn ⊃ Bmn	3, UI
	7. ~Amn	5, 6, MT
	8. (∃y)~Amy	7, EG
	9. (∃x)(∃y)~Axy	8, EG
	10. (∃x)~(y)Axy	9, CQ
	11. ~(x)(y)Axy	10, CQ

(10)	1. (x)(∃y)Axy ⊃ (x)(∃y)Bxy	
	2. (∃x)(y)~Bxy	/ (∃x)(y)~Axy
	3. (∃x)~(∃y)Bxy	2, CQ
	4. ~(x)(∃y)Bxy	3, CQ
	5. ~(x)(∃y)Axy	1, 4, MT
	6. (∃x)~(∃y)Axy	5, CQ
	7. (∃x)(y)~Axy	6, CQ

(11)	1. (∃x){Ax • [(∃y)By ⊃ Cx]}	
	2. (x)(Ax ⊃ Bx)	/ (∃x)Cx
	3. Am • [(∃y)By ⊃ Cm]	1, EI
	4. Am	3, Simp
	5. Am ⊃ Bm	2, UI

6. Bm	4, 5, MP
7. $(\exists y)By$	6, EG
8. $(\exists y)By \supset Cm$	3, Com, Simp
9. Cm	7, 8, MP
10. $(\exists x)Cx$	9, EG

(12)

1. $(\exists x)(y)[(Ay \cdot By) \supset Cxy]$	
2. $(y)(Ay \supset By)$	/ $(y)[Ay \supset (\exists x)Cxy]$
3. Ay	ACP
4. $Ay \supset By$	2, UI
5. By	3, 4, MP
6. $Ay \cdot By$	3, 5, Conj
7. $(y)[(Ay \cdot By) \supset Cmy]$	1, EI
8. $(Ay \cdot By) \supset Cmy$	7, UI
9. Cmy	6, 8, MP
10. $(\exists x)Cxy$	9, EG
11. $Ay \supset (\exists x)Cxy$	3-10, CP
12. $(y)[Ay \supset (\exists x)Cxy]$	11, UG

(13)

1. $(\exists x)\{Ax \cdot (y)[(By \vee Cy) \supset Dxy]\}$	
2. $(\exists x)Ax \supset (\exists y)By$	/ $(\exists x)(\exists y)Dxy$
3. $Am \cdot (y)[(By \vee Cy) \supset Dmy]$	1, EI
4. Am	3, Simp
5. $(\exists x)Ax$	4, EG
6. $(\exists y)By$	2, 5, MP
7. Bn	6, EI
8. $(y)[(By \vee Cy) \supset Dmy]$	3, Com, Simp
9. $(Bn \vee Cn) \supset Dmn$	8, UI
10. $Bn \vee Cn$	7, Add
11. Dmn	9, 10, MP
12. $(\exists y)Dmy$	11, EG
13. $(\exists x)(\exists y)Dxy$	12, EG

(14)

1. $(x)\{Ax \supset [(\exists y)(By \cdot Cy) \supset Dx]\}$	
2. $(x)(Bx \supset Cx)$	/ $(x)[Ax \supset (Bx \supset Dx)]$
3. Ax	ACP
4. Bx	ACP
5. $Ax \supset [(\exists y)(By \cdot Cy) \supset Dx]$	1, UI
6. $(\exists y)(By \cdot Cy) \supset Dx$	3, 5, MP
7. $Bx \supset Cx$	2, UI
8. Cx	4, 7, MP
9. $Bx \cdot Cx$	4, 8, Conj
10. $(\exists y)(By \cdot Cy)$	9, EG
11. Dx	6, 10, MP
12. $Bx \supset Dx$	4-11, CP
13. $Ax \supset (Bx \supset Dx)$	3-12, CP
14. $(x)[Ax \supset (Bx \supset Dx)]$	13, UG

(15)

1. (∃x)(y)(Ayx ⊃ ~Axy)	/ ~(x)Axx
2. (y)(Aym ⊃ ~Amy)	1, EI
3. Amm ⊃ ~Amm	2, UI
4. ~Amm ∨ ~Amm	3, Impl
5. ~Amm	4, Taut
6. (∃x)~Axx	5, EG
7. ~(x)Axx	6, CQ

(16)

1. (x)(∃y)(Ax · By)	/ (∃y)(x)(Ax · By)
2. (∃y)(Ax · By)	1, UI
3. Ax · Ba	2, EI
4. Ax	3, Simp
5. (x)Ax	4, UG
6. Az	5, UI
7. Ba	3, Com, Simp
8. Az · Ba	6, 7, Conj
9. (x)(Ax · Ba)	8, UG
10. (∃y)(x)(Ax · By)	9, EG

(17)

1. (x)(∃y)(Ax ∨ By)	/ (∃y)(x)(Ax ∨ By)
2. ~(∃y)(x)(Ax ∨ By)	AIP
3. (y)~(x)(Ax ∨ By)	2, CQ
4. (y)(∃x)~(Ax ∨ By)	3, CQ
5. (∃x)~(Ax ∨ By)	4, UI
6. ~(Am ∨ By)	5, EI
7. ~Am • ~By	6, DM
8. (∃y)(Am ∨ By)	1, UI
9. Am ∨ Bn	8, EI
10. ~Am	7, Simp
11. Bn	9, 10, DS
12. ~By	7, Com, Simp
13. (y)~By	12, UG
14. ~Bn	13, UI
15. Bn • ~Bn	11, 14, Conj
16. (∃y)(x)(Ax ∨ By)	2-15, IP, DN

(Note: Attempts to use direct proof on this argument violate the second restriction on UG.)

(18)

1. (x)[Ax ⊃ (∃y)(By • Cxy)]	
2. (∃x)[Ax • (y)(By ⊃ Dxy)]	/ (∃x)(∃y)(Cxy • Dxy)
3. Am • (y)(By ⊃ Dmy)	2, EI
4. Am ⊃ (∃y)(By • Cmy)	1, UI
5. Am	3, Simp
6. (∃y)(By • Cmy)	4, 5, MP

7. $Bn \cdot Cmn$	6, EI
8. $(y)(By \supset Dmy)$	3, Com, Simp
9. $Bn \supset Dmn$	8, UI
10. Bn	7, Simp
11. Dmn	9, 10, MP
12. Cmn	7, Com, Simp
13. $Cmn \cdot Dmn$	11, 12, Conj
14. $(\exists y)(Cmy \cdot Dmy)$	13, EG
15. $(\exists x)(\exists y)(Cxy \cdot Dxy)$	14, EG

(19)

1. $(x)(\exists y)Axy \vee (x)(y)Bxy$	
2. $(x)(\exists y)(Cx \supset {\sim}Bxy)$	/ $(x)(\exists y)(Cx \supset Axy)$
3. Cx	ACP
4. $(\exists y)(Cx \supset {\sim}Bxy)$	2, UI
5. $Cx \supset {\sim}Bxm$	4, EI
6. ${\sim}Bxm$	3, 5, MP
7. $(\exists y){\sim}Bxy$	6, EG
8. $(\exists x)(\exists y){\sim}Bxy$	7, EG
9. $(\exists x){\sim}(y)Bxy$	8, CQ
10. ${\sim}(x)(y)Bxy$	9, CQ
11. $(x)(\exists y)Axy$	1, 10, Com, DS
12. $(\exists y)Axy$	11, UI
13. Axn	12, EI
14. $Cx \supset Axn$	3-13, CP
15. $(\exists y)(Cx \supset Axy)$	14, EG
16. $(x)(\exists y)(Cx \supset Axy)$	15, UG

(20)

1. $(x)(y)[Axy \supset (Bx \cdot Cy)]$	
2. $(x)(y)[(Bx \vee Dy) \supset {\sim}Axy]$	/ ${\sim}(\exists x)(\exists y)Axy$
3. $(\exists x)(\exists y)Axy$	AIP
4. $(\exists y)Amy$	3, EI
5. Amn	4, EI
6. $(y)[Amy \supset (Bm \cdot Cy)]$	1, UI
7. $Amn \supset (Bm \cdot Cn)$	6, UI
8. $Bm \cdot Cn$	5, 7, MP
9. Bm	8, Simp
10. $(y)[(Bm \vee Dy) \supset {\sim}Amy]$	2, UI
11. $(Bm \vee Dn) \supset {\sim}Amn$	10, UI
12. $Bm \vee Dn$	9, Add
13. ${\sim}Amn$	11, 12, MP
14. $Amn \cdot {\sim}Amn$	5, 13, Conj
15. ${\sim}(\exists x)(\exists y)Axy$	3-14, IP

Part III

(1) 1. (x)[Px ⊃ (y)(Ay ⊃ Oxy)]
2. Pj • ~Ojm / ~Am
3. Pj ⊃ (y)(Ay ⊃ Ojy) 1, UI
4. Pj 2, Simp
5. (y)(Ay ⊃ Ojy) 3, 4, MP
6. Am ⊃ Ojm 5, UI
7. ~Ojm 2, Com, Simp
8. ~Am 6, 7, MT

(2) 1. (x)[(Fxm ∨ Fxp) ⊃ Rx]
2. (∃x)Fxm ⊃ Fem / Fam ⊃ Re
3. Fam ACP
4. (∃x)Fxm 3, EG
5. Fem 2, 4, MP
6. (Fem ∨ Fep) ⊃ Re 1, UI
7. Fem ∨ Fep 5, Add
8. Re 6, 7, MP
9. Fam ⊃ Re 3-8, CP

(3) 1. (x)(Hx ⊃ Ax) / (x)[(∃y)(Hy • Oxy) ⊃ (∃y)(Ay • Oxy)]
2. (∃y)(Hy • Oxy) ACP
3. Hm • Oxm 2, EI
4. Hm 3, Simp
5. Hm ⊃ Am 1, UI
6. Am 4, 5, MP
7. Oxm 3, Com, Simp
8. Am • Oxm 6, 7, Conj
9. (∃y)(Ay • Oxy) 8, EG
10. (∃y)(Hy • Oxy) ⊃ (∃y)(Ay • Oxy) 2-9, CP
11. (x)[(∃y)(Hy • Oxy) ⊃ (∃y)(Ay • Oxy)] 10, UG

(4) 1. Po
2. (x)[(Px • Cx) ⊃ Sox]
3. (x)(Px ⊃ ~Sxx) / ~Co
4. Co AIP
5. (Po • Co) ⊃ Soo 2, UI
6. Po • Co 1, 4, Conj
7. Soo 5, 6, MP
8. Po ⊃ ~Soo 3, UI
9. ~Soo 1, 8, MP
10. Soo • ~Soo 7, 9, Conj
11. ~Co 4-10, IP

(5)

1. (x){(Hx • Px) ⊃ [(y)(By ⊃ Cy) ⊃ Rx]}	
2. (∃x)[(Hx • Px) • ~Rx]	/ (∃x)(Bx • ~Cx)
3. (Hm • Pm) • ~Rm	2, EI
4. Hm • Pm	3, Simp
5. (Hm • Pm) ⊃ [(y)(By ⊃ Cy) ⊃ Rm]	1, UI
6. (y)(By ⊃ Cy) ⊃ Rm	4, 5, MP
7. ~Rm	3, Com, Simp
8. ~(y)(By ⊃ Cy)	6, 7, MT
9. (∃y)~(By ⊃ Cy)	8, CQ
10. ~(Bn ⊃ Cn)	9, EI
11. ~(~Bn ∨ Cn)	10, Impl
12. Bn • ~Cn	11, Dm, DN
13. (∃x)(Bx • ~Cx)	12, EG

(6)

1. (x)[(Px • ~Cxx) ⊃ Crx]	
2. Pr	/ Crr
4. (Pr • ~Crr) ⊃ Crr	1, UI
5. Pr ⊃ (~Crr ⊃ Crr)	4, Exp
6. ~Crr ⊃ Crr	2, 5, MP
7. Crr ∨ Crr	6, Impl, DN
8. Crr	7, Taut

(7)

1. (∃x){Px • (y)[(Py • Kxy) ⊃ Fxy]}	
2. (x)[Px ⊃ (∃y)(Py • Kxy)]	/ (∃x)(∃y)[(Px • Py) • Fxy]
3. Pm • (y)[(Py • Kmy) ⊃ Fmy]	1, EI
4. Pm ⊃ (∃y)(Py • Kmy)	2, UI
5. Pm	3, Simp
6. (∃y)(Py • Kmy)	4, 5, MP
7. Pn • Kmn	6, EI
8. (y)[(Py • Kmy) ⊃ Fmy]	3, Com, Simp
9. (Pn • Kmn) ⊃ Fmn	8, UI
10. Fmn	7, 9, MP
11. Pn	7, Simp
12. Pm • Pn	5, 11, Conj
13. (Pm • Pn) • Fmn	10, 12, Conj
14. (∃y)[(Pm • Py) • Fmy]	13, EG
15. (∃x)(∃y)[(Px • Py) • Fxy]	14, EG

(8)

1. (x)[Px ⊃ (y)(Ry ⊃ Axy)]	
2. (x){[Rx • (∃y)(Py • Ayx)] ⊃ Jx}	
3. (∃x)Px • Rm	/ Jm
4. (∃x)Px	3, Simp
5. Pc	4, EI
6. Pc ⊃ (y)(Ry ⊃ Acy)	1, UI
7. (y)(Ry ⊃ Acy)	5, 6, MP
8. Rm ⊃ Acm	7, UI

9. Rm	3, Com, Simp
10. Acm	8, 9, MP
11. [Rm • (∃y)(Py • Aym)] ⊃ Jm	2, UI
12. Pc • Acm	5, 10, Conj
13. (∃y)(Py • Aym)	12, EG
14. Rm • (∃y)(Py • Aym)	9, 13, Conj
15. Jm	11, 14, MP

(9)

1. (x)[Mx ⊃ (∃y)(Py • Syx)]	
2. (x)[Dx ⊃ (∃y)(Py • Byx)]	
3. (∃x)(Mx ∨ Dx)	/ (∃x)[Px • (∃y)(Sxy ∨ Bxy)]
4. Mm ∨ Dm	3, EI
5. Mm ⊃ (∃y)(Py • Sym)	1, UI
6. Dm ⊃ (∃y)(Py • Bym)	2, UI
7. [Mm ⊃ (∃y)(Py • Sym)] • [Dm ⊃ (∃y)(Py • Bym)]	5, 6, Conj
8. (∃y)(Py • Sym) ∨ (∃y)(Py • Bym)	4, 7, CD
9. ~(∃y)[(Py • Sym) ∨ (Py • Bym)]	AIP
10. (y)~[(Py • Sym) ∨ (Py • Bym)]	9, CQ
11. ~[(Py • Sym) ∨ (Py • Bym)]	10, UI
12. ~(Py • Sym) • ~(Py • Bym)	11, DM
13. ~(Py • Sym)	12, Simp
14. (y)~(Py • Sym)	13, UG
15. ~(∃y)(Py • Sym)	14, CQ
16. (∃y)(Py • Bym)	8, 15, DS
17. ~(Py • Bym)	12, Com, Simp
18. (y)~(Py • Bym)	17, UG
19. ~(∃y)(Py • Bym)	18, CQ
20. (∃y)(Py • Bym) • ~(∃y)(Py • Bym)	16, 19, Conj
21. (∃y)[(Py • Sym) ∨ (Py • Bym)]	9-20, IP, DN
22. (Pn • Snm) ∨ (Pn • Bnm)	21, EI
23. Pn • (Snm ∨ Bnm)	22, Dist
24. Snm ∨ Bnm	23, Com, Simp
25. (∃y)(Sny ∨ Bny)	24, EG
26. Pn	23, Simp
27. Pn • (∃y)(Sny ∨ Bny)	25, 26, Conj
28. (∃x)[Px • (∃y)(Sxy ∨ Bxy)]	27, EG

(10)

1. (x){Ix ⊃ [(∃y)(Cy • Ay) ⊃ Ex]}	
2. [(∃x)Tx ∨ (∃x)Wx] ⊃ [(∃x)Ix • (∃x)Cx]	
3. (x)(Cx ⊃ Ax)	/ (∃x)Tx ⊃ (∃x)(Ix • Ex)
4. (∃x)Tx	ACP
5. (∃x)Tx ∨ (∃x)Wx	4, Add
6. (∃x)Ix • (∃x)Cx	2, 5, MP
7. (∃x)Ix	6, Simp
8. Im	7, EI
9. Im ⊃ [(∃y)(Cy • Ay) ⊃ Em]	1, UI

10. $(\exists y)(Cy \cdot Ay) \supset Em$	8, 9, MP
11. $(\exists x)Cx$	6, Com, Simp
12. Cn	11, EI
13. $Cn \supset An$	3, UI
14. An	12, 13, MP
15. $Cn \cdot An$	12, 14, Conj
16. $(\exists y)(Cy \cdot Ay)$	15, EG
17. Em	10, 16, MP
18. $Im \cdot Em$	8, 17, Conj
19. $(\exists x)(Ix \cdot Ex)$	18, EG
20. $(\exists x)Tx \supset (\exists x)(Ix \cdot Ex)$	4-19, CP

Exercise 8.7
Part I.

Simple identity statements:
1. $s = g$
2. $r \neq m$
3. $m = b$
4. $h \neq g$

Statements involving "only," "the only," and "no except":
5. $Wp \cdot (x)(Wx \supset x = p)$
6. $Pl \cdot (x)(Px \supset x = l)$
7. $Na \cdot Ma \cdot (x)[(Nx \cdot Mx) \supset x = a]$
8. $Nc \cdot Mc \cdot (x)[(Nx \cdot Mx) \supset x = c]$
9. $Uw \cdot Fw \cdot Ua \cdot Fa \cdot (x)[(Ux \cdot Fx) \supset (x = w \vee x = a)]$
10. $Sh \cdot Wh \cdot (x)[(Sx \cdot Wx) \supset x = h]$
11. $Sh \cdot Ph \cdot (x)[(Sx \cdot Px) \supset x = h]$

Superlative statements:
12. $Eh \cdot (x)[(Ex \cdot x \neq h) \supset Lhx]$
13. $Pp \cdot (x)[(Px \cdot x \neq p) \supset Spx]$
14. $Ah \cdot Uh \cdot (x)[(Ax \cdot Ux \cdot x \neq h) \supset Ohx]$
15. $Rd \cdot Nd \cdot (x)[(Rx \cdot Nx \cdot x \neq d) \supset Ldx]$

Statements involving "all except":
16. $Al \cdot {\sim}Sl \cdot (x)[(Ax \cdot x \neq l) \supset Sx]$
17. $Uf \cdot {\sim}Wf \cdot (x)[(Ux \cdot x \neq f) \supset Wx]$
18. $Ns \cdot Ps \cdot {\sim}Ms \cdot (x)[(Nx \cdot Px \cdot x \neq s) \supset Mx]$
19. $Pc \cdot {\sim}Wc \cdot (x)[(Px \cdot x \neq c) \supset Wx]$

Numerical statements:
20. $(x)(y)[(Cx \cdot Bx \cdot Cy \cdot By) \supset x = y]$
21. $(x)(y)(z)(Nx \cdot Sx \cdot Ny \cdot Sy \cdot Nz \cdot Sz) \supset (x = y \vee x = z \vee y = z)]$

22. $(x)(y)[(Nx \cdot Jx \cdot Ny \cdot Jy) \supset x = y]$
23. $(x)(y)(z)[(Cx \cdot Mx \cdot Cy \cdot My \cdot Cz \cdot Mz) \supset (x = y \vee x = z \vee y = z)]$
24. $(\exists x)(Qx \cdot Fx)$
25. $(\exists x)(\exists y)(Ax \cdot Wx \cdot Ay \cdot Wy \cdot x \neq y)$
26. $(\exists x)(\exists y)(\exists z)(Cx \cdot Cy \cdot Cz \cdot x \neq y \cdot x \neq z \cdot y \neq z)$
27. $(\exists x)[Ux \cdot (y)(Uy \supset y = x)]$
28. $(\exists x)\{Sx \cdot Nx \cdot (y)[(Sy \cdot Ny) \supset y = x]\}$
29. $(\exists x)(\exists y)\{Sx \cdot Bx \cdot Gx \cdot Sy \cdot By \cdot Gy \cdot x \neq y \cdot (z)[(Sz \cdot Bz \cdot Gz) \supset (z = x \vee z = y)]\}$

Statements containing definite descriptions:
30. $(\exists x)[Wxv \cdot (y)(Wyv \supset y = x) \cdot Bx]$
31. $(\exists x)[Wxo \cdot (y)(Wyo \supset y = x) \cdot x = d]$
32. $(\exists x)\{Mx \cdot Cxn \cdot (y)[(My \cdot Cyn) \supset y = x] \cdot Rx\}$
33. $(\exists x)\{Ax \cdot Pxa \cdot (y)[(Ay \cdot Pya) \supset y = x] \cdot x = b\}$
34. $(\exists x)[Cxg \cdot (y)(Cyg \supset y = x) \cdot x \neq s)]$

Assorted statements:
35. $Sr \cdot (x)[(Sx \cdot x \neq r) \supset Srx]$
36. $(\exists x)(Nx \cdot Sx)$
37. $g = l$
38. $Ar \cdot Er \cdot (x)[(Ax \cdot Ex) \supset x = r]$
39. $(\exists x)(\exists y)(Cx \cdot Qx \cdot Cy \cdot Qy \cdot x \neq y)$
40. $Pb \cdot (x)(Px \supset x = b)$
41. $(x)(y)[(Sxh \cdot Syh) \supset x = y]$
42. $Mb \cdot Hb \cdot (x)[(Mx \cdot Hx) \supset x = b]$
43. $(x)(y)(z)[(Sx \cdot Nx \cdot Sy \cdot Ny \cdot Sz \cdot Nz) \supset (x = y \vee x = z \vee y = z)]$
44. $m \neq b$
45. $(\exists x)\{Ex \cdot Dxn \cdot (y)[(Ey \cdot Dyn) \supset y = x] \cdot x = a\}$
46. $Rh \cdot (x)[(Rx \cdot x \neq h) \supset Ohx]$
47. $(\exists x)(\exists y)\{Tx \cdot Cx \cdot Ty \cdot Cy \cdot x \neq y \cdot (z)[(Tz \cdot Cz) \supset (z = x \vee z = y)]\}$
48. $Pj \cdot Rj \cdot {\sim}Ij \cdot (x)[(Px \cdot Rx \cdot x \neq j) \supset Ix]$
49. $(\exists x)\{Px \cdot Dxr \cdot (y)[(Py \cdot Dyr) \supset y = x] \cdot Ex\}$
50. $(\exists x)(\exists y)(\exists z)(Sx \cdot Ox \cdot Sy \cdot Oy \cdot Sz \cdot Oz \cdot x \neq y \cdot x \neq z \cdot y \neq z)$

Part II

(1)

1.	$(x)(x = a)$	
2.	$(\exists x)Rx$	/ Ra
3.	Ri	2, EI
4.	$i = a$	1, UI
5.	Ra	3, 4, Id

Exercise 8.7

(2)

1. Ke	
2. $\sim Kn$	$/ e \neq n$
3. $e = n$	AIP
4. Kn	1, 3, Id
5. $Kn \bullet \sim Kn$	1, 2, Conj
6. $e \neq n$	3-5, IP

(3)

1. $(x)(x = c \supset Nx)$	$/ Nc$
2. $c = c \supset Nc$	1, UI
3. $c = c$	Id
4. Nc	2, 3, MP

(4)

1. $(\exists x)(x = g)$	
2. $(x)(x = i)$	$/ g = i$
3. $n = g$	1, EI
4. $n = i$	2, UI
5. $g = n$	3, Id
6. $g = i$	4, 5, Id

(5)

1. $(x)(Gx \supset x = a)$	
2. $(\exists x)(Gx \bullet Hx)$	$/ Ha$
3. $Ge \bullet He$	2, EI
4. $Ge \supset e = a$	1, UI
5. Ge	3, Simp
6. $e = a$	4, 5, MP
7. $He \bullet Ge$	3, Com
8. He	7, Simp
9. Ha	6, 9, Id

(6)

1. $(x)(Ax \supset Bx)$	
2. $Ac \bullet \sim Bi$	$/ c \neq i$
3. $c = i$	AIP
4. $Ac \supset Bc$	1, UI
5. Ac	2, Simp
6. Bc	4, 5, MP
7. Bi	3, 6, Id
8. $\sim Bi \bullet Ac$	2, Com
9. $\sim Bi$	8, Simp
10. $Bi \bullet \sim Bi$	7, 9, Conj
11. $c \neq i$	3-10, IP

(7)

1. $(x)(x = a)$	
2. Fa	$/ Fm \bullet Fn$
3. $m = a$	1, UI
4. $a = m$	3, Id
5. Fm	2, 4, Id
6. $n = a$	1, Ui

	7. $a = n$	6, Id
	8. Fn	2, 7, Id
	9. $Fm \cdot Fn$	5, 8, Conj
(8)	1. $(x)(x = r)$	
	2. $Hr \cdot Kn$	/ $Hn \cdot Kr$
	3. $n = r$	1, UI
	4. Hr	2, Simp
	5. $r = n$	3, Id
	6. Hn4, 5, Id	
	7. $Kn \cdot Hr$	2, Com
	8. Kn	7, Simp
	9. Kr	3, 8, Id
	10. $Hn \cdot Kr$	6, 9, Conj
(9)	1. $(x)(Lx \supset x = e)$	
	2. $(x)(Sx \supset x = i)$	
	3. $(\exists x)(Lx \cdot Sx)$	$i = e$
	4. $Ln \cdot Sn$	3, EI
	5. $Ln \supset n = e$	1, UI
	6. Ln	4, Simp
	7. $n = e$	5, 6, MP
	8. $Sn \supset n = i$	2, UI
	9. $Sn \cdot Ln$	4, Com
	10. Sn	9, Simp
	11. $n = i$	8, 10, MP
	12. $i = n$	11, Id
	13. $i = e$	7, 12, Id
(10)	1. $(x)(Px \supset x = a)$	
	2. $(x)(x = c \supset Qx$	
	3. $a = c$	/ $(x)(Px \supset Qx)$
	4. Px	ACP
	5. $Px \supset x = a$	1, UI
	6. $x = a$	4, 5, MP
	7. $x = c$	3, 6, Id
	8. $x = c \supset Qx$	2, UI
	9. Qx	7, 8, MP
	10. $Px \supset Qx$	4-9, CP
	11. $(x)(Px \supset Qx)$	10, UG
(11)	1. $(x)(y)(Txy \supset x = e)$	
	2. $(\exists x)Txi$	/ Tei
	3. Tni	2, EI
	4. $(y)(Tny \supset n = e)$	1, UI
	5. $Tni \supset n = e$	4, UI

6. n = e	3, 5, MP
7. Tei	3, 6, Id

(12)

1. (x)[Rx ⊃ (Hx • x = m)]	/ Rc ⊃ Hm
2. Rc	ACP
3. Rc ⊃ (Hc • c = m)	1, UI
4. Hc • c = m	2, 3, MP
5. Hc	4, Simp
6. c = m • Hc	4, Com
7. c = m	6, Simp
8. Hm	5, 7, Id
9. Rc ⊃ Hm	2-8, CP

(13)

1. (x)(Ba ⊃ x ≠ a)	
2. Bc	/ a ≠ c
3. a = c	AIP
4. c = a	3, Id
5. Ba	2, 4, Id
6. Ba ⊃ c ≠ a	1, UI
7. c ≠ a	5, 6, MP
8. c = a • c ≠ a	4, 7, Conj
9. a ≠ c	3-8, IP

(14)

1. (∃x)Gx ⊃ (∃x)(Kx • x = i)	/ Gn ⊃ Ki
2. Gn	ACP
3. (∃x)Gx	2, EG
4. (∃x)(Kx • x = i)	1, 3, MP
5. Ks • s = i	4, EI
6. Ks	5, Simp
7. s = i • Ks	5, Com
8. s = i	7, Simp
9. Ki	6, 8, Id
10. Gn ⊃ Ki	2-9, CP

(15)

1. (x)(Rax ⊃ ~Rxc)	
2. (x)Rxx	/ c ≠ a
3. c = a	AIP
4. Rac ⊃ ~Rcc	1, IU
5. Rcc	2, UI
6. Rac	3, 5, Id
7. ~Rcc	4, 6, MP
8. Rcc • ~Rcc	5, 7, Conj
9. c ≠ a	3-8, IP

(16)

1.	(x)[Nx ⊃ (Px • x = m)]	
2.	~Pm	/ ~Ne
3.	Ne	AIP
4.	Ne ⊃ (Pe • e = m)	1, UI
5.	Pe • e = m	3, 4, MP
6.	Pe	5, Simp
7.	e = m • Pe	5, Com
8.	e = m	7, Simp
9.	Pm	6, 8, Id
10.	Pm • ~Pm	2, 9, Conj
11.	~Ne	3-10, IP

(17)

1.	(x)(Fx ⊃ x = e)	
2.	(∃x)(Fx • x = a)	/ a = e
3.	Fn • n = a	2, EI
4.	Fn ⊃ n = e	1, UI
5.	Fn	3, Simp
6.	n = e	4, 5, MP
7.	n = a • Fn	3, Com
8.	n = a	7, Simp
9.	a = e	6, 8, Id

(18)

1.	(x)[Ex ⊃ (Hp • x = e)]	
2.	(∃x)(Ex • x = p)	/ He
3.	Es • s = p	2, EI
4.	Es ⊃ (Hp • s = e)	1, UI
5.	Es	3, Simp
6.	Hp • s = e	4, 5, MP
7.	Hp6, Simp	
8.	s = p • Es	3, Com
9.	s = p	8, Simp
10.	p = s	9, Id
11.	Hs	7, 10, Id
12.	s = e • Hp	6, Com
13.	s = e	12, Simp
14.	He	11, 13, Id

(19)

1.	(x)(∃y)(Cxy ⊃ x = y)	
2.	(∃x)(y)(Cxy • x = a)	/ Caa
3.	(y)(Cny • n = a)	2, EI
4.	(∃y)(Cay ⊃ a = y)	1, UI
5.	Cam ⊃ a = m	4, EI
6.	Cnm • n = a	3, UI
7.	Cnm	6, Simp
8.	n = a • Cnm	6, Com
9.	n = a	8, Simp
10.	Cam	7, 9, Id

11. $a = m$	5, 10, MP
12. $m = a$	11, Id
13. Caa	10, 12, Id

(20) 1. $(x)[Fx \supset (Gx \cdot x = n)]$	
2. $Gn \supset (\exists x)(Hx \cdot x = e)$	/ $Fm \supset He$
3. Fm	ACP
4. $Fm \supset (Gm \cdot m = n)$	1, UI
5. $Gm \cdot m = n$	3, 4, MP
6. Gm	5, Simp
7. $m = n \cdot Gm$	5, Com
8. $m = n$	7, Simp
9. Gn	6, 8, Id
10. $(\exists x)(Hx \cdot x = e)$	2, 9, MP
11. $Ha \cdot a = e$	10, EI
12. Ha	11, Simp
13. $a = e \cdot Ha$	11, Com
14. $a = e$	13, Simp
15. He	12, 14, Id
16. $Fm \supset He$	3-15, CP

Part III.

(1) 1. $(\exists x)(Nx \cdot Wjx \cdot Ix)$	
2. $Nc \cdot Wjc \cdot (x)[(Nx \cdot Wjx) \supset x = c]$	/ Ic
3. $Na \cdot Wja \cdot Ia$	1, EI
4. $(x)[(Nx \cdot Wjx) \supset x = c]$	2, Com, Simp
5. $(Na \cdot Wja) \supset a = c$	4, UI
6. $Na \cdot Wja$	3, Simp
7. $a = c$	5, 6, MP
8. Ia	3, Com, Simp
9. Ic	7, 8, Id

(2) 1. $Ur \cdot (x)[(Ux \cdot x \neq r) \supset Orx]$	
2. Uw	
3. $w \neq r$	/ Orw
4. $(x)[(Ux \cdot x \neq r) \supset Orx]$	1, Com, Simp
5. $(Uw \cdot w \neq r) \supset Orw$	4, UI
6. $Uw \cdot w \neq r$	2, 3, Conj
7. Orw	5, 6, MP

(3) 1. $(\exists x)\{Ax \cdot Pxm \cdot (y)[(Ay \cdot Pym) \supset y = x] \cdot Fx\}$	
2. $(\exists x)\{Ax \cdot Pxm \cdot (y)[(Ay \cdot Pym) \supset y = x] \cdot x = l\}$	/ Fl
3. $Aa \cdot Pam \cdot (y)[(Ay \cdot Pym) \supset y = a] \cdot Fa$	1, EI
4. $Ac \cdot Pcm \cdot (y)[(Ay \cdot Pym) \supset y = c] \cdot c = l$	2, EI
5. $(y)[(Ay \cdot Pym) \supset y = c]$	4, Com, Simp
6. $(Aa \cdot Pam) \supset a = c$	5, UI

7. $Aa \cdot Pam$	3, Simp
8. $a = c$	6, 7, MP
9. $c = l$	4, Com, Simp
10. $a = l$	8, 9, Id
11. Fa	3, Com, Simp
12. Fl	10, 11, Id

(4)

1. $(\exists x)\{Nx \cdot Tx \cdot (y)[(Ny \cdot Ty) \supset y = x] \cdot Wmx\}$	
2. $Ng \cdot Wmg \cdot (x)[(Nx \cdot Wmx) \supset x = g]$	
/ $(\exists x)\{Nx \cdot Tx \cdot (y)[(Ny \cdot Ty) \supset y = x] \cdot x = g\}$	
3. $Na \cdot Ta \cdot (y)[(Ny \cdot Ty) \supset y = a] \cdot Wma$	1, EI
4. $(x)[(Nx \cdot Wmx) \supset x = g]$	2, Com, Simp
5. $(Na \cdot Wma) \supset a = g$	4, UI
6. Na	3, Simp
7. Wma	3, Com, Simp
8. $Na \cdot Wma$	6, 7, Conj
9. $a = g$	5, 8, MP
10. $Na \cdot Ta \cdot (y)[(Ny \cdot Ty) \supset y = a]$	3, Simp
11. $Na \cdot Ta \cdot (y)[(Ny \cdot Ty) \supset y = a] \cdot a = g$	9, 10, Conj
12. $(\exists x)\{Nx \cdot Tx \cdot (y)[(Ny \cdot Ty) \supset y = x] \cdot x = g\}$	11, EG

(5)

1. $(\exists x)[Wxk \cdot (y)(Wyk \supset y = x) \cdot Ex \cdot Ax]$	
2. $Em \cdot {\sim}Am$	/ ${\sim}Wmk$
3. $Wak \cdot (y)(Wyk \supset y = a) \cdot Ea \cdot Aa$	1, EI
4. $(y)(Wyk \supset y = a$	3, Com, Simp
5. $Wmk \supset m = a$	4, UI
6. Wmk	5, AIP
7. $m = a$	5, 6, MP
8. Aa	3, Com, Simp
9. $a = m$	7, Id
10. Am	8, 9, Id
11. ${\sim}Am$	2, Com, Simp
12. $Am \cdot {\sim}Am$	10, 11, Conj
13. ${\sim}Wmk$	6-12, IP

(6)

1. $(\exists x)\{(Dx \cdot Bx) \cdot (y)[(Dy \cdot By) \supset y = x] \cdot Lx \cdot Tx\}$	/ ${\sim}Ba$
2. ${\sim}La \cdot Da$	
3. $Dc \cdot Bc \cdot (y)[(Dy \cdot By) \supset y = c] \cdot Lc \cdot Tc$	1, EI
4. $(y)(Dy \cdot By) \supset y = c$	3, Com, Simp
5. $(Da \cdot Ba) \supset a = c$	4, UI
6. $a = c$	AIP
7. Lc	3, Com, Simp
8. La	6, 7, Id
9. ${\sim}La$	2, Simp
10. $La \cdot {\sim}La$	8, 9, Conj
11. $a \neq c$	6-10, IP

12. ~(Da • Ba)	5, 11, MT
13. ~Da ∨ ~Ba	12, DM
14. Da	2, Com, Simp
15. ~Ba	13, 14, DN, DS

(7)

1. Me • ~Se • (x)[(Mx • x ≠ e) ⊃ Sx]	
2. Mn • ~Gn • (x)[(Mx • x ≠ n) ⊃ Gx]	
3. e ≠ n	/ Ge • Sn
4. (x)[(Mx • x ≠ e) ⊃ Sx]	1, Com, Simp
5. (Mn • n ≠ e) ⊃ Sn	4, UI
6. Mn	2, Simp
7. n ≠ e	3, Id
8. Mn • n ≠ e	6, 7, Conj
9. Sn	5, 8, MP
10. (x)[(Mx • x ≠ n) ⊃ Gx]	2, Com, Simp
11. (Me • e ≠ n) ⊃ Ge	10, UI
12. Me	1, Simp
13. Me • e ≠ n	3, 12, Conj
14. Ge	11, 13, MP
15. Ge • Sn	9, 14, Conj

(8)

1. Pa • Oa • (y)[(Py • Oy) ⊃ y = a]	
2. Pw • Sw • (y)[(Py • Sy) ⊃ y = w]	
3. (∃x)(Px • Ox • Sx)	/ a = w
4. Pc • Oc • Sc	3, EI
5. (y)(Py • Oy) ⊃ y = a	1, Com, Simp
6. (Pc • Oc) ⊃ c = a	5, UI
7. Pc • Oc	4, Simp
8. c = a	6, 7, MP
9. (y)[(Py • Sy) ⊃ y = w]	2, Com, Simp
10. (Pc • Sc) ⊃ c = w	9, UI
11. Pc • Sc	4, Com, Simp
12. c = w	10, 11, MP
13. a = c	8, Id
14. a = w	12, 13, Id

(9)

1. (∃x){Mx • Tx • (y)[(My • y ≠ x) ⊃ Hxy]}	
/ (∃x){Mx • Tx • (y)[(My • ~Ty) ⊃ Hxy]}	
2. Ma • Ta • (y)[(My • y ≠ a) ⊃ Hay]	1, EI
3. Ma • Ta	2, Simp
4. ~(y)[(My • ~Ty) ⊃ Hay]	AIP
5. (∃x)~[(My • ~Ty) ⊃ Hay]	4, CQ
6. ~[(Mc • ~Tc) ⊃ Hac]	5, EI
7. ~[~(Mc • ~Tc) ∨ Hac]	6, Impl
8. Mc • ~Tc • ~Hac	11, DM, DN
9. (y)[(My • y ≠ a) ⊃ Tay]	2, Com, Simp

	10. $(Mc \cdot c \neq a) \supset Hac$	13, UI
	11. $Mc \supset (c \neq a \supset Hac)$	10, Exp
	12. Mc	8, Simp
	13. $c \neq a \supset Hac$	11, 12, MP
	14. $\sim Hac$	8, Com, Simp
	15. $c = a$	13, 14, MT, DN
	16. Ta	3, Com, Simp
	17. $\sim Tc$	8, Com, Simp
	18. $\sim Ta$	15, 17, Id
	19. $Ta \cdot \sim Ta$	16, 18, Conj
	20. $(y)[(My \cdot \sim Ty) \supset Hay]$	4-19, IP, DN
	21. $Ma \cdot Ta \cdot (y)[(My \cdot \sim Ty) \supset Hay]$	3, 20, Conj
	22. $(\exists x)\{Mx \cdot Tx \cdot (y)[(My \cdot \sim Ty) \supset Hxy]\}$	21, EG

(10)	1. $Bs \cdot (x)[(Bx \cdot x \neq s) \supset Tsx]$	
	2. $(\exists x)\{Bx \cdot (y)[(By \cdot y \neq x) \supset Txy] \cdot Cx\}$	
	3. $(x)(y)(Txy \supset \sim Tyx)$	/ Cs
	4. $Ba \cdot (y)[(By \cdot y \neq a) \supset Tay] \cdot Ca$	2, EI
	5. $(x)[(Bx \cdot x \neq s) \supset Tsx]$	1, Com, Simp
	6. $(Ba \cdot a \neq s) \supset Tsa$	5, UI
	7. $a \neq s$	AIP
	8. Ba	4, Simp
	9. $Ba \cdot a \neq s$	7, 8, Conj
	10. Tsa	6, 9, MP
	11. $(y)[(By \cdot y \neq a) \supset Tay]$	4, Com, Simp
	12. $(Bs \cdot s \neq a) \supset Tas$	11, UI
	13. Bs	1, Simp
	14. $s \neq a$	7, Id
	15. $Bs \cdot s \neq a$	13, 14, Conj
	16. Tas	12, 15, MP
	17. $(y)(Tay \supset \sim Tya)$	3, UI
	18. $Tas \supset \sim Tsa$	17, UI
	19. $\sim Tsa$	16, 18, MP
	20. $Tsa \cdot \sim Tsa$	10, 19, Conj
	21. $a = s$	7-20, IP, DN
	22. Ca	4, Com, Simp
	23. Cs	21, 22, Id

(11)	1. $(\exists x)(\exists y)(Px \cdot Lx \cdot Py \cdot Ly \cdot x \neq y)$	
	2. $Pr \cdot Fr \cdot Lr \cdot (y)[(Py \cdot Fy \cdot Ly) \supset y = r]$	/ $(\exists x)(Px \cdot Lx \cdot \sim Fx)$
	3. $\sim(\exists x)(Px \cdot Lx \cdot \sim Fx)$	AIP
	4. $(\exists y)(Pa \cdot La \cdot Py \cdot Ly \cdot a \neq y)$	1, EI
	5. $Pa \cdot La \cdot Pc \cdot Lc \cdot a \neq c$	4, EI
	6. $(y)[(Py \cdot Fy \cdot Ly) \supset y = r]$	2, Com, Simp
	7. $(Pa \cdot Fa \cdot La) \supset a = r$	6, UI
	8. $(Pc \cdot Fc \cdot Lc) \supset c = r$	6, UI

9. (x)~(Px • Lx • ~Fx)	3, CQ
10. ~(Pa • La • ~Fa)	9, UI
11. ~(Pa • La) ∨ Fa	10, DM, DN
12. Pa • La	5, ComExercise 8.7
13. Fa	11, 12, DN, DS
14. Pa • Fa • La	12, 13, Conj, Com
15. a = r	7, 14, MP
16. ~(Pc • Lc • ~Fc)	9, UI
17. ~(Pc • Lc) ∨ Fc	16, DM, DN
18. Pc • Lc	5, Com, Simp
19. Fc	17, 18, DN, DS
20. Pc • Fc • Lc	18, 19, Conj, Com
21. c = r	8, 20, MP
22. r = c	21, Id
23. a = c	15, 22, Id
24. a ≠ c	5, Com, Simp
25. a = c • a ≠ c	23, 24, Conj
26. (∃x)(Px • Lx • ~Fx)	3-25, IP

(12) 1. Df • Bf • Dp • Bp • (x)[(Dx • Bx) ⊃ (x = f ∨ x = p)]
2. f ≠ p
3. Df • ~Rf • (x)[(Dx • x ≠ f) ⊃ Rx]
/ (∃x){Dx • Bx • Rx • (y)[(Dy • By • Ry) ⊃ y = x]}

4. (x)[(Dx • x ≠ f) ⊃ Rx]	3, Com, Simp
5. (Dp • p ≠ f) ⊃ Rp	4, UI
6. Dp	1, Com, Simp
7. p ≠ f	2, Id
8. Dp • p ≠ f	6, 7, Conj
9. Rp	5, 8, MP
10. Bp	1, Com, Simp
11. Dp • Bp • Rp	6, 9, 10, Conj
12. ~(y)[(Dy • By • Ry) ⊃ y = p]	AIP
13. (∃y)~[(Dy • By • Ry) ⊃ y = p]	12, CQ
14. ~[(Da • Ba • Ra] ⊃ a = p]	13, EI
15. ~[~(Da • Ba • Ra) ∨ a = p]	14, Impl
16. Da • Ba • Ra • a ≠ p	15, DM, DN
17. (x)[(Dx • Bx) ⊃ (x = f ∨ x = p)]	1, Com, Simp
18. (Da • Ba) ⊃ (a = f ∨ a = p)	17, UI
19. Da • Ba	16, Simp
20. a = f ∨ a = p	18, 19, MP
21. a ≠ p	16, Com, Simp
22. a = f	20, 21, Com, DS
23. Ra	16, Com, Simp
24. Rf	22, 23, Id
25. ~Rf	3, Com, Simp
26. Rf • ~Rf	24, 25, Conj

27. $(y)[(Dy \cdot By \cdot Ry) \supset y = p$ — 12-26, IP
28. $Dp \cdot Bp \cdot Rp \cdot (y)[(Dy \cdot By \cdot Ry) \supset y = p]$ — 11, 27, Conj
29. $(\exists x)\{Dx \cdot Bx \cdot Rx \cdot (y)[(Dy \cdot By \cdot Ry) \supset y = x]\}$ — 28, EG

(13)
1. $(\exists x)(\exists y)(Ax \cdot Ox \cdot Ay \cdot Oy \cdot x \neq y)$
2. $(x)(Ax \supset Px)$
3. $(x)(y)(z)[(Px \cdot Ox \cdot Py \cdot Oy \cdot Pz \cdot Oz) \supset (x = y \vee x = z \vee y = z)]$ / $(\exists x)(\exists y)\{Px \cdot Ox \cdot Py \cdot Oy \cdot x \neq y \cdot (z)[(Pz \cdot Oz) \supset (z = x \vee z = y]\}$
4. $(\exists y)(Aa \cdot Oa \cdot Ay \cdot Oy \cdot a \neq y)$ — 1, EI
5. $Aa \cdot Oa \cdot Ab \cdot Ob \cdot a \neq b$ — 4, EI
6. $Aa \supset Pa$ — 2, UI
7. Aa — 5, Simp
8. Pa — 6, 7, MP
9. $Ab \supset Pb$ — 2, UI
10. Ab — 5, Com, Simp
11. Pb — 9, 10, MP
12. Oa — 5, Com, Simp
13. Ob — 5, Com, Simp
14. $a \neq b$ — 5, Com, Simp
15. $Pa \cdot Oa \cdot Pb \cdot Ob \cdot a \neq b$ — 8, 11-14, Conj
    16. $\sim(z)[(Pz \cdot Oz) \supset (z = a \vee z = b)]$ — AIP
    17. $(\exists z)\sim[(Pz \cdot Oz) \supset (z = a \vee z = b)]$ — 16, CQ
    18. $\sim[(Pc \cdot Oc) \supset (c = a \vee c = b)]$ — 17, EI
    19. $\sim[\sim(Pc \cdot Oc) \vee (c = a \vee c = b)]$ — 18, Impl
    20. $Pc \cdot Oc \cdot \sim(c = a \vee c = b)$ — 19, DM, DN
    21. $(y)(z)[(Pa \cdot Oa \cdot Py \cdot Oy \cdot Pz \cdot Oz) \supset (a = y \vee a = z \vee y = z)]$ — 3, UI
    22. $(z)[(Pa \cdot Oa \cdot Pb \cdot Ob \cdot Pz \cdot Oz) \supset (a = b \vee a = z \vee b = z)]$ — 21, UI
    23. $(Pa \cdot Oa \cdot Pb \cdot Ob \cdot Pc \cdot Oc) \supset (a = b \vee a = c \vee b = c)$ — 22, UI
    24. $Pc \cdot Oc$ — 20, Simp
    25. $Pa \cdot Oa \cdot Pb \cdot Ob$ — 15, Simp
    26. $Pa \cdot Oa \cdot Pb \cdot Ob \cdot Pc \cdot Oc$ — 24, 25, Conj
    27. $a = b \vee a = c \vee b = c$ — 23, 26, MP
    28. $a = c \vee b = c$ — 14, 27, DS
    29. $\sim(c = a \vee c = b)$ — 20, Com, Simp
    30. $\sim(a = c \vee b = c)$ — 29, Id, Id
    31. $(a = c \vee b = c) \cdot \sim(a = c \vee b = c)$ — 28, 30, Conj
32. $(z)[(Pz \cdot Oz) \supset z = a \vee z = b)]$ — 16-31, IP, DN
33. $Pa \cdot Oa \cdot Pb \cdot Ob \cdot a \neq b \cdot (z)[(Pz \cdot Oz) \supset (z = a \vee z = b)]$ — 15, 32, Conj
34. $(\exists y)\{Pa \cdot Oa \cdot Py \cdot Oy \cdot a \neq y \cdot (z)[(Pz \cdot Oz) \supset (z = a \vee z = y)]\}$ — 33, EG
35. $(\exists x)(\exists y)\{Px \cdot Ox \cdot Py \cdot Oy \cdot x \neq y \cdot (z)[(Pz \cdot Oz) \supset (z = x \vee z = y)]\}$ — 34, EG

(14) 1. $(x)(y)(z)[(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (x = y \lor x = z \lor y = z)]$

2. $(\exists x)(\exists y)(Sx \cdot Lx \cdot Rx \cdot Sy \cdot Ly \cdot Ry \cdot x \neq y)$

3. $(x)(Rx \supset \sim Cx)$ / $(Sn \cdot Cn) \supset \sim Ln$

4. $(\exists y)(Sa \cdot La \cdot Ra \cdot Sy \cdot Ly \cdot Ry \cdot a \neq y)$ 2, EI

5. $Sa \cdot La \cdot Ra \cdot Sc \cdot Lc \cdot Rc \cdot a \neq c$ 4, EI

6. $(y)(z)[(Sa \cdot La \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset$
$(a = y \lor a = z \lor y = z)]$ 1, UI

7. $(z)[(Sa \cdot La \cdot Sc \cdot Lc \cdot Sz \cdot Lz) \supset$
$(a = c \lor a = z \lor c = z)]$ 6, UI

8. $(Sa \cdot La \cdot Sc \cdot Lc \cdot Sn \cdot Ln) \supset$
$(a = c \lor a = n \lor c = n)$ 7, UI

9. $Sn \cdot Cn \cdot Ln$ AIP

10. $Sn \cdot Ln$ 9, Com, Simp

11. $Sa \cdot La$ 5, Simp

12. $Sc \cdot Lc$ 5, Com, Simp

13. $Sa \cdot La \cdot Sc \cdot Lc \cdot Sn \cdot Ln$ 10-12. Conj

14. $a = c \lor a = n \lor c = n$ 8, 13, MP

15. $a \neq c$ 5, Com, Simp

16. $a = n \lor c = n$ 14, 15, DS

17. $Ra \supset \sim Ca$ 3, UI

18. Ra 5, Com, Simp

19. $\sim Ca$ 17, 18, MP

20. $a = n$ AIP

21. $\sim Cn$ 19, 20, Id

22. Cn 9, Com, Simp

23. $Cn \cdot \sim Cn$ 21, 22, Conj

24. $a \neq n$ 20-23, IP

25. $c = n$ 16, 24, DS

26. $Rc \supset \sim Cc$ 3, UI

27. Rc 5, Com, Simp

28. $\sim Cc$ 26, 27, MP

29. $\sim Cn$ 25, 28, Id

30. Cn 9, Com, Simp

31. $Cn \cdot \sim Cn$ 29, 30, Conj

32. $\sim(Sn \cdot Cn \cdot Ln)$ 9-31, IP

33. $\sim(Sn \cdot Cn) \lor \sim Ln$ 32, DM

34. $(Sn \cdot Cn) \supset \sim Ln$ 33, Impl

(15) 1. $Cm \cdot \sim Em \cdot (x)[(Cx \cdot x \neq m) \supset Ex]$

2. $Cr \cdot Er \cdot (x)[(Cx \cdot Ex) \supset x = r]$

3. $m \neq r$

/ $(\exists x)(\exists y)\{Cx \cdot Cy \cdot x \neq y \cdot (z)[Cz \supset (z = x \lor z = y)]\}$

4. Cm1, Simp

5. Cr 2, Simp

6. $\sim(z)[Cz \supset (z = m \lor z = r)]$ AIP

7. $(\exists z)\sim[Cx \supset (z = m \lor z = r)]$ 6, CQ

8. $\sim[Ca \supset (a = m \vee a = r)]$ — 7, EI
9. $\sim(\sim Ca \vee a = m \vee a = r)$ — 8, Impl
10. $Ca \cdot a \neq m \cdot a \neq r$ — 9, DM, DN
11. $(x)[(Cx \cdot x \neq m)\ Ex]$ — 1, Com, Simp
12. $(Ca \cdot a \neq m) \supset Ea$ — 11, UI
13. $Ca \cdot a \neq m$ — 10, Simp
14. Ea — 12, 13, MP
15. $(x)[(Cx \cdot Ex) \supset x = r]$ — 2, Com, Simp
16. $(Ca \cdot Ea) \supset a = r$ — 15, UI
17. Ca — 10, Simp
18. $Ca \cdot Ea$ — 14, 17, Conj
19. $a = r$ — 16, 18, MP
20. $a \neq r$ — 10, Com, Simp
21. $a = r \cdot a \neq r$ — 19, 20, Conj
22. $(z)[Cz \supset (z = m \vee z = r)]$ — 6-21, IP, DN
23. $Cm \cdot Cr \cdot m \neq r \cdot (z)[Cz \supset (z = m \vee z = r)]$ — 3-5, 22, Conj
24. $(\exists y)\{Cm \cdot Cy \cdot m \neq y \cdot (z)[Cz \supset (z = m \vee z = y)]\}$ — 23, EG
25. $(\exists x)(\exists y)\{Cx \cdot Cy \cdot x \neq y \cdot (z)[Cz \supset (z = x \vee z = y)]\}$ — 24, EG

(16) 1. $Sc \cdot \sim Pc \cdot Sn \cdot \sim Pn \cdot (x)[(Sx \cdot x \neq c \cdot x \neq n) \supset Px]$
2. $Sn \cdot Dn \cdot (x)[(Sx \cdot Dx) \supset x = n]$
3. $(x)\{Sx \supset [Rx \equiv (\sim Dx \cdot \sim Px)]\}$
4. $c \neq n$ — / $(\exists x)\{(Sx \cdot Rx) \cdot (y)[(Sy \cdot Ry) \supset y = x]\}$
5. Sc — 1, Simp
6. $Sc \supset [Rc \equiv (\sim Dc \cdot \sim Pc)]$ — 3, UI
7. $Rc \equiv (\sim Dc \cdot \sim Pc)$ — 5, 6, MP
8. $[Rc \supset (\sim Dc \cdot \sim Pc)] \cdot [(\sim Dc \cdot \sim Pc) \supset Rc]$ — 7, Equiv
9. $(\sim Dc \cdot \sim Pc) \supset Rc$ — 8, Com, Simp
10. $(x)[(Sx \cdot Dx) \supset x = n]$ — 2, Com, Simp
11. $(Sc \cdot Dc) \supset c = n$ — 10, UI
12. $\sim(Sc \cdot Dc)$ — 4, 11, MT
13. $\sim Sc \vee \sim Dc$ — 12, DM
14. $\sim Dc$ — 5, 13, DN, DS
15. $\sim Pc$ — 1, Com, Simp
16. $\sim Dc \cdot \sim Pc$ — 14, 15, Conj
17. Rc — 9, 16, MP
18. $Sc \cdot Rc$ — 5, 17, Conj
19. $\sim(y)[(Sy \cdot Ry) \supset y = c]$ — AIP
20. $(\exists y)\sim[(Sy \cdot Ry) \supset y = c]$ — 19, CQ
21. $\sim[(Sa \cdot Ra) \supset a = c]$ — 20, EI
22. $\sim[\sim(Sa \cdot Ra) \vee a = c]$ — 21, Impl
23. $Sa \cdot Ra \cdot a \neq c$ — 22, DM, DN
24. $Sa \supset [Ra \equiv (\sim Da \cdot \sim Pa)]$ — 3, UI
25. Sa — 23, Simp
26. $Ra \equiv (\sim Da \cdot \sim Pa)$ — 24, 25, MP
27. $[Ra \supset (\sim Da \cdot \sim Pa)] \cdot [(\sim Da \cdot \sim Pa) \supset Ra]$ — 26, Equiv

28. $Ra \supset (\sim Da \cdot \sim Pa)$	27, Simp
29. Ra	23, Com, Simp
30. $\sim Da \cdot \sim Pa$	28, 29, MP
31. $(x)[(Sx \cdot x \neq c \cdot x \neq n) \supset Px]$	1, Com, Simp
32. $(Sa \cdot a \neq c \cdot a \neq n) \supset Pa$	31, UI
33. $(Sa \cdot a \neq c) \supset (a \neq n \supset Pa)$	32, Exp
34. $a \neq c$	23, Com, Simp
35. $Sa \cdot a \neq c$	25, 34, Conj
36. $a \neq n \supset Pa$	33, 35, MP
37. $\sim Pa$	30, Com, Simp
38. $a = n$	36, 37, MT, DN
39. $n = a$	38, Id
40. Dn	2, Com, Simp
41. Da	39, 40, Id
42. $\sim Da$	30, Simp
43. $Da \cdot \sim Da$	41, 42, Conj
44. $(y)[(Sy \cdot Ry) \supset y = c]$	19-43, IP, DN
45. $Sc \cdot Rc \cdot (y)[(Sy \cdot Ry) \supset y = c]$	18, 44, Conj
46. $(\exists x)\{Sx \cdot Rx \cdot (y)[(Sy \cdot Ry) \supset y = x]\}$	45, EG

Exercise 9.1
Part II

1. a. Has no affect.
   b. Strengthens.
   c. Weakens.
   d. Weakens.
   e. Strengthens.
   f. Strengthens.
   g. Weakens.
   h. Strengthens.
   i. Weakens.
   j. Weakens.

2. a. Strengthens.
   b. Weakens.
   c. Weakens.
   d. Weakens.
   e. Strengthens.
   f. Strengthens.
   g. Has no affect.
   h. Weakens.
   i. Weakens.
   j. Strengthens.

3. a. Strengthens.
   b. Has no affect.
   c. Strengthens.
   d. Weakens.
   e. Strengthens.
   f. Weakens.
   g. Strengthens.
   h. Weakens.
   i. Weakens.
   j. Strengthens.

4. a. Has no effect.
   b. Weakens.
   c. Strengthens.
   d. Weakens.
   e. Strengthens.
   f. Has no effect.
   g. Strengthens
   h. Weakens.
   i. Weakens
   j. Strengthens.