

- 4, Com, Simp
5, DN, Impl
2, Impl
6, 7, HS
8, Impl, DN
9, Taut
3-10, CP
2-11, CP
1-12, CP
5. $Q \vee R$
6. $\sim Q \supset R$
7. $R \supset Q$
8. $\sim Q \supset Q$
9. $Q \vee Q$
10. Q
11. $P \supset Q$
12. $(\sim R \vee Q) \supset (P \supset Q)$
13. $(P \cdot Q) \vee R \supset (\sim R \vee Q) \supset (P \supset Q)$

(19) $/P \equiv [P \vee (Q \cdot \sim Q)]$

- ACP
1, Add
1-2, CP
ACP
AIP
4, 5, DS
5-6, IP, DN
4-7, CP
3, 8, Conj
9, Equiv
1. P
2. $P \vee (Q \cdot \sim Q)$
3. $P \supset [P \vee (Q \cdot \sim Q)]$
4. $P \vee (Q \cdot \sim Q)$
5. $\sim P$
6. $Q \cdot \sim Q$
7. P
8. $[P \vee (Q \cdot \sim Q)] \supset P$
9. {line 3} $\cdot$ {line 8}
10. $P \equiv [P \vee (Q \cdot \sim Q)]$

(20)

1. P
2. Q
3. $Q \vee \sim P$
4. $\sim P \vee Q$
5. $P \supset Q$
6. $Q \supset (P \supset Q)$
7. $P \supset Q$
8. Q
9. $(P \supset Q) \supset Q$
10. $[Q \supset (P \supset Q)] \cdot [(P \supset Q) \supset Q]$
11. $Q \equiv (P \supset Q)$
12. $P \supset [Q \equiv (P \supset Q)]$
- ACP
ACP
2, Add
3, Com
4, Impl
2-5, CP
ACP
1, 7, MP
7-8, CP
6, 9, Conj
10, Equiv
1-11, CP

Exercise 8.1

1. Ce

2. $\sim S_n$ 3. $\sim(Iw \vee Is)$ 4. $Jr \vee Nr$ 5. $Di \supset Dm$

Exercise 8.1

6. $(Sb \cdot Sf) \supset (Ea \vee Eg)$
7. $(x)(Mx \supset Tx)$
8. $(\exists x)(Gx \cdot Sx)$
9. $(x)(Nx \supset \sim Bx)$
10. $(\exists x)(Hx \cdot \sim Rx)$
11. $Cg \supset Mt$
12. $\sim Cg \supset Gt$
13. $(\exists x)Tx$
14. $(x)(Lx \supset Wx)$
15. $(\exists x)(Ax \cdot Px)$
16. $(\exists x)(Sx \cdot \sim Gx)$
17. $(x)(Px \supset Lx)$
18. $(\exists x)(Tx \cdot Rx)$
19. $(x)(Sx \supset Vx)$
20. $(x)(Cx \cdot Mx) \supset (Vx \cdot Nx)$
21. $(x)[Tx \supset \sim(Rx \vee Ex)]$
22. $(x)(Cx \supset \sim Hx)$
23. $(x)Ix$
24. $(x)\sim Gx$
25. $(x)(Tx \supset Hx)$
26. $(\exists x)(Tx \cdot Wx)$
27. $(\exists x)(Mx \cdot \sim Ex)$
28. $(x)(Hx \supset \sim Ex)$

29. $(\exists x)(Gx \cdot Ax)$
30. $(x)(Px \supset Gx)$
31. $(\exists x)[Cx \cdot \sim(Sx \vee Bx)]$
32. $(x)(Cx \supset \sim Tx)$
33. $(x)[Sx \supset (Hx \vee Fx)]$
34. $(\exists x)[(Dx \cdot Bx) \equiv Tx]$
35. $(x)[Ax \supset (Sx \equiv Px)]$
36. $(\exists x)[Cx \cdot (Sx \supset Bx)]$
37. $(\exists x)[Cx \cdot (Ax \supset Tx)]$
38. $(x)[(Mx \cdot Ax) \supset Tx]$
39. $(x)[(Mx \cdot Px) \supset Tx]$
40. $(x)[(Wx \cdot Cx) \supset Rx]$
41. $(\exists x)[(Fx \cdot Cx) \cdot \sim Rx]$
42. $(x)[(Gx \cdot Vx) \supset (Rx \cdot Ex)]$
43. $(x)[(Vx \vee Cx) \supset (Sx \cdot Ix)]$
44. $(\exists x)[(Rx \cdot Vx) \cdot Ax]$
45. $(x)[(Rx \cdot Vx) \supset Ex]$
46. $(\exists x)[(Fx \cdot Rx) \cdot Ex]$
47. $(\exists x)[(Fx \cdot Cx) \cdot \sim Rx]$
48. $(x)[(Hx \vee Ex) \supset (Vx \cdot Dx)]$
49. $Gt \equiv (x)(Wx \supset Cx)$
50. $(\exists x)(Wx \cdot Tx) \supset (Gp \vee Gh)$
51. $(x)(Mx \supset Ix) \supset Ir$
52. $(\exists x)(Ix \cdot Mx) \supset Ir$

Exercise 8.2

53. $(x)[(Sx \vee Dx) \supset (Ix \cdot Ex)]$
54. $(x)[(Sx \cdot Wx) \supset (\sim Ex \vee Mx)]$
55. $(x)[(Bx \cdot Mx) \supset Sx] \supset Sc$
56. $(x)[(Ex \cdot Mx) \supset Wx] \supset (x)[(Mx \cdot \sim Ex) \supset Lx]$
57. $(x)[(Sx \cdot Bx) \supset \sim Cx] \vee (x)[(Sx \cdot Ox) \supset Tx]$
58. $(\exists x)(Ex \cdot Rx) \equiv (\exists x)(Mx \cdot Ox)$
59. $(x)\{[(Px \vee Ax) \cdot Sx] \supset [(Cx \vee Rx) \supset Lx]\}$
60. $(x)[(Sx \vee Tx) \supset (Cx \cdot Ex)] \supset (\exists x)[Mx \cdot (Px \vee Dx)]$

Exercise 8.2

Part I

- (1)
 1. $(x)(Ax \supset Bx)$
 2. $(x)(Bx \supset Cx)$ / $(x)(Ax \supset Cx)$
 3. $Ax \supset Bx$ 1, UI
 4. $Bx \supset Cx$ 2, UI
 5. $Ax \supset Cx$ 3, 4, HS
 6. $(x)(Ax \supset Cx)$ 5, UG
- (2)
 1. $(x)(Bx \supset Cx)$
 2. $(\exists x)(Ax \cdot Bx)$ / $(\exists x)(Ax \cdot Cx)$
 3. $Am \cdot Bm$ 2, EI
 4. $Bm \supset Cm$ 1, UI
 5. Bm 3, Com, Simp
 6. Cm 4, 5, MP
 7. Am 3, Simp
 8. $Am \cdot Cm$ 6, 7, Conj
 9. $(\exists x)(Ax \cdot Cx)$ 8, EG
- (3)
 1. $(x)(Ax \supset Bx)$
 2. $\sim Bm$ / $(\exists x)\sim Ax$
 3. $Am \supset Bm$ 1, UI
 4. $\sim Am$ 2, 3, MT
 5. $(\exists x)\sim Ax$ 4, EG

- (4) 1. $(x)[Ax \supset (Bx \vee Cx)]$
 2. $Ag \cdot \sim Bg$ / Cg
 3. $Ag \supset (Bg \vee Cg)$ 1, UI
 4. Ag 2, Simp
 5. $Bg \vee Cg$ 3, 4, MP
 6. $\sim Bg$ 2, Com, Simp
 7. Cg 5, 6, DS
- (5) 1. $(x)[(Ax \vee Bx) \supset Cx]$
 2. $(\exists y)(Ay \cdot Dy)$ / $(\exists y)Cy$
 3. $Am \cdot Dm$ 2, EI
 4. Am 3, Simp
 5. $(Am \vee Bm) \supset Cm$ 1, UI
 6. $Am \vee Bm$ 4, Add
 7. Cm 5, 6, MP
 8. $(\exists y)Cy$ 7, EG
- (6) 1. $(x)[Jx \supset (Kx \cdot Lx)]$
 2. $(\exists y)\sim Ky$ / $(\exists z)\sim Jz$
 3. $\sim Km$ 2, EI
 4. $Jm \supset (Km \cdot Lm)$ 1, UI
 5. $\sim Km \vee \sim Lm$ 3, Add
 6. $\sim (Km \cdot Lm)$ 5, DM
 7. $\sim Jm$ 4, 6, MT
 8. $(\exists z)\sim Jz$ 7, EG
- (7) 1. $(x)[Ax \supset (Bx \vee Cx)]$
 2. $(\exists x)(Ax \cdot \sim Cx)$ / $(\exists x)Bx$
 3. $Am \cdot \sim Cm$ 2, EI
 4. $Am \supset (Bm \vee Cm)$ 1, UI
 5. Am 3, Simp
 6. $Bm \vee Cm$ 4, 5, MP
 7. $\sim Cm$ 3, Com, Simp
 8. Bm 6, 7, Com, DS
 9. $(\exists x)Bx$ 8, EG
- (8) 1. $(x)(Ax \supset Bx)$
 2. $Am \cdot An$ / $Bm \cdot Bn$
 3. $Am \supset Bm$ 1, UI
 4. Am 3, Simp
 5. Bm 3, 4, MP
 6. $An \supset Bn$ 1, UI
 7. An 3, Com, Simp
 8. Bn 6, 7, MP
 9. $Bm \cdot Bn$ 5, 8, Conj

Exercise 8.2

- (9)
- | | |
|--|----------------|
| 1. $(x)(Ax \supset Bx)$ | |
| 2. $Am \vee An$ | / $Bm \vee Bn$ |
| 3. $Am \supset Bm$ | 1, UI |
| 4. $An \supset Bn$ | 1, UI |
| 5. $(Am \supset Bm) \cdot (An \supset Bn)$ | 3, 4, Conj |
| 6. $Bm \vee Bn$ | 2, 5, CD |
- (10)
- | | |
|-------------------------|-------------|
| 1. $(x)(Bx \vee Ax)$ | |
| 2. $(x)(Bx \supset Ax)$ | / $(x)Ax$ |
| 3. $Bx \vee Ax$ | 1, UI |
| 4. $Bx \supset Ax$ | 2, UI |
| 5. $Ax \vee Bx$ | 3, Com |
| 6. $\sim Ax \supset Bx$ | 5, DN, Impl |
| 7. $\sim Ax \supset Ax$ | 4, 6, HS |
| 8. $Ax \vee Ax$ | 7, Impl, DN |
| 9. Ax | 8, Taut |
| 10. $(x)Ax$ | 9, UG |
- (11)
- | | |
|------------------------------------|------------------------|
| 1. $(x)[(Ax \cdot Bx) \supset Cx]$ | |
| 2. $(\exists x)(Bx \cdot \sim Cx)$ | / $(\exists x)\sim Ax$ |
| 3. $Bm \cdot \sim Cm$ | 2, EI |
| 4. $(Am \cdot Bm) \supset Cm$ | 1, UI |
| 5. $\sim Cm$ | 3, Com, Simp |
| 6. $\sim(Am \cdot Bm)$ | 4, 5, MT |
| 7. $\sim Am \vee \sim Bm$ | 6, DM |
| 8. Bm | 3, Simp |
| 9. $\sim Am$ | 7, 8, Com, DN, DS |
| 10. $(\exists x)\sim Ax$ | 9, EG |
- (12)
- | | |
|---|--------------|
| 1. $(\exists x)Ax \supset (x)(Bx \supset Cx)$ | |
| 2. $Am \cdot Bm$ | / Cm |
| 3. Am | 2, Simp |
| 4. $(\exists x)Ax$ | 3, EG |
| 5. $(x)(Bx \supset Cx)$ | 1, 4, MP |
| 6. $Bm \supset Cm$ | 5, UI |
| 7. Bm | 2, Com, Simp |
| 8. Cm | 6, 7, MP |
- (13)
- | | |
|--|------------------------------|
| 1. $(\exists x)Ax \supset (x)Bx$ | |
| 2. $(\exists x)Cx \supset (\exists x)Dx$ | |
| 3. $An \cdot Cn$ | / $(\exists x)(Bx \cdot Dx)$ |
| 4. An | 2, Simp |
| 5. $(\exists x)Ax$ | 4, EG |
| 6. $(x)Bx$ | 1, 5, MP |
| 7. Cn | 3, Com, Simp |
| 8. $(\exists x)Cx$ | 7, EG |

- | | |
|--------------------------------|--------------|
| 9. $(\exists x)Dx$ | 2, 8, MP |
| 10. Dm | 9, EI |
| 11. Bm | 6, UI |
| 12. $Bm \cdot Dm$ | 10, 11, Conj |
| 13. $(\exists x)(Bx \cdot Dx)$ | 12, EG |

- (14)
- | | |
|---|------------------------|
| 1. $(\exists x)Ax \supset (x)(Cx \supset Bx)$ | |
| 2. $(\exists x)(Ax \vee Bx)$ | |
| 3. $(x)(Bx \supset Ax)$ | / $(x)(Cx \supset Ax)$ |
| 4. $Am \vee Bm$ | 2, EI |
| 5. $Bm \supset Am$ | 3, UI |
| 6. $\sim Am \supset Bm$ | 4, DN, Impl |
| 7. $\sim Am \supset Am$ | 5, 6, HS |
| 8. $Am \vee Am$ | 7, Impl, DN |
| 9. Am | 8, Taut |
| 10. $(\exists x)Ax$ | 9, EG |
| 11. $(x)(Cx \supset Bx)$ | 1, 10, MP |
| 12. $Cx \supset Bx$ | 11, UI |
| 13. $Bx \supset Ax$ | 3, UI |
| 14. $Cx \supset Ax$ | 12, 13, HS |
| 15. $(x)(Cx \supset Ax)$ | 14, UG |

- (15)
- | | |
|---|------------------------|
| 1. $(\exists x)Ax \supset (x)(Bx \supset Cx)$ | |
| 2. $(\exists x)Dx \supset (\exists x)\sim Cx$ | |
| 3. $(\exists x)(Ax \cdot Dx)$ | / $(\exists x)\sim Bx$ |
| 4. $Am \cdot Dm$ | 3, EI |
| 5. Am | 4, Simp |
| 6. Dm | 4, Com, Simp |
| 7. $(\exists x)Ax$ | 5, EG |
| 8. $(\exists x)Dx$ | 6, EG |
| 9. $(x)(Bx \supset Cx)$ | 1, 7, MP |
| 10. $(\exists x)\sim Cx$ | 2, 8, MP |
| 11. $\sim Cn$ | 10, EI |
| 12. $Bn \supset Cn$ | 9, UI |
| 13. $\sim Bn$ | 11, 12, MT |
| 14. $(\exists x)\sim Bx$ | 13, EG |

Part II

- (1)
- | | |
|-------------------------|-----------------------------------|
| 1. $(x)(Ox \supset Sx)$ | |
| 2. $(x)(Ox \supset Fx)$ | / $(x)[Ox \supset (Sx \cdot Fx)]$ |
| 3. $Ox \supset Sx$ | 1, UI |
| 4. $Ox \supset Fx$ | 2, UI |
| 5. $\sim Ox \vee Sx$ | 3, Impl |
| 6. $\sim Ox \vee Fx$ | 4, Impl |

Exercise 8.2

- | | |
|--|------------|
| 7. $(\sim O x \vee S x) \cdot (\sim O x \vee F x)$ | 5, 6, Conj |
| 8. $\sim O x \vee (S x \cdot F x)$ | 7, Dist |
| 9. $O x \supset (S x \cdot F x)$ | 8, Impl |
| 10. $(x)[O x \supset (S x \cdot F x)]$ | 9, UG |
-
- | | |
|--|--------------------------------------|
| (2) 1. $(x)(T x \supset V x)$ | $/ (x)[(T x \cdot G x) \supset V x]$ |
| 2. $T x \supset V x$ | 1, UI |
| 3. $\sim T x \vee V x$ | 2, Impl |
| 4. $V x \vee \sim T x$ | 3, Com |
| 5. $(V x \vee \sim T x) \vee \sim G x$ | 4, Add |
| 6. $V x \vee (\sim T x \vee \sim G x)$ | 5, Assoc |
| 7. $V x \vee \sim(T x \cdot G x)$ | 6, DM |
| 8. $\sim(T x \cdot G x) \vee V x$ | 7, Com |
| 9. $(T x \cdot G x) \supset V x$ | 8, Impl |
| 10. $(x)[(T x \cdot G x) \supset V x]$ | 9, UG |
-
- | | |
|--|--------------------------|
| (3) 1. $(x)[(A x \vee P x) \supset G x]$ | $/ (x)(A x \supset G x)$ |
| 2. $(A x \vee P x) \supset G x$ | 1, UI |
| 3. $\sim(A x \vee P x) \vee G x$ | 2, Impl |
| 4. $(\sim A x \cdot \sim P x) \vee G x$ | 3, DM |
| 5. $(\sim A x \vee G x) \cdot (\sim P x \vee G x)$ | 4, Com, Dist, Com |
| 6. $\sim A x \vee G x$ | 5, Simp |
| 7. $A x \supset G x$ | 6, Impl |
| 8. $(x)(A x \supset G x)$ | 7, UG |
-
- | | |
|---|---|
| (4) 1. $(x)(C x \supset V x) \cdot (x)(P x \supset F x)$ | |
| 2. $(\exists x)(C x \cdot G x) \cdot (\exists x)(P x \cdot G x)$ | $/ (\exists x)(V x \cdot G x) \cdot (\exists x)(F x \cdot G x)$ |
| 3. $(\exists x)(C x \cdot G x)$ | 2, Simp |
| 4. $C m \cdot G m$ | 3, EI |
| 5. $(\exists x)(P x \cdot G x)$ | 2, Com, Simp |
| 6. $P n \cdot G n$ | 5, EI |
| 7. $(x)(C x \supset V x)$ | 1, Simp |
| 8. $C m \supset V m$ | 7, UI |
| 9. $C m$ | 4, Simp |
| 10. $V m$ | 8, 9, MP |
| 11. $G m$ | 4, Com, Simp |
| 12. $V m \cdot G m$ | 10, 11, Conj |
| 13. $(\exists x)(V x \cdot G x)$ | 12, EG |
| 14. $(x)(P x \supset F x)$ | 1, Com, Simp |
| 15. $P n \supset F n$ | 14, UI |
| 16. $P n$ | 6, Simp |
| 17. $F n$ | 15, 16, MP |
| 18. $G n$ | 6, Com, Simp |
| 19. $F n \cdot G n$ | 17, 18, Conj |
| 20. $(\exists x)(F x \cdot G x)$ | 19, EG |
| 21. $(\exists x)(V x \cdot G x) \cdot (\exists x)(F x \cdot G x)$ | 13, 20, Conj |

- (5)
- | | |
|--|-----------------------------|
| 1. $(x)[(Bx \vee Px) \supset Lx]$ | |
| 2. $(x)(Gx \supset \sim Lx)$ | $/ (x)(Gx \supset \sim Bx)$ |
| 3. $(Bx \vee Px) \supset Lx$ | 1, UI |
| 4. $Gx \supset \sim Lx$ | 2, UI |
| 5. $\sim Lx \supset \sim(Bx \vee Px)$ | 3, Trans |
| 6. $Gx \supset \sim(Bx \vee Px)$ | 4, 5, HS |
| 7. $Gx \supset (\sim Bx \cdot \sim Px)$ | 6, DM |
| 8. $\sim Gx \vee (\sim Bx \cdot \sim Px)$ | 7, Impl |
| 9. $(\sim Gx \vee \sim Bx) \cdot (\sim Gx \vee \sim Px)$ | 8, Dist |
| 10. $\sim Gx \vee \sim Bx$ | 9, Simp |
| 11. $Gx \supset \sim Bx$ | 10, Impl |
| 12. $(x)(Gx \supset \sim Bx)$ | 11, UG |

- (6)
- | | |
|---|------------------------------|
| 1. $(\exists x)(Cx \cdot Gx)$ | |
| 2. $(\exists x)Cx \supset (\exists x)(Px \cdot Gx)$ | $/ (\exists x)(Vx \cdot Gx)$ |
| 3. $(x)(Px \supset Vx)$ | 1, EI |
| 4. $Cm \cdot Gm$ | 4, Simp |
| 5. Cm | 5, EG |
| 6. $(\exists x)Cx$ | 2, 6, MP |
| 7. $(\exists x)(Px \cdot Gx)$ | 7, EI |
| 8. $Pn \cdot Gn$ | 8, Simp |
| 9. Pn | 3, UI |
| 10. $Pn \supset Vn$ | 9, 10, MP |
| 11. Vn | 8, Com, Simp |
| 12. Gn | 11, 12, Conj |
| 13. $Vn \cdot Gn$ | 13, EG |
| 14. $(\exists x)(Vx \cdot Gx)$ | |

- (7)
- | | |
|------------------------------------|-----------------|
| 1. $(x)[Gx \supset (Ix \cdot Px)]$ | |
| 2. $(x)[(Ix \cdot Px) \supset Rx]$ | $/ Ra \cdot Rc$ |
| 3. $Ga \cdot Gc$ | 1, UI |
| 4. $Gx \supset (Ix \cdot Px)$ | 2, UI |
| 5. $(Ix \cdot Px) \supset Rx$ | 4, 5, HS |
| 6. $Gx \supset Rx$ | 6, UG |
| 7. $(x)(Gx \supset Rx)$ | 7, UI |
| 8. $Ga \supset Ra$ | 3, Simp |
| 9. Ga | 8, 9, MP |
| 10. Ra | 7, UI |
| 11. $Gc \supset Rc$ | 3, Com, Simp |
| 12. Gc | 11, 12, MP |
| 13. Rc | 10, 13, Conj |
| 14. $Ra \cdot Rc$ | |

Exercise 8.2

- (8)
- | | |
|--|------------------------------|
| 1. $(\exists x)(Hx \cdot Rx)$ | |
| 2. $(\exists x)(Bx \cdot Sx)$ | |
| 3. $(\exists x)Hx \supset (x)[Bx \supset (Sx \supset Ex)]$ | / $(\exists x)(Bx \cdot Ex)$ |
| 4. $Hm \cdot Rm$ | 1, EI |
| 5. $Bn \cdot Sn$ | 2, EI |
| 6. Hm | 4, Simp |
| 7. $(\exists x)Hx$ | 6, EG |
| 8. $(x)[Bx \supset (Sx \supset Ex)]$ | 3, 7, MP |
| 9. $Bn \supset (Sn \supset En)$ | 8, UI |
| 10. $(Bn \cdot Sn) \supset En$ | 9, Exp |
| 11. En | 5, 10, MP |
| 12. Bn | 5, Simp |
| 13. $Bn \cdot En$ | 11, 12, Conj |
| 14. $(\exists x)(Bx \cdot Ex)$ | 13, EG |

- (9)
- | | |
|--|------------------------|
| 1. $(\exists x)(Rx \cdot Wx) \supset (x)(Cx \supset Px)$ | |
| 2. $(\exists x)(Lx \cdot Wx) \supset (x)(Px \supset Bx)$ | |
| 3. $(\exists x)[(Lx \cdot Rx) \cdot Wx]$ | / $(x)(Cx \supset Bx)$ |
| 4. $(Lm \cdot Rm) \cdot Wm$ | 3, EI |
| 5. $Lm \cdot Rm$ | 4, Simp |
| 6. Lm | 5, Simp |
| 7. Rm | 5, Com, Simp |
| 8. Wm | 4, Com, Simp |
| 9. $Rm \cdot Wm$ | 7, 8, Conj |
| 10. $Lm \cdot Wm$ | 6, 8, Conj |
| 11. $(\exists x)(Rx \cdot Wx)$ | 9, EG |
| 12. $(\exists x)(Lx \cdot Wx)$ | 10, EG |
| 13. $(x)(Cx \supset Px)$ | 1, 11, MP |
| 14. $(x)(Px \supset Bx)$ | 2, 12, MP |
| 15. $Cx \supset Px$ | 13, UI |
| 16. $Px \supset Bx$ | 14, UI |
| 17. $Cx \supset Bx$ | 15, 16, HS |
| 18. $(x)(Cx \supset Bx)$ | 17, UG |

- (10)
- | | |
|--|-----------------------------------|
| 1. $(x)[(Ax \cdot Kx) \supset Rx] \supset (x)(Gx \supset Sx)$ | |
| 2. $(x)[(Ax \cdot Kx) \supset Fx] \supset (x)(Gx \supset Px)$ | |
| 3. $(x)[(Ax \cdot Kx) \supset (Rx \cdot Fx)]$ | / $(x)[Gx \supset (Sx \cdot Px)]$ |
| 4. $(Ax \cdot Kx) \supset (Rx \cdot Fx)$ | 3, UI |
| 5. $\sim(Ax \cdot Kx) \vee (Rx \cdot Fx)$ | 4, Impl |
| 6. $[\sim(Ax \cdot Kx) \vee Rx] \cdot [\sim(Ax \cdot Kx) \vee Fx]$ | 5, Dist |
| 7. $\sim(Ax \cdot Kx) \vee Rx$ | 6, Simp |
| 8. $\sim(Ax \cdot Kx) \vee Fx$ | 6, Com, Simp |
| 9. $(Ax \cdot Kx) \supset Rx$ | 7, Impl |
| 10. $(Ax \cdot Kx) \supset Fx$ | 8, Impl |
| 11. $(x)[(Ax \cdot Kx) \supset Rx]$ | 9, UG |
| 12. $(x)[(Ax \cdot Kx) \supset Fx]$ | 10, UG |

13. $(x)(Gx \supset Sx)$	1, 11, MP
14. $(x)(Gx \supset Px)$	2, 12, MP
15. $Gx \supset Sx$	13, UI
16. $Gx \supset Px$	14, UI
17. $\sim Gx \vee Sx$	15, Impl
18. $\sim Gx \vee Px$	16, Impl
19. $(\sim Gx \vee Sx) \cdot (\sim Gx \vee Px)$	17, 18, Conj
20. $\sim Gx \vee (Sx \cdot Px)$	19, Dist
21. $Gx \supset (Sx \cdot Px)$	20, Impl
22. $(x)[Gx \supset (Sx \cdot Px)]$	21, UG

Exercise 8.3

Part I

- (1) 1. $(x)Ax \supset (\exists x)Bx$
 2. $(x)\sim Bx$ / $(\exists x)\sim Ax$
 3. $\sim(\exists x)Bx$ 2, CQ
 4. $\sim(x)Ax$ 1, 3, MT
 5. $(\exists x)\sim Ax$ 4, CQ
- (2) 1. $(\exists x)\sim Ax \vee (\exists x)\sim Bx$
 2. $(x)Bx$ / $\sim(x)Ax$
 3. $\sim(\exists x)\sim Bx$ 2, CQ
 4. $(\exists x)\sim Ax$ 1, 3, Com, DS
 5. $\sim(x)Ax$ 4, CQ
- (3) 1. $\sim(\exists x)Ax$ / $(x)(Ax \supset Bx)$
 2. $(x)\sim Ax$ 1, CQ
 3. $\sim Ax$ 2, UI
 4. $\sim Ax \vee Bx$ 3, Add
 5. $Ax \supset Bx$ 4, Impl
 6. $(x)(Ax \supset Bx)$ 5, UG
- (4) 1. $(\exists x)Ax \vee (\exists x)(Bx \cdot Cx)$
 2. $\sim(\exists x)Bx$ / $(\exists x)Ax$
 3. $(x)\sim Bx$ 2, CQ
 4. $\sim Bx$ 3, UI
 5. $\sim Bx \vee \sim Cx$ 4, Add
 6. $\sim(Bx \cdot Cx)$ 5, DM
 7. $(x)\sim(Bx \cdot Cx)$ 6, UG
 8. $\sim(\exists x)(Bx \cdot Cx)$ 7, CQ
 9. $(\exists x)Ax$ 1, 8, Com, DS

Exercise 8.3

- (5)
- | | |
|---|---------------|
| 1. $(x)(Ax \cdot Bx) \vee (x)(Cx \cdot Dx)$ | |
| 2. $\sim(x)Dx$ | / $(x)Bx$ |
| 3. $(\exists x)\sim Dx$ | 2, CQ |
| 4. $\sim Dm$ | 3, EI |
| 5. $\sim Dm \vee \sim Cm$ | 4, Add |
| 6. $\sim Cm \vee \sim Dm$ | 5, Com |
| 7. $\sim(Cm \cdot Dm)$ | 6, DM |
| 8. $(\exists x)\sim(Cx \cdot Dx)$ | 7, EG |
| 9. $\sim(x)(Cx \cdot Dx)$ | 8, CQ |
| 10. $(x)(Ax \cdot Bx)$ | 1, 9, Com, DS |
| 11. $Ax \cdot Bx$ | 10, UI |
| 12. Bx | 11, Com, Simp |
| 13. $(x)Bx$ | 12, UG |

- (6)
- | | |
|--|---------------|
| 1. $(\exists x)\sim Ax \supset (x)(Bx \supset Cx)$ | |
| 2. $\sim(x)(Ax \vee Cx)$ | / $\sim(x)Bx$ |
| 3. $(\exists x)\sim(Ax \vee Cx)$ | 2, CQ |
| 4. $\sim(Am \vee Cm)$ | 3, EI |
| 5. $\sim Am \cdot \sim Cm$ | 4, DM |
| 6. $\sim Am$ | 5, Simp |
| 7. $(\exists x)\sim Ax$ | 6, EG |
| 8. $(x)(Bx \supset Cx)$ | 1, 7, MP |
| 9. $Bm \supset Cm$ | 8, UI |
| 10. $\sim Cm$ | 5, Com, Simp |
| 11. $\sim Bm$ | 9, 10, MT |
| 12. $(\exists x)\sim Bx$ | 11, EG |
| 13. $\sim(x)Bx$ | 12, CQ |

- (7)
- | | |
|---------------------------|------------------------|
| 1. $(x)(Ax \supset Bx)$ | |
| 2. $\sim(x)Cx \vee (x)Ax$ | |
| 3. $\sim(x)Bx$ | / $(\exists x)\sim Cx$ |
| 4. $(\exists x)\sim Bx$ | 3, CQ |
| 5. $\sim Bm$ | 4, EI |
| 6. $Am \supset Bm$ | 1, UI |
| 7. $\sim Am$ | 5, 6, MT |
| 8. $(\exists x)\sim Ax$ | 7, EG |
| 9. $\sim(x)Ax$ | 8, CQ |
| 10. $\sim(x)Cx$ | 2, 9, Com, DS |
| 11. $(\exists x)\sim Cx$ | 10, CQ |

- (8)
- | | |
|---|--------------------------------------|
| 1. $(x)Ax \supset (\exists x)\sim Bx$ | |
| 2. $\sim(x)Bx \supset (\exists x)\sim Cx$ | / $(x)Cx \supset (\exists x)\sim Ax$ |
| 3. $(x)Ax \supset \sim(x)Bx$ | 1, CQ |
| 4. $(x)Ax \supset (\exists x)\sim Cx$ | 2, 3, HS |

5. $\sim(\exists x)\sim Cx \supset \sim(x)Ax$ 4, Trans
 6. $(x)Cx \supset (\exists x)\sim Ax$ 5, CQ

- (9) 1. $(\exists x)(Ax \vee Bx) \supset (x)Cx$
 2. $(\exists x)\sim Cx$ / $\sim(\exists x)Ax$
 3. $\sim(x)Cx$ 2, CQ
 4. $\sim(\exists x)(Ax \vee Bx)$ 1, 3, MT
 5. $(x)\sim(Ax \vee Bx)$ 4, CQ
 6. $\sim(Ax \vee Bx)$ 5, UI
 7. $\sim Ax \cdot \sim Bx$ 6, DM
 8. $\sim Ax$ 7, Simp
 9. $(x)\sim Ax$ 8, UG
 10. $\sim(\exists x)Ax$ 9, CQ

- (10) 1. $\sim(\exists x)(Ax \cdot \sim Bx)$
 2. $\sim(\exists x)(Bx \cdot \sim Cx)$ / $(x)(Ax \supset Cx)$
 3. $(x)\sim(Ax \cdot \sim Bx)$ 1, CQ
 4. $(x)\sim(Bx \cdot \sim Cx)$ 2, CQ
 5. $\sim(Ax \cdot \sim Bx)$ 3, UI
 6. $\sim(Bx \cdot \sim Cx)$ 4, UI
 7. $\sim Ax \vee Bx$ 5, DM, DN
 8. $\sim Bx \vee Cx$ 6, DM, DN
 9. $Ax \supset Bx$ 7, Impl
 10. $Bx \supset Cx$ 8, Impl
 11. $Ax \supset Cx$ 9, 10, HS
 12. $(x)(Ax \supset Cx)$ 11, UG

- (11) 1. $\sim(\exists x)(Ax \cdot \sim Bx)$
 2. $\sim(\exists x)(Ax \cdot \sim Cx)$ / $(x)[Ax \supset (Bx \cdot Cx)]$
 3. $(x)\sim(Ax \cdot \sim Bx)$ 1, CQ
 4. $(x)\sim(Ax \cdot \sim Cx)$ 2, CQ
 5. $\sim(Ax \cdot \sim Bx)$ 3, UI
 6. $\sim(Ax \cdot \sim Cx)$ 4, UI
 7. $\sim Ax \vee Bx$ 5, DM, DN
 8. $\sim Ax \vee Cx$ 6, DM, DN
 9. $(\sim Ax \vee Bx) \cdot (\sim Ax \vee Cx)$ 7, 8, Conj
 10. $\sim Ax \vee (Bx \cdot Cx)$ 9, Dist
 11. $Ax \supset (Bx \cdot Cx)$ 10, Impl
 12. $(x)[Ax \supset (Bx \cdot Cx)]$ 11, UG

- (12) 1. $(x)[(Ax \cdot Bx) \supset Cx]$
 2. $\sim(x)(Ax \supset Cx)$ / $\sim(x)Bx$
 3. $(\exists x)\sim(Ax \supset Cx)$ 2, CQ
 4. $\sim(Am \supset Cm)$ 3, EI
 5. $\sim(\sim Am \vee Cm)$ 4, Impl

Exercise 8.3

6. $A_m \cdot \sim C_m$	5, DM, DN
7. $(A_m \cdot B_m) \supset C_m$	1, UI
8. $\sim C_m$	6, Com, Simp
9. $\sim(A_m \cdot B_m)$	7, 8, MT
10. $\sim A_m \vee \sim B_m$	9, DM
11. A_m	6, Simp
12. $\sim B_m$	10, 11, DN, DS
13. $(\exists x)\sim B_x$	12, EG
14. $\sim(x)B_x$	13, CQ

(13) 1. $(x)(A_x \cdot \sim B_x) \supset (\exists x)C_x$	
2. $\sim(\exists x)(C_x \vee B_x)$	/ $\sim(x)A_x$
3. $(x)\sim(C_x \vee B_x)$	2, CQ
4. $\sim(C_x \vee B_x)$	3, UI
5. $\sim C_x \cdot \sim B_x$	4, DM,
6. $\sim C_x$	5, Simp
7. $(x)\sim C_x$	6, UG
8. $\sim(\exists x)C_x$	7, CQ
9. $\sim(x)(A_x \cdot \sim B_x)$	1, 8, MT
10. $(\exists x)\sim(A_x \cdot \sim B_x)$	9, CQ
11. $\sim(A_m \cdot \sim B_m)$	10, EI
12. $\sim A_m \vee B_m$	11, DM, DN
13. $\sim B_x$	5, Com, Simp
14. $(x)\sim B_x$	13, UG
15. $\sim B_m$	14, UI
16. $\sim A_m$	12, 15, Com, DS
17. $(\exists x)\sim A_x$	16, EG
18. $\sim(x)A_x$	17, CQ

(14) 1. $(\exists x)\sim A_x \supset (x)\sim B_x$	
2. $(\exists x)\sim A_x \supset (\exists x)B_x$	
3. $(x)(A_x \supset C_x)$	/ $(x)C_x$
4. $\sim(x)\sim B_x \supset \sim(\exists x)\sim A_x$	1, Trans
5. $(\exists x)B_x \supset \sim(\exists x)\sim A_x$	4, CQ
6. $(\exists x)\sim A_x \supset \sim(\exists x)\sim A_x$	2, 5, HS
7. $\sim(\exists x)\sim A_x \vee \sim(\exists x)\sim A_x$	6, Impl
8. $\sim(\exists x)\sim A_x$	7, Taut
9. $(x)A_x$	8, CQ
10. A_x	9, UI
11. $A_x \supset C_x$	3, UI
12. C_x	10, 11, MP
13. $(x)C_x$	12, UG

- (15)
- | | | |
|-----|---------------------------------------|---------------------------------|
| 1. | $\sim(\exists x)(Ax \vee Bx)$ | |
| 2. | $(\exists x)Cx \supset (\exists x)Ax$ | |
| 3. | $(\exists x)Dx \supset (\exists x)Bx$ | $/ \sim(\exists x)(Cx \vee Dx)$ |
| 4. | $(x)\sim(Ax \vee Bx)$ | 1, CQ |
| 5. | $\sim(Ax \vee Bx)$ | 4, UI |
| 6. | $\sim Ax \cdot \sim Bx$ | 5, DM |
| 7. | $\sim Ax$ | 6, Simp |
| 8. | $\sim Bx$ | 6, Com, Simp |
| 9. | $(x)\sim Ax$ | 7, UG |
| 10. | $(x)\sim Bx$ | 8, UG |
| 11. | $\sim(\exists x)Ax$ | 9, CQ |
| 12. | $\sim(\exists x)Bx$ | 10, CQ |
| 13. | $\sim(\exists x)Cx$ | 2, 11, MT |
| 14. | $\sim(\exists x)Dx$ | 3, 12, MT |
| 15. | $(x)\sim Cx$ | 13, CQ |
| 16. | $(x)\sim Dx$ | 14, CQ |
| 17. | $\sim Cx$ | 15, UI |
| 18. | $\sim Dx$ | 16, UI |
| 19. | $\sim Cx \cdot \sim Dx$ | 17, 18, Conj |
| 20. | $\sim(Cx \vee Dx)$ | 19, DM |
| 21. | $(x)\sim(Cx \vee Dx)$ | 20, UG |
| 22. | $\sim(\exists x)(Cx \vee Dx)$ | 21, CQ |

Part II

- (1)
- | | | |
|-----|--|-----------------------------------|
| 1. | $(x)[Px \supset (Hx \vee Nx)] \supset \sim(\exists x)Cx$ | |
| 2. | Cf | $/ (\exists x)(Px \cdot \sim Nx)$ |
| 3. | $(\exists x)Cx$ | 2, EG |
| 4. | $\sim(x)[Px \supset (Hx \vee Nx)]$ | 1, 3, DN, MT |
| 5. | $(\exists x)\sim[Px \supset (Hx \vee Nx)]$ | 4, CQ |
| 6. | $\sim[Pm \supset (Hm \vee Nm)]$ | 5, EI |
| 7. | $\sim[\sim Pm \vee (Hm \vee Nm)]$ | 6, Impl |
| 8. | $Pm \cdot \sim(Hm \vee Nm)$ | 7, DM, DN |
| 9. | $Pm \cdot (\sim Hm \cdot \sim Nm)$ | 8, DM |
| 10. | Pm | 9, Simp |
| 11. | $\sim Hm \cdot \sim Nm$ | 9, Com, Simp |
| 12. | $\sim Nm$ | 11, Com, Simp |
| 13. | $Pm \cdot \sim Nm$ | 10, 12, Conj |
| 14. | $(\exists x)(Px \cdot \sim Nx)$ | 13, EG |
- (2)
- | | | |
|----|------------------------------|-------------|
| 1. | $Ia \vee (x)(Px \supset Ix)$ | |
| 2. | $\sim(\exists x)Ix$ | $/ \sim Pa$ |
| 3. | $(x)\sim Ix$ | 2, CQ |
| 4. | $\sim Ia$ | 3, UI |

Exercise 8.3

- | | |
|-------------------------|----------|
| 5. $(x)(Px \supset Ix)$ | 1, 4, DS |
| 6. $Pa \supset Ia$ | 5, UI |
| 7. $\sim Pa$ | 4, 6, MT |

- (3) 1. $(\exists x)(Sx \cdot Ax) \supset (\exists x)(Px \cdot Rx)$
 2. $(x)(Px \supset \sim Rx)$ / $(x)(Sx \supset \sim Ax)$
 3. $(x)(\sim Px \vee \sim Rx)$ 2, Impl
 4. $(x)\sim(Px \cdot Rx)$ 3, DM
 5. $\sim(\exists x)(Px \cdot Rx)$ 4, CQ
 6. $\sim(\exists x)(Sx \cdot Ax)$ 1, 5, MT
 7. $(x)\sim(Sx \cdot Ax)$ 6, CQ
 8. $(x)(\sim Sx \vee \sim Ax)$ 7, DM
 9. $(x)(Sx \supset \sim Ax)$ 8, Impl

- (4) 1. $(\exists x)(Gx \cdot Px) \vee (\exists x)(Sx \cdot Ex)$
 2. $\sim(\exists x)Ex$ / $(\exists x)Px$
 3. $(x)\sim Ex$ 2, CQ
 4. $\sim Ex$ 3, UI
 5. $\sim Ex \vee \sim Sx$ 4, Add
 6. $\sim Sx \vee \sim Ex$ 5, Com
 7. $\sim(Sx \cdot Ex)$ 6, DM
 8. $(x)\sim(Sx \cdot Ex)$ 7, UG
 9. $\sim(\exists x)(Sx \cdot Ex)$ 8, CQ
 10. $(\exists x)(Gx \cdot Px)$ 1, Com, DS
 11. $Gm \cdot Pm$ 10, EI
 12. Pm 11, Com, Simp
 13. $(\exists x) Px$ 12, EG

- (5) 1. $(x)[(Px \cdot \sim Ax) \supset Ix]$
 2. $\sim(\exists x)(Px \cdot Ix)$ / $(x)(Px \supset Ax)$
 3. $(x)\sim(Px \cdot Ix)$ 2, CQ
 4. $\sim(Px \cdot Ix)$ 3, UI
 5. $\sim Px \vee \sim Ix$ 4, DM
 6. $\sim Ix \vee \sim Px$ 5, Com
 7. $Ix \supset \sim Px$ 6, Impl
 8. $(Px \cdot \sim Ax) \supset Ix$ 1, UI
 9. $(Px \cdot \sim Ax) \supset \sim Px$ 7, 8, HS
 10. $(\sim Ax \cdot Px) \supset \sim Px$ 9, Com
 11. $\sim Ax \supset (Px \supset \sim Px)$ 10, Exp
 12. $\sim Ax \supset (\sim Px \vee \sim Px)$ 11, Impl
 13. $\sim Ax \supset \sim Px$ 12, Taut
 14. $Px \supset Ax$ 13, Trans
 15. $(x)(Px \supset Ax)$ 14, UG

- (6)
- | | |
|--|------------------------|
| 1. $\sim(\exists x)(Ix \cdot \sim Px)$ | |
| 2. $\sim(\exists x)(Px \cdot \sim Dx)$ | / $(x)(Ix \supset Dx)$ |
| 3. $(x)\sim(Ix \cdot \sim Px)$ | 1, CQ |
| 4. $(x)\sim(Px \cdot \sim Dx)$ | 2, CQ |
| 5. $\sim(Ix \cdot \sim Px)$ | 3, UI |
| 6. $\sim(Px \cdot \sim Dx)$ | 4, UI |
| 7. $\sim Ix \vee Px$ | 5, DM, DN |
| 8. $\sim Px \vee Dx$ | 6, DM, DN |
| 9. $Ix \supset Px$ | 7, Impl |
| 10. $Px \supset Dx$ | 8, Impl |
| 11. $Ix \supset Dx$ | 9, 10, HS |
| 12. $(x)(Ix \supset Dx)$ | 11, UG |
- (7)
- | | |
|---|----------------------------------|
| 1. $(x)(Px \supset Sx) \cdot (x)(Ix \supset Gx)$ | |
| 2. $\sim(\exists x)(Sx \cdot Gx)$ | / $\sim(\exists x)(Px \cdot Ix)$ |
| 3. $(x)\sim(Sx \cdot Gx)$ | 2, CQ |
| 4. $\sim(Sx \cdot Gx)$ | 3, UI |
| 5. $\sim Sx \vee \sim Gx$ | 4, DM |
| 6. $(x)(Px \supset Sx)$ | 1, Simp |
| 7. $(x)(Ix \supset Gx)$ | 1, Com, Simp |
| 8. $Px \supset Sx$ | 6, UI |
| 9. $Ix \supset Gx$ | 7, UI |
| 10. $\sim Sx \supset \sim Px$ | 8, Trans |
| 11. $\sim Gx \supset \sim Ix$ | 9, Trans |
| 12. $(\sim Sx \supset \sim Px) \cdot (\sim Gx \supset \sim Ix)$ | 10, 11, Conj |
| 13. $\sim Px \vee \sim Ix$ | 5, 12, CD |
| 14. $\sim(Px \cdot Ix)$ | 13, DM |
| 15. $(x)\sim(Px \cdot Ix)$ | 14, UG |
| 16. $\sim(\exists x)(Px \cdot Ix)$ | 15, CQ |
- (8)
- | | |
|---|-----------------------|
| 1. $(\exists x)(Ox \cdot \sim Gx) \supset (\exists x)(Hx \cdot Rx)$ | |
| 2. $\sim(\exists x)(Hx \vee Gx)$ | / $\sim(\exists x)Ox$ |
| 3. $(x)\sim(Hx \vee Gx)$ | 2, CQ |
| 4. $\sim(Hx \vee Gx)$ | 3, UI |
| 5. $\sim Hx \cdot \sim Gx$ | 4, DM |
| 6. $\sim Hx$ | 5, Simp |
| 7. $\sim Hx \vee \sim Rx$ | 6, Add |
| 8. $\sim(Hx \cdot Rx)$ | 7, DM |
| 9. $(x)\sim(Hx \cdot Rx)$ | 8, UG |
| 10. $\sim(\exists x)(Hx \cdot Rx)$ | 9, CQ |
| 11. $\sim(\exists x)(Ox \cdot \sim Gx)$ | 1, 10, MT |
| 12. $(x)\sim(Ox \cdot \sim Gx)$ | 11, CQ |
| 13. $\sim(Ox \cdot \sim Gx)$ | 12, UI |
| 14. $\sim Ox \vee Gx$ | 13, DM, DN |

Exercise 8.3

- | | |
|-------------------------|-----------------|
| 15. $\sim Gx$ | 5, Com, Simp |
| 16. $\sim Ox$ | 14, 15, Com, DS |
| 17. $(x)\sim Ox$ | 16, UI |
| 18. $\sim(\exists x)Ox$ | 17, CQ |

- (9)
- | | |
|---|----------------------------------|
| 1. $(x)\{[(Ax \vee Dx) \cdot Px] \supset (Ux \cdot Sx)\}$ | |
| 2. $\sim(\exists x)(Ux \cdot Sx)$ | / $\sim(\exists x)(Dx \cdot Px)$ |
| 3. $(x)\sim(Ux \cdot Sx)$ | 2, CQ |
| 4. $\sim(Ux \cdot Sx)$ | 3, UI |
| 5. $[(Ax \vee Dx) \cdot Px] \supset (Ux \cdot Sx)$ | 1, UI |
| 6. $\sim[(Ax \vee Dx) \cdot Px]$ | 4, 5, MT |
| 7. $\sim(Ax \vee Dx) \vee \sim Px$ | 6, DM |
| 8. $(\sim Ax \cdot \sim Dx) \vee \sim Px$ | 7, DM |
| 9. $(\sim Ax \vee \sim Px) \cdot (\sim Dx \vee \sim Px)$ | 8, Com, Dist, Com |
| 10. $\sim Dx \vee \sim Px$ | 9, Com, Simp |
| 11. $\sim(Dx \cdot Px)$ | 10, DM |
| 12. $(x)\sim(Dx \cdot Px)$ | 11, UG |
| 13. $\sim(\exists x)(Dx \cdot Px)$ | 12, CQ |

- (10)
- | | |
|---|-----------------------------------|
| 1. $\sim(\exists x)[Px \cdot (Gx \vee Hx)]$ | |
| 2. $(x)[Nx \supset (Px \cdot Hx)]$ | |
| 3. $(\exists x)(Px \cdot Cx) \vee (\exists x)(Px \cdot Nx)$ | / $(\exists x)(Cx \cdot \sim Gx)$ |
| 4. $(x)\sim[Px \cdot (Gx \vee Hx)]$ | 1, CQ |
| 5. $\sim[Px \cdot (Gx \vee Hx)]$ | 4, UI |
| 6. $\sim Px \vee \sim(Gx \vee Hx)$ | 5, DM |
| 7. $\sim Px \vee (\sim Gx \cdot \sim Hx)$ | 6, DM |
| 8. $(\sim Px \vee \sim Gx) \cdot (\sim Px \vee \sim Hx)$ | 7, Dist |
| 9. $\sim Px \vee \sim Gx$ | 8, Simp |
| 10. $\sim Px \vee \sim Hx$ | 8, Com, Simp |
| 11. $\sim(Px \cdot Hx)$ | 10, DM |
| 12. $Nx \supset (Px \cdot Hx)$ | 2, UI |
| 13. $\sim Nx$ | 11, 12, MT |
| 14. $\sim Nx \vee \sim Px$ | 13, Add |
| 15. $\sim Px \vee \sim Nx$ | 14, Com |
| 16. $\sim(Px \cdot Nx)$ | 15, DM |
| 17. $(x)\sim(Px \cdot Nx)$ | 16, UG |
| 18. $\sim(\exists x)(Px \cdot Nx)$ | 17, CQ |
| 19. $(\exists x)(Px \cdot Cx)$ | 3, 18, Com, DS |
| 20. $Pm \cdot Cm$ | 19, EI |
| 21. $(x)(\sim Px \vee \sim Gx)$ | 9, UG |
| 22. $\sim Pm \vee \sim Gm$ | 21, UI |
| 23. Pm | 20, Simp |
| 24. $\sim Gm$ | 22, 23, DN, DS |
| 25. Cm | 20, Com, Simp |

26. $Cm \cdot \sim Gm$ 24, 25, Conj
 27. $(\exists x)(Cx \cdot \sim Gx)$ 26, EG

Exercise 8.4

Part I

- (1) 1. $(x)(Ax \supset Bx)$
 2. $(x)(Ax \supset Cx)$ / $(x)[(Ax \supset (Bx \cdot Cx))]$
 3. Ax ACP
 4. $Ax \supset Bx$ 1, UI
 5. $Ax \supset Cx$ 2, UI
 6. Bx 3, 4, MP
 7. Cx 3, 5, MP
 8. $Bx \cdot Cx$ 6, 7, Conj
 9. $Ax \supset (Bx \cdot Cx)$ 3-8, CP
 10. $(x)[Ax \supset (Bx \cdot Cx)]$ 9, UG
- (2) 1. $(\exists x)Ax \supset (\exists x)(Bx \cdot Cx)$
 2. $(\exists x)(Cx \vee Dx) \supset (x)Ex$ / $(x)(Ax \supset Ex)$
 3. Ax ACP
 4. $(\exists x)Ax$ 3, EG
 5. $(\exists x)(Bx \cdot Cx)$ 1, 4, MP
 6. $Bm \cdot Cm$ 5, EI
 7. Cm 6, Com, Simp
 8. $Cm \vee Dm$ 7, Add
 9. $(\exists x)(Cx \vee Dx)$ 8, EG
 10. $(x)Ex$ 2, 9, MP
 11. Ex 10, UI
 12. $Ax \supset Ex$ 3-11, CP
 13. $(x)(Ax \supset Ex)$ 12, UG
- (3) 1. $(\exists x)Ax \supset (\exists x)(Bx \cdot Cx)$
 2. $\sim(\exists x)Cx$ / $(x)\sim Ax$
 3. $\sim(x)\sim Ax$ AIP
 4. $(\exists x)Ax$ 3, CQ
 5. $(\exists x)(Bx \cdot Cx)$ 1, 4, MP
 6. $Bm \cdot Cm$ 5, EI
 7. Cm 6, Com, Simp
 8. $(x)\sim Cx$ 2, CQ
 9. $\sim Cm$ 8, UI
 10. $Cm \cdot \sim Cm$ 7, 9, Conj
 11. $(x)\sim Ax$ 3-10, IP, DN

Exercise 8.4

- (4)
- | | |
|---|---|
| 1. $(x)(Ax \supset Cx)$ | |
| 2. $(\exists x)Cx \supset (\exists x)(Bx \cdot Dx)$ | $/ (\exists x)Ax \supset (\exists x)Bx$ |
| 3. $(\exists x)Ax$ | ACP |
| 4. Am | 3, EI |
| 5. $Am \supset Cm$ | 1, UI |
| 6. Cm | 4, 5, MP |
| 7. $(\exists x)Cx$ | 6, EG |
| 8. $(\exists x)(Bx \cdot Dx)$ | 2, 7, MP |
| 9. $Bn \cdot Dn$ | 8, EI |
| 10. Bn | 9, Simp |
| 11. $(\exists x)Bx$ | 10, EG |
| 12. $(\exists x)Ax \supset (\exists x)Bx$ | 3-11, CP |

- (5)
- | | |
|------------------------------------|------------------------|
| 1. $(x)(Ax \supset Bx)$ | |
| 2. $(x)[(Ax \cdot Bx) \supset Cx]$ | $/ (x)(Ax \supset Cx)$ |
| 3. Ax | ACP |
| 4. $Ax \supset Bx$ | 1, UI |
| 5. $(Ax \cdot Bx) \supset Cx$ | 2, UI |
| 6. Bx | 3, 4, MP |
| 7. $Ax \cdot Bx$ | 3, 6, Conj |
| 8. Cx | 5, 7, MP |
| 9. $Ax \supset Cx$ | 3-8, CP |
| 10. $(x)(Ax \supset Cx)$ | 9, UG |

- (6)
- | | |
|----------------------------------|-------------|
| 1. $(\exists x)Ax \supset (x)Bx$ | |
| 2. $An \supset \sim Bn$ | $/ \sim An$ |
| 3. An | AIP |
| 4. $\sim Bn$ | 2, 3, MP |
| 5. $(\exists x)Ax$ | 3, EG |
| 6. $(x)Bx$ | 1, 5, MP |
| 7. Bn | 6, UI |
| 8. $Bn \cdot \sim Bn$ | 4, 7, Conj |
| 9. $\sim An$ | 3-8, IP |

- (7)
- | | |
|-----------------------------------|------------------------|
| 1. $(x)[(Ax \vee Bx) \supset Cx]$ | |
| 2. $(x)[(Cx \vee Dx) \supset Ex]$ | $/ (x)(Ax \supset Ex)$ |
| 3. Ax | ACP |
| 4. $(Ax \vee Bx) \supset Cx$ | 1, UI |
| 5. $(Cx \vee Dx) \supset Ex$ | 2, UI |
| 6. $Ax \vee Bx$ | 3, Add |
| 7. Cx | 4, 6, MP |
| 8. $Cx \vee Dx$ | 7, Add |
| 9. Ex | 5, 8, MP |
| 10. $Ax \supset Ex$ | 3-9, CP |
| 11. $(x)(Ax \supset Ex)$ | 10, UG |

- (8) 1. $(\exists x)(Ax \vee Bx) \supset \sim(\exists x)Ax$ / $(x)\sim Ax$
 2. $\sim(x)\sim Ax$ AIP
 3. $(\exists x)Ax$ 2, CQ
 4. Am 3, EI
 5. $Am \vee Bm$ 4, Add
 6. $(\exists x)(Ax \vee Bx)$ 5, EG
 7. $\sim(\exists x)Ax$ 1, 6, MP
 8. $(x)\sim Ax$ 7, CQ
 9. $\sim Am$ 8, UI
 10. $Am \cdot \sim Am$ 4, 9, Conj
 11. $(x)\sim Ax$ 2-10, IP, DN
- (9) 1. $(x)(Ax \supset Bx)$
 2. $(x)(Cx \supset Dx)$ / $(\exists x)(Ax \vee Cx) \supset (\exists x)(Bx \vee Dx)$
 3. $(\exists x)(Ax \vee Cx)$ ACP
 4. $Am \vee Cm$ 3, EI
 5. $Am \supset Bm$ 1, UI
 6. $Cm \supset Dm$ 2, UI
 7. $(Am \supset Bm) \cdot (Cm \supset Dm)$ 5, 6, Conj
 8. $Bm \vee Dm$ 4, 7, CD
 9. $(\exists x)(Bx \vee Dx)$ 8, EG
 10. $(\exists x)(Ax \vee Cx) \supset (\exists x)(Bx \vee Dx)$ 3-9, CP
- (10) 1. $(x)(Ax \supset Bx)$
 2. $Am \vee An$ / $(\exists x)Bx$
 3. $\sim(\exists x)Bx$ AIP
 4. $(x)\sim Bx$ 3, CQ
 5. $Am \supset Bm$ 1, UI
 6. $An \supset Bn$ 1, UI
 7. $(Am \supset Bm) \cdot (An \supset Bn)$ 5, 6, Conj
 8. $Bm \vee Bn$ 2, 7, CD
 9. $\sim Bm$ 4, UI
 10. Bn 8, 9, DS
 11. $\sim Bn$ 4, UI
 12. $Bn \cdot \sim Bn$ 10, 11, Conj
 13. $\sim(\exists x)Bx$ 3-12, IP
 14. $(\exists x)Bx$ 13, DN
- (11) 1. $(x)[(Ax \vee Bx) \supset Cx]$
 2. $(x)[(Cx \vee Dx) \supset \sim Ax]$ / $(x)\sim Ax$
 3. $\sim(x)\sim Ax$ AIP
 4. $(\exists x)Ax$ 3, CQ
 5. Am 4, EI
 6. $(Am \vee Bm) \supset Cm$ 1, UI
 7. $Am \vee Bm$ 5, Add

Exercise 8.4

8. Cm	6, 7, MP
9. $(Cm \vee Dm) \supset \sim Am$	2, UI
10. $Cm \vee Dm$	8, Add
11. $\sim Am$	9, 10, MP
12. $Am \bullet \sim Am$	5, 11, Conj
13. $(x)\sim Ax$	3-12, IP, DN
 (12)	
1. $(\exists x)Ax \supset (x)(Bx \supset Cx)$	
2. $(\exists x)Dx \supset (x)\sim Cx$	$/ (x)[(Ax \bullet Dx) \supset \sim Bx]$
3. $Ax \bullet Dx$	ACP
4. Ax	3, Simp
5. $(\exists x)Ax$	4, EG
6. $(x)(Bx \supset Cx)$	1, 5, MP
7. $Bx \supset Cx$	6, UI
8. Dx	3, Com, Simp
9. $(\exists x)Dx$	8, EG
10. $(x)\sim Cx$	2, 9, MP
11. $\sim Cx$	10, UI
12. $\sim Bx$	7, 11, MT
13. $(Ax \bullet Dx) \supset \sim Bx$	3-12, CP
14. $(x)[(Ax \bullet Dx) \supset \sim Bx]$	13, UG
 (13)	
1. $(\exists x)Ax \supset (x)(Bx \supset Cx)$	
2. $(\exists x)Dx \supset (\exists x)Bx$	$/ (\exists x)(Ax \bullet Dx) \supset (\exists x)Cx$
3. $(\exists x)(Ax \bullet Dx)$	ACP
4. $Am \bullet Dm$	3, EI
5. Am	4, Simp
6. $(\exists x)Ax$	5, EG
7. $(x)(Bx \supset Cx)$	1, 6, MP
8. Dm	4, Com, Simp
9. $(\exists x)Dx$	8, EG
10. $(\exists x)Bx$	2, 9, MP
11. Bn	10, EI
12. $Bn \supset Cn$	7, UI
13. Cn	11, 12, MP
14. $(\exists x)Cx$	13, EG
15. $(\exists x)(Ax \bullet Dx) \supset (\exists x)Cx$	3-14, CP
 (14)	
1. $(\exists x)Ax \vee (\exists x)(Bx \bullet Cx)$	
2. $(x)(Ax \supset Cx)$	$/ (\exists x)Cx$
3. $\sim (\exists x)Cx$	AIP
4. $(x)\sim Cx$	3, CQ
5. $\sim Cx$	4, UI
6. $Ax \supset Cx$	2, UI
7. $\sim Ax$	5, 6, MT
8. $(x)\sim Ax$	7, UG

9. $\sim(\exists x)Ax$	8, CQ
10. $(\exists x)(Bx \cdot Cx)$	1, 9, DS
11. $Bm \cdot Cm$	10, EI
12. Cm	11, Com, Simp
13. $\sim Cm$	4, UI
14. $Cm \cdot \sim Cm$	12, 13, Conj
15. $(\exists x)Cx$	3-14, IP, DN

(15) 1. $(\exists x)Ax \supset (\exists x)(Bx \cdot Cx)$	
2. $(\exists x)Cx \supset (x)(Dx \cdot Ex)$	$/ (x)(Ax \supset Ex)$
3. Ax	ACP
4. $(\exists x)Ax$	3, EG
5. $(\exists x)(Bx \cdot Cx)$	1, 4, MP
6. $Bm \cdot Cm$	5, EI
7. Cm	6, Com, Simp
8. $(\exists x)Cx$	7, EG
9. $(x)(Dx \cdot \supset Ex)$	2, 8, MP
10. $Dx \cdot Ex$	9, UI
11. Ex	10, Com, Simp
12. $Ax \supset Ex$	3-11, CP
13. $(x)(Ax \supset Ex)$	12, UG

(16) 1. $(x)[(Ax \vee Bx) \supset Cx]$	
2. $(\exists x)(\sim Ax \vee Dx) \supset (x)Ex$	$/ (x)Cx \vee (x)Ex$
3. $\sim[(x)Cx \vee (x)Ex]$	AIP
4. $\sim(x)Cx \cdot \sim(x)Ex$	3, DM
5. $\sim(x)Cx$	4, Simp
6. $(\exists x)\sim Cx$	5, CQ
7. $\sim Cm$	6, EI
8. $(Am \vee Bm) \supset Cm$	1, UI
9. $\sim(Am \vee Bm)$	7, 8, MT
10. $\sim Am \cdot \sim Bm$	9, DM
11. $\sim Am$	10, Simp
12. $\sim Am \vee Dm$	11, Add
13. $(\exists x)(\sim Ax \vee Dx)$	12, EG
14. $(x)Ex$	2, 13, MP
15. $\sim(x)Ex$	4, Com, Simp
16. $(x)Ex \cdot \sim(x)Ex$	14, 15, Conj
17. $(x)Cx \vee (x)Ex$	3-16, IP, DN

(17) 1. $(x)Ax \equiv (\exists x)(Bx \cdot Cx)$	
2. $(x)(Cx \supset Bx)$	$/ (x)Ax \equiv (\exists x)Cx$
3. $[(x)Ax \supset (\exists x)(Bx \cdot Cx)] \cdot [(\exists x)(Bx \cdot Cx) \supset (x)Ax]$	1, Equiv
4. $(x)Ax$	ACP
5. $(x)Ax \supset (\exists x)(Bx \cdot Cx)$	3, Simp
6. $(\exists x)(Bx \cdot Cx)$	4, 5, MP

Exercise 8.4

7. $Bx \cdot Cm$	6, EI
8. Cm	7, Com, Simp
9. $(\exists x)Cx$	8, EG
10. $(x)Ax \supset (\exists x)Cx$	4-9, CP
11. $(\exists x)Cx$	ACP
12. Cn	11, EI
13. $Cn \supset Bn$	2, UI
14. Bn	12, 13, MP
15. $Bn \cdot Cn$	12, 14, Conj
16. $(\exists x)(Bx \cdot Cx)$	15, EG
17. $(\exists x)(Bx \cdot Cx) \supset (x)Ax$	3, Com, Simp
18. $(x)Ax$	16, 17, MP
19. $(\exists x)Cx \supset (x)Ax$	11-18, CP
20. $[(x)Ax \supset (\exists x)Cx] \cdot [(\exists x)Cx \supset (x)Ax]$	10, 19, Conj
21. $(x)Ax \equiv (\exists x)Cx$	20, Equiv
 (18)	
1. $(x)(Ax \equiv Bx)$	
2. $(x)[Ax \supset (Bx \supset Cx)]$	
3. $(\exists x)Ax \vee (\exists x)Bx$	$/ (\exists x)Cx$
4. $\sim(\exists x)Cx$	AIP
5. $(x)\sim Cx$	4, CQ
6. $\sim Cx$	5, UI
7. $Ax \supset (Bx \supset Cx)$	2, UI
8. $(Ax \cdot Bx) \supset Cx$	7, Exp
9. $\sim(Ax \cdot Bx)$	6, 8, MT
10. $Ax \equiv Bx$	1, UI
11. $(Ax \cdot Bx) \vee (\sim Ax \cdot \sim Bx)$	10, Equiv
12. $\sim Ax \cdot \sim Bx$	9, 11, DS
13. $\sim Ax$	12, Simp
14. $(x)\sim Ax$	13, UG
15. $\sim(\exists x)Ax$	14, CQ
16. $(\exists x)Bx$	3, 15, DS
17. $\sim Bx$	12, Com, Simp
18. $(x)\sim Bx$	17, UG
19. $\sim(\exists x)Bx$	18, CQ
20. $(\exists x)Bx \cdot \sim(\exists x)Bx$	16, 19, Conj
21. $(\exists x)Cx$	4-20, IP, DN
 (19)	
1. $(x)[Bx \supset (Cx \cdot Dx)]$	$/ (x)(Ax \supset Bx) \supset (x)(Ax \supset Dx)$
2. $(x)(Ax \supset Bx)$	ACP
3. Ax	ACP
4. $Ax \supset Bx$	2, UI
5. Bx	3, 4, MP
6. $Bx \supset (Cx \cdot Dx)$	1, UI
7. $Cx \cdot Dx$	5, 6, MP
8. Dx	7, Com, Simp

- | | |
|---|----------|
| 9. $Ax \supset Dx$ | 3-8, CP |
| 10. $(x)(Ax \supset Dx)$ | 9, UG |
| 11. $(x)(Ax \supset Bx) \supset (x)(Ax \supset Dx)$ | 2-10, CP |
- (20)
- | | |
|---|---|
| 1. $(x)[Ax \supset (Bx \cdot Cx)]$ | |
| 2. $(x)[Dx \supset (Ex \cdot Fx)]$ | $/ (x)(Cx \supset Dx) \supset (x)(Ax \supset Fx)$ |
| 3. $(x)(Cx \supset Dx)$ | ACP |
| 4. Ax | ACP |
| 5. $Ax \supset (Bx \cdot Cx)$ | 1, UI |
| 6. $Bx \cdot Cx$ | 4, 5, MP |
| 7. Cx | 6, Com, Simp |
| 8. $Cx \supset Dx$ | 3, UI |
| 9. Dx | 7, 8, MP |
| 10. $Dx \supset (Ex \cdot Fx)$ | 2, UI |
| 11. $Ex \cdot Fx$ | 9, 10, MP |
| 12. Fx | 11, Com, Simp |
| 13. $Ax \supset Fx$ | 4-12, CP |
| 14. $(x)(Ax \supset Fx)$ | 13, UG |
| 15. $(x)(Cx \supset Dx) \supset (x)(Ax \supset Fx)$ | 3-14, CP |
- (21)
- | | |
|---|-------------------|
| 1. $(\exists x)(Ax \vee Bx)$ | |
| 2. $(\exists x)Ax \supset (x)(Cx \supset Bx)$ | |
| 3. $(\exists x)Cx$ | $/ (\exists x)Bx$ |
| 4. $Am \vee Bm$ | 1, EI |
| 5. $\sim(\exists x)Bx$ | AIP |
| 6. $(x)\sim Bx$ | 5, CQ |
| 7. $\sim Bm$ | 6, UI |
| 8. Am | 4, 7, Com, DS |
| 9. $(\exists x)Ax$ | 8, EG |
| 10. $(x)(Cx \supset Bx)$ | 2, 9, MP |
| 11. Cn | 3, EI |
| 12. $Cn \supset Bn$ | 10, UI |
| 13. Bn | 11, 12, MP |
| 14. $\sim Bn$ | 6, UI |
| 15. $Bn \cdot \sim Bn$ | 13, 14, Conj |
| 16. $(\exists x)Bx$ | 5-15, IP, DN |

Part II

- | | |
|-----------------------------|--|
| (1) 1. $(x)(Ax \supset Wx)$ | |
| 2. $(x)(Rx \supset Cx)$ | $/ (x)[(Rx \cdot Ax) \supset (Cx \cdot Wx)]$ |
| 3. $Rx \cdot Ax$ | ACP |
| 4. Rx | 3, Simp |
| 5. Ax | 3, Com, Simp |
| 6. $Ax \supset Wx$ | 1, UI |

Exercise 8.4

- | | |
|--|------------|
| 7. $Rx \supset Cx$ | 2, UI |
| 8. Cx | 4, 7, MP |
| 9. Wx | 5, 6, MP |
| 10. $Cx \cdot Wx$ | 8, 9, Conj |
| 11. $(Rx \cdot Ax) \supset (Cx \cdot Wx)$ | 3-10, CP |
| 12. $(x)[(Rx \cdot Ax) \supset (Cx \cdot Wx)]$ | 11, UG |
- (2)
- | | |
|--|------------------------------|
| 1. $(x)(Sx \supset Wx)$ | |
| 2. $(\exists x)(Wx \cdot Sx) \supset Vo$ | / $(\exists x)Sx \supset Vo$ |
| 3. $(\exists x)Sx$ | ACP |
| 4. Sm | 3, EI |
| 5. $Sm \supset Wm$ | 1, UI |
| 6. Wm | 4, 5, MP |
| 7. $Wm \cdot Sm$ | 4, 6, Conj |
| 8. $(\exists x)(Wx \cdot Sx)$ | 7, EG |
| 9. Vo | 2, 8, MP |
| 10. $(\exists x)Sx \supset Vo$ | 3-9, CP |
- (3)
- | | |
|---|------------------------------|
| 1. $(x)(Jx \supset Wx) \supset (\exists x)(Ax \cdot Rx)$ | |
| 2. $(\exists x)(Jx \cdot \sim Wx) \supset (\exists x)(Ax \cdot Rx)$ | / $(\exists x)(Ax \cdot Rx)$ |
| 3. $\sim(\exists x)(Ax \cdot Rx)$ | AIP |
| 4. $\sim(x)(Jx \supset Wx)$ | 1, 3, MT |
| 5. $(\exists x)\sim(Jx \supset Wx)$ | 4, CQ |
| 6. $\sim(Jm \supset Wm)$ | 5, EI |
| 7. $\sim(\sim Jm \vee Wm)$ | 6, Impl |
| 8. $Jm \cdot \sim Wm$ | 7, DM, DN |
| 9. $\sim(\exists x)(Jx \cdot \sim Wx)$ | 2, 3, MT |
| 10. $(x)\sim(Jx \cdot \sim Wx)$ | 9, CQ |
| 11. $\sim(Jm \cdot \sim Wm)$ | 10, UI |
| 12. $\sim Jm \vee Wm$ | 11, DM, DN |
| 13. Jm | 8, Simp |
| 14. Wm | 12, 13, DN, DS |
| 15. $\sim Wm$ | 8, Com, Simp |
| 16. $Wm \cdot \sim Wm$ | 14, 15, Conj |
| 17. $(\exists x)(Ax \cdot Rx)$ | 3-16, IP, DN |
- (4)
- | | |
|--|------------------------|
| 1. $(x)[(Sx \vee Ux) \supset (Ix \cdot Cx)]$ | |
| 2. $(x)[(Cx \vee Vx) \supset (Rx \cdot Ax)]$ | / $(x)(Sx \supset Ax)$ |
| 3. Sx | ACP |
| 4. $Sx \vee Ux$ | 3, Add |
| 5. $(Sx \vee Ux) \supset (Ix \cdot Cx)$ | 1, UI |
| 6. $Ix \cdot Cx$ | 4, 5, MP |
| 7. Cx | 6, Com, Simp |
| 8. $Cx \vee Vx$ | 7, Add |
| 9. $(Cx \vee Vx) \supset (Rx \cdot Ax)$ | 2, UI |
| 10. $Rx \cdot Ax$ | 8, 9, MP |

- | | |
|--------------------------|---------------|
| 11. Ax | 10, Com, Simp |
| 12. $Sx \supset Ax$ | 3-11, CP |
| 13. $(x)(Sx \supset Ax)$ | 12, UG |
- (5)
- | | |
|--|-----------------------------------|
| 1. $(x)(Ax \supset Dx)$ | |
| 2. $(x)[(Ex \cdot Ax) \supset Cx] \cdot (x)[(Cx \cdot Dx) \supset Fx]$ | $/ (x)[(Ex \cdot Ax) \supset Fx]$ |
| 3. $Ex \cdot Ax$ | ACP |
| 4. $(x)[(Ex \cdot Ax) \supset Cx]$ | 2, Simp |
| 5. $(x)[(Cx \cdot Dx) \supset Fx]$ | 2, Com, Simp |
| 6. $(Ex \cdot Ax) \supset Cx$ | 4, UI |
| 7. $(Cx \cdot Dx) \supset Fx$ | 5, UI |
| 8. Cx | 3, 6, MP |
| 9. Ax | 3, Com, Simp |
| 10. $Ax \supset Dx$ | 1, UI |
| 11. Dx | 9, 10, MP |
| 12. $Cx \cdot Dx$ | 8, 11, Conj |
| 13. Fx | 7, 12, MP |
| 14. $(Ex \cdot Ax) \supset Fx$ | 3-13, CP |
| 15. $(x)[(Ex \cdot Ax) \supset Fx]$ | 14, UG |
- (6)
- | | |
|---|-------------------|
| 1. $(\exists x)Sx \supset (\exists x)(Ex \cdot Wx)$ | |
| 2. $(\exists x)(Ex \vee Vx) \supset (\exists x)Lx$ | |
| 3. $(\exists x)Vx \vee (\exists x)Sx$ | $/ (\exists x)Lx$ |
| 4. $\sim(\exists x)Lx$ | AIP |
| 5. $\sim(\exists x)(Ex \vee Vx)$ | 2, 4, MT |
| 6. $(x)\sim(Ex \vee Vx)$ | 5, CQ |
| 7. $\sim(Ex \vee Vx)$ | 6, UI |
| 8. $\sim Ex \cdot \sim Vx$ | 7, DM |
| 9. $\sim Ex$ | 8, Simp |
| 10. $\sim Ex \vee \sim Wx$ | 9, Add |
| 11. $\sim(Ex \cdot Wx)$ | 10, DM |
| 12. $(x)\sim(Ex \cdot Wx)$ | 11, UG |
| 13. $\sim(\exists x)(Ex \cdot Wx)$ | 12, CQ |
| 14. $\sim(\exists x)Sx$ | 1, 13, MT |
| 15. $(\exists x)Vx$ | 3, 14, Com, DS |
| 16. $\sim Vx$ | 8, Com, Simp |
| 17. $(x)\sim Vx$ | 16, UG |
| 18. $\sim(\exists x)Vx$ | 17, CQ |
| 19. $(\exists x)Vx \cdot \sim(\exists x)Vx$ | 15, 18, Conj |
| 20. $(\exists x)Lx$ | 4-19, IP, DN |
- (7)
- | | |
|---|------------------------------|
| 1. $(\exists x)Cx \supset (x)[Ax \supset (Sx \cdot Dx)]$ | |
| 2. $(x)(Cx \supset \sim Ax) \supset (\exists x)(Dx \cdot Sx)$ | $/ (\exists x)(Dx \cdot Sx)$ |
| 3. $\sim(\exists x)(Dx \cdot Sx)$ | AIP |
| 4. $\sim(x)(Cx \supset \sim Ax)$ | 2, 3, MT |

Exercise 8.4

5. $(\exists x)\sim(Cx \supset \sim Ax)$	4, CQ
6. $\sim(Cm \supset \sim Am)$	5, EI
7. $\sim(\sim Cm \vee \sim Am)$	6, Impl
8. $Cm \cdot Am$	7, DM, DN
9. Cm	8, Simp
10. $(\exists x)Cx$	9, EG
11. $(x)[Ax \supset (Sx \cdot Dx)]$	1, 10, MP
12. $Am \supset (Sm \cdot Dm)$	11, UI
13. Am	8, Com, Simp
14. $Sm \cdot Dm$	12, 13, MP
15. $Dm \cdot Sm$	14, Com
16. $(\exists x)(Dx \cdot Sx)$	15, EG
17. $(\exists x)(Dx \cdot Sx) \cdot \sim(\exists x)(Dx \cdot Sx)$	3, 16, Conj
18. $(\exists x)(Dx \cdot Sx)$	3-17, IP, DN

(8) 1. $(\exists x)Vx \supset (x)(Px \supset Ax)$	
2. $(\exists x)Px \supset (x)(Ax \supset Cx)$	/ $(\exists x)Vx \supset (x)(Px \supset Cx)$
3. $(\exists x)Vx$	ACP
4. Px	ACP
5. $(x)(Px \supset Ax)$	1, 3, MP
6. $(\exists x)Px$	4, EG
7. $(x)(Ax \supset Cx)$	2, 6, MP
8. $Px \supset Ax$	5, UI
9. Ax	4, 8, MP
10. $Ax \supset Cx$	7, UI
11. Cx	9, 10, MP
12. $Px \supset Cx$	4-11, CP
13. $(x)(Px \supset Cx)$	12, UG
14. $(\exists x)Vx \supset (x)(Px \supset Cx)$	3-13, CP

(9) 1. $(x)(Sx \supset \sim Px) \vee (x)(Rx \supset \sim Px)$	
2. $(\exists x)(Sx \cdot Px) \vee (x)(Wx \supset \sim Px)$	/ $\sim(\exists x)[(Rx \cdot Px) \cdot Wx]$
3. $(\exists x)[(Rx \cdot Px) \cdot Wx]$	AIP
4. $(Rm \cdot Pm) \cdot Wm$	3, EI
5. $Rm \cdot Pm$	4, Simp
6. $\sim(\sim Rm \vee \sim Pm)$	5, DN, DM
7. $\sim(Rm \supset \sim Pm)$	6, Impl
8. $(\exists x)\sim(Rx \supset \sim Px)$	7, EG
9. $\sim(x)(Rx \supset \sim Px)$	8, CQ
10. $(x)(Sx \supset \sim Px)$	1, 9, Com, DS
11. Wm	4, Com, Simp
12. Pm	5, Com, Simp
13. $Wm \cdot Pm$	11, 12, Conj
14. $\sim(\sim Wm \vee \sim Pm)$	13, DN, DM
15. $\sim(Wm \supset \sim Pm)$	14, Impl
16. $(\exists x)\sim(Wx \supset \sim Px)$	15, EG

- | | |
|---|----------------|
| 17. $\sim(x)(Wx \supset \sim Px)$ | 16, CQ |
| 18. $(\exists x)(Sx \cdot Px)$ | 2, 17, Com, DS |
| 19. $Sn \cdot Pn$ | 18, EI |
| 20. $Sn \supset \sim Pn$ | 10, UI |
| 21. Sn | 19, Simp |
| 22. $\sim Pn$ | 20, 21, MP |
| 23. Pn | 19, Com, Simp |
| 24. $Pn \cdot \sim Pn$ | 22, 23, Conj |
| 25. $\sim(\exists x)[(Rx \cdot Px) \cdot Wx]$ | 3-24, IP |

- | | |
|--|------------------------------|
| (10) 1. $(\exists x)(Gx \cdot Px) \vee (\exists x)(Ax \cdot Px)$ | |
| 2. $(\exists x)Px \supset (\exists x)[Ax \cdot (Cx \cdot Dx)]$ | / $(\exists x)(Dx \cdot Cx)$ |
| 3. $\sim(\exists x)(Dx \cdot Cx)$ | AIP |
| 4. $(x)\sim(Dx \cdot Cx)$ | 3, CQ |
| 5. $\sim(Dx \cdot Cx)$ | 4, UI |
| 6. $\sim(Cx \cdot Dx)$ | 5, Com |
| 7. $\sim(Cx \cdot Dx) \vee \sim Ax$ | 6, Add |
| 8. $\sim Ax \vee \sim(Cx \cdot Dx)$ | 7, Com |
| 9. $\sim[Ax \cdot (Cx \cdot Dx)]$ | 8, DM |
| 10. $(x)\sim[Ax \cdot (Cx \cdot Dx)]$ | 9, UG |
| 11. $\sim(\exists x)[(Ax \cdot (Cx \cdot Dx))]$ | 10, CQ |
| 12. $\sim(\exists x)Px$ | 2, 11, MT |
| 13. $(x)\sim Px$ | 12, CQ |
| 14. $\sim Px$ | 13, UI |
| 15. $\sim Px \vee \sim Gx$ | 14, Add |
| 16. $\sim Gx \vee \sim Px$ | 15, Com |
| 17. $\sim(Gx \cdot Px)$ | 16, DM |
| 18. $(x)\sim(Gx \cdot Px)$ | 17, UG |
| 19. $\sim(\exists x)(Gx \cdot Px)$ | 18, CQ |
| 20. $(\exists x)(Ax \cdot Px)$ | 1, 19, DS |
| 21. $Am \cdot Pm$ | 20, EI |
| 22. Pm | 21, Com, Simp |
| 23. $\sim Pm$ | 13, UI |
| 24. $Pm \cdot \sim Pm$ | 22, 23, Conj |
| 25. $\sim(\exists x)(Dx \cdot Cx)$ | 3-24, IP |
| 26. $(\exists x)(Dx \cdot Cx)$ | 25, DN |

(An alternate method appears in the back of the text.)

Exercise 8.5

Part I

1. All cats are animals.
No cats are dogs.
Therefore, no dogs are animals.