

|                                             |            |
|---------------------------------------------|------------|
| 2. $\sim E \vee (G \cdot W)$                |            |
| 3. $\sim P \vee \sim W$                     | / $\sim E$ |
| 4. $(\sim E \vee B) \cdot (\sim E \vee P)$  | 1, Dist    |
| 5. $(\sim E \vee P) \cdot (\sim E \vee B)$  | 4, Com     |
| 6. $\sim E \vee P$                          | 5, Simp    |
| 7. $(\sim E \vee G) \cdot (\sim E \vee W)$  | 2, Dist    |
| 8. $(\sim E \vee W) \cdot (\sim E \vee G)$  | 7, Com     |
| 9. $\sim E \vee W$                          | 8, Simp    |
| 10. $(\sim E \vee P) \cdot (\sim E \vee W)$ | 6, 9, Conj |
| 11. $\sim E \vee (P \cdot W)$               | 10, Dist   |
| 12. $(P \cdot W) \vee \sim E$               | 11, Com    |
| 13. $\sim(P \cdot W)$                       | 3, DM      |
| 14. $\sim E$                                | 12, 13, DS |

## Exercise 7.4

## Part I

|                                 |   |                              |   |
|---------------------------------|---|------------------------------|---|
| 1. $G \supset Q$                | 3 | 9. $D \equiv H$              | 1 |
| 2. $(R \cdot S) \supset N$      | 1 | 10. $S \supset G$            | 3 |
| 3. $\sim P \supset \sim H$      | 2 | 11. $J \supset F$            | 2 |
| 4. $B \equiv N$                 | 1 | 12. $(C \cdot H) \supset A$  | 3 |
| 5. $A$                          | 2 | 13. $W \equiv \sim T$        | 2 |
| 6. $\sim Q \supset \sim \sim L$ | 3 | 14. $(K \supset M) \equiv S$ | 1 |
| 7. $\sim \sim C \vee F$         | 1 | 15. $S \supset D$            | 1 |
| 8. $G \supset (N \supset Z)$    | 2 |                              |   |

## Part II

|                                        |                        |                                        |                      |
|----------------------------------------|------------------------|----------------------------------------|----------------------|
| 1. $J \supset M$                       | 1, Impl<br>2, 3, HS    | 5. $(G \supset B) \cdot (H \supset C)$ | 1, Trans<br>2, 3, CD |
| 2. $J \supset (F \supset N)$           | 1, Exp<br>2, 3, MP     | 6. $(J \cdot M) \supset Q$             | 1, Exp<br>2, 3, MP   |
| 3. $(C \supset A) \cdot (A \supset C)$ | 1, 2, Conj<br>3, Equiv | 7. $H \supset (C \supset R)$           | 1, Impl<br>2, Exp    |
| 4. $K \vee K$                          | 1, 2, CD<br>3, Taut    | 8. $T \supset G$                       | 1, Trans<br>2, 3, HS |

# Exercise 7.4

|                             |                    |                                              |                      |
|-----------------------------|--------------------|----------------------------------------------|----------------------|
| 9. $(K \cdot A) \supset F$  | 1, Exp<br>2, 3, MT | 13. $(N \supset A) \cdot (A \supset N)$      | 1, Trans<br>2, Equiv |
| 10. $\sim H \vee \sim H$    | 1, Impl<br>2, Taut | 14. $(E \cdot R) \vee (\sim E \cdot \sim R)$ | 1, Add<br>2, Equiv   |
| 11. $\sim S \vee \sim K$    | 1, Add<br>2, Impl  | 15. $Q \supset (G \supset W)$                | 1, Trans<br>2, Exp   |
| 12. $(M \cdot M) \supset D$ | 1, Exp<br>2, Taut  |                                              |                      |

## Part III

- (1) 1.  $(S \cdot K) \supset R$   
     2.  $K$  /  $S \supset R$   
     3.  $(K \cdot S) \supset R$  1, Com  
     4.  $K \supset (S \supset R)$  3, Exp  
     5.  $S \supset R$  2, 5, MP
- (2) 1.  $G \supset E$   
     2.  $H \supset \sim E$  /  $G \supset \sim H$   
     3.  $\sim E \supset \sim H$  2, Trans  
     4.  $E \supset \sim H$  3, DN  
     5.  $G \supset \sim H$  1, 4, HS
- (3) 1.  $\sim N \vee P$   
     2.  $(N \supset P) \supset T$  /  $T$   
     3.  $N \supset P$  1, Impl  
     4.  $T$  2, 3, MP
- (4) 1.  $(B \supset M) \cdot (D \supset M)$   
     2.  $B \vee D$  /  $M$   
     3.  $M \vee M$  1, 2, CD  
     4.  $M$  3, Taut
- (5) 1.  $Q \supset (F \supset A)$   
     2.  $R \supset (A \supset F)$   
     3.  $Q \cdot R$  /  $F \equiv A$   
     4.  $Q$  3, Simp  
     5.  $F \supset A$  1, 4, MP  
     6.  $R \cdot Q$  3, Com  
     7.  $R$  6, Simp  
     8.  $A \supset F$  2, 7, MP  
     9.  $(F \supset A) \cdot (A \supset F)$  5, 8, Conj  
     10.  $F \equiv A$  9, Equiv

- (6) 1.  $(B \supset G) \cdot (F \supset N)$   
     2.  $\sim(G \cdot N)$  /  $\sim(B \cdot F)$   
     3.  $\sim G \vee \sim N$  2, DM  
     4.  $(\sim G \supset \sim B) \cdot (\sim N \supset \sim F)$  1, Trans  
     5.  $\sim B \vee \sim F$  3, 4, CD  
     6.  $\sim(B \cdot F)$  5, DM
- (7) 1.  $(J \cdot R) \supset H$   
     2.  $(R \supset H) \supset M$   
     3.  $\sim(P \vee \sim J)$  /  $M \cdot \sim P$   
     4.  $J \supset (R \supset H)$  1, Exp  
     5.  $J \supset M$  2, 4, HS  
     6.  $\sim P \cdot J$  3, DM, DN  
     7.  $\sim P$  6, Simp  
     8.  $J$  6, Com, Simp  
     9.  $M$  5, 8, MP  
     10.  $M \cdot \sim P$  7, 9, Conj
- (8) 1.  $T$  /  $S \supset T$   
     2.  $T \vee \sim S$  1, Add  
     3.  $\sim S \vee T$  2, Com  
     4.  $S \supset T$  3, Impl
- (9) 1.  $(O \supset C) \cdot (\sim S \supset \sim D)$   
     2.  $(E \supset D) \cdot (\sim E \supset \sim C)$  /  $O \supset S$   
     3.  $O \supset C$  1, Simp  
     4.  $\sim S \supset \sim D$  1, Com, Simp  
     5.  $D \supset S$  4, Trans  
     6.  $E \supset D$  2, Simp  
     7.  $\sim E \supset \sim C$  2, Com, Simp  
     8.  $C \supset E$  7, Trans  
     9.  $O \supset E$  3, 8, HS  
     10.  $O \supset D$  6, 9, HS  
     11.  $O \supset S$  5, 10, HS
- (10) 1.  $\sim(U \cdot W) \supset X$   
     2.  $U \supset \sim U$  /  $\sim(U \vee \sim X)$   
     3.  $\sim U \vee \sim U$  2, Impl  
     4.  $\sim U$  3, Taut  
     5.  $\sim U \vee \sim W$  4, Add  
     6.  $\sim(U \cdot W)$  5, DM  
     7.  $X$  1, 6, MP  
     8.  $\sim U \cdot X$  4, 7, Conj  
     9.  $\sim U \cdot \sim \sim X$  8, DN  
     10.  $\sim(U \vee \sim X)$  9, DM

## Exercise 7.4

- (11)
- |    |                      |              |
|----|----------------------|--------------|
| 1. | $T \supset R$        |              |
| 2. | $T \supset \sim R$   | / $\sim T$   |
| 3. | $R \supset \sim T$   | 2, Trans, DN |
| 4. | $T \supset \sim T$   | 1, 3, HS     |
| 5. | $\sim T \vee \sim T$ | 4, Impl      |
| 6. | $\sim T$             | 5, Taut      |
- (12)
- |    |                 |                 |
|----|-----------------|-----------------|
| 1. | $S \vee \sim N$ |                 |
| 2. | $\sim S \vee Q$ | / $N \supset Q$ |
| 3. | $\sim N \vee S$ | 1, Com          |
| 4. | $N \supset S$   | 3, Impl         |
| 5. | $S \supset Q$   | 2, Impl         |
| 6. | $N \supset Q$   | 4, 5, HS        |
- (13)
- |    |                             |                 |
|----|-----------------------------|-----------------|
| 1. | $M \supset (U \supset H)$   |                 |
| 2. | $(H \vee \sim U) \supset F$ | / $M \supset F$ |
| 3. | $(\sim U \vee H) \supset F$ | 2, Com          |
| 4. | $(U \supset H) \supset F$   | 3, Impl         |
| 5. | $M \supset F$               | 1, 4, HS        |
- (14)
- |    |                                     |                |
|----|-------------------------------------|----------------|
| 1. | $\sim R \vee P$                     |                |
| 2. | $R \vee \sim P$                     | / $R \equiv P$ |
| 3. | $R \supset P$                       | 1, Impl        |
| 4. | $\sim P \vee R$                     | 2, Com         |
| 5. | $P \supset R$                       | 4, Impl        |
| 6. | $(R \supset P) \cdot (P \supset R)$ | 3, 5, Conj     |
| 7. | $R \equiv P$                        | 6, Equiv       |
- (15)
- |    |                                               |            |
|----|-----------------------------------------------|------------|
| 1. | $\sim B \supset H$                            |            |
| 2. | $\sim D \supset H$                            |            |
| 3. | $\sim (B \cdot D)$                            | / H        |
| 4. | $\sim B \vee \sim D$                          | 3, DM      |
| 5. | $(\sim B \supset H) \cdot (\sim D \supset H)$ | 1, 2, Conj |
| 6. | $H \vee H$                                    | 4, 5, CD   |
| 7. | $H$                                           | 6, Taut    |
- (16)
- |    |                           |                             |
|----|---------------------------|-----------------------------|
| 1. | $J \supset (G \supset L)$ | / $G \supset (J \supset L)$ |
| 2. | $(J \cdot G) \supset L$   | 1, Exp                      |
| 3. | $(G \cdot J) \supset L$   | 2, Com                      |
| 4. | $G \supset (J \supset L)$ | 3, Exp                      |

- (17) 1.  $S \supset (L \cdot M)$   
 2.  $M \supset (L \supset R)$  /  $S \supset R$   
 3.  $(M \cdot L) \supset R$  2, Exp  
 4.  $(L \cdot M) \supset R$  3, Com  
 5.  $S \supset R$  1, 4, HS
- (18) 1.  $F \supset (A \cdot K)$   
 2.  $G \supset (\sim A \cdot \sim K)$   
 3.  $F \vee G$  /  $A \equiv K$   
 4.  $[F \supset (A \cdot K)] \cdot [G \supset (\sim A \cdot \sim K)]$  1, 2, Conj  
 5.  $(A \cdot K) \vee (\sim A \cdot \sim K)$  3, 4, CD  
 6.  $A \equiv K$  5, Equiv
- (19) 1.  $(I \supset E) \supset C$   
 2.  $C \supset \sim C$  / I  
 3.  $\sim C \vee \sim C$  2, Impl  
 4.  $\sim C$  3, Taut  
 5.  $\sim(I \supset E)$  1, 4, MT  
 6.  $\sim(\sim I \vee E)$  5, Impl  
 7.  $I \cdot \sim E$  6, DM, DN  
 8. I 7, Simp
- (20) 1.  $T \supset G$   
 2.  $S \supset G$  /  $(T \vee S) \supset G$   
 3.  $\sim T \vee G$  1, Impl  
 4.  $\sim S \vee G$  2, Impl  
 5.  $(\sim T \vee G) \cdot (\sim S \vee G)$  3, 4, Conj  
 6.  $(\sim T \cdot \sim S) \vee G$  5, Com, Dist, Com  
 7.  $\sim(T \vee S) \vee G$  6, DM  
 8.  $(T \vee S) \supset G$  7, Impl
- (21) 1.  $H \supset U$  /  $H \supset (U \vee T)$   
 2.  $\sim H \vee U$  1, Impl  
 3.  $(\sim H \vee U) \vee T$  2, Add  
 4.  $\sim H \vee (U \vee T)$  3, Assoc  
 5.  $H \supset (U \vee T)$  4, Impl
- (22) 1.  $Q \supset (W \cdot D)$  /  $Q \supset W$   
 2.  $\sim Q \vee (W \cdot D)$  1, Impl  
 3.  $(\sim Q \vee W) \cdot (\sim Q \vee D)$  2, Dist  
 4.  $\sim Q \vee W$  3, Simp  
 5.  $Q \supset W$  4, Impl

# Exercise 7.4

(23) 1.  $P \supset (\sim E \supset B)$

- |                             |             |
|-----------------------------|-------------|
| 2. $\sim(B \vee E)$         | / $\sim P$  |
| 3. $\sim(E \vee B)$         | 2, Com      |
| 4. $\sim(\sim E \supset B)$ | 3, DN, Impl |
| 5. $\sim P$                 | 1, 4, MT    |

(24) 1.  $(G \supset J) \supset (H \supset Q)$

- |                     |              |
|---------------------|--------------|
| 2. $J \cdot \sim Q$ | / $\sim H$   |
| 3. $J$              | 2, Simp      |
| 4. $J \vee \sim G$  | 3, Add       |
| 5. $\sim G \vee J$  | 4, Com       |
| 6. $G \supset J$    | 5, Impl      |
| 7. $H \supset Q$    | 1, 6, MP     |
| 8. $\sim Q$         | 2, Com, Simp |
| 9. $\sim H$         | 7, 8, MT     |

(25) 1.  $I \vee (N \cdot F)$

- |                                  |              |
|----------------------------------|--------------|
| 2. $I \supset F$                 | / F          |
| 3. $(I \vee N) \cdot (I \vee F)$ | 1, Dist      |
| 4. $I \vee F$                    | 3, Com, Simp |
| 5. $\sim I \supset F$            | 4, DN, Impl  |
| 6. $\sim F \supset \sim I$       | 2, Trans     |
| 7. $\sim F \supset F$            | 5, 6, HS     |
| 8. $F \vee F$                    | 7, Impl, DN  |
| 9. $F$                           | 8, Taut      |

(26) 1.  $K \equiv R$

- |                                             |            |
|---------------------------------------------|------------|
| 2. $K \supset (R \supset P)$                |            |
| 3. $\sim P$                                 | / $\sim R$ |
| 4. $(K \cdot R) \vee (\sim K \cdot \sim R)$ | 1, Equiv   |
| 5. $(K \cdot R) \supset P$                  | 2, Exp     |
| 6. $\sim(K \cdot R)$                        | 3, 5, MT   |
| 7. $\sim K \cdot \sim R$                    | 4, 6, DS   |
| 8. $\sim R \cdot \sim K$                    | 7, Com     |
| 9. $\sim R$                                 | 8, Simp    |

(27) 1.  $C \supset (\sim L \supset Q)$

- |                                 |            |
|---------------------------------|------------|
| 2. $L \supset \sim C$           |            |
| 3. $\sim Q$                     | / $\sim C$ |
| 4. $(C \cdot \sim L) \supset Q$ | 1, Exp     |
| 5. $\sim(C \cdot \sim L)$       | 3, 4, MT   |
| 6. $\sim C \vee \sim \sim L$    | 5, DM      |
| 7. $\sim C \vee L$              | 6, DN      |
| 8. $C \supset L$                | 7, Impl    |
| 9. $C \supset \sim C$           | 2, 8, HS   |

10.  $\sim C \vee \sim C$ 

9, Impl

11.  $\sim C$ 

10, Taut

(28) 1.  $(E \supset A) \cdot (F \supset A)$ 2.  $E \vee G$ 3.  $F \vee \sim G$ 

/ A

4.  $\sim E \vee G$ 

2, DN

5.  $\sim E \supset G$ 

4, Impl

6.  $\sim F \vee G$ 

3, DN

7.  $\sim F \supset \sim G$ 

6, Impl

8.  $G \supset F$ 

7, Trans

9.  $\sim E \supset F$ 

5, 8, HS

10.  $\sim E \vee F$ 

9, Impl

11.  $E \vee F$ 

10, DN

12.  $A \vee A$ 

1, 11, CD

13. A

12, Taut

(29) 1.  $(F \cdot H) \supset N$ 2.  $F \vee S$ 

3. H

/  $N \vee S$ 4.  $(H \cdot F) \supset N$ 

1, Com

5.  $H \supset (F \supset N)$ 

4, Exp

6.  $F \supset N$ 

3, 5, MP

7.  $\sim N \supset \sim F$ 

6, Trans

8.  $\sim F \vee S$ 

2, DN

9.  $\sim F \supset S$ 

8, Impl

10.  $\sim N \supset S$ 

7, 9, HS

11.  $\sim \sim N \vee S$ 

10, Impl

12.  $N \vee S$ 

11, DN

(30) 1.  $T \supset (H \cdot J)$ 2.  $(H \vee N) \supset T$ /  $T \equiv H$ 3.  $\sim T \vee (H \cdot J)$ 

1, Impl

4.  $(\sim T \vee H) \cdot (\sim T \vee J)$ 

3, Dist

5.  $\sim T \vee H$ 

4, Simp

6.  $T \supset H$ 

5, Impl

7.  $\sim(H \vee N) \vee T$ 

2, Impl

8.  $(\sim H \cdot \sim N) \vee T$ 

7, DM

9.  $T \vee (\sim H \cdot \sim N)$ 

8, Com

10.  $(T \vee \sim H) \cdot (T \vee \sim N)$ 

9, Dist

11.  $(\sim H \vee T) \cdot (T \vee \sim N)$ 

10, Com

12.  $\sim H \vee T$ 

11, Simp

13.  $H \supset T$ 

12, Impl

14.  $(T \supset H) \cdot (H \supset T)$ 

6, 13, Conj

15.  $T \equiv H$ 

14, Equiv

# Exercise 7.4

- (31) 1.  $T \supset \sim(A \supset N)$   
 2.  $T \vee N$  /  $T \equiv \sim N$   
 3.  $\sim T \vee \sim(A \supset N)$  1, Impl  
 4.  $\sim T \vee \sim(\sim A \vee N)$  3, Impl  
 5.  $\sim T \vee (A \cdot \sim N)$  4, DM, DN  
 6.  $(\sim T \vee A) \cdot (\sim T \vee \sim N)$  5, Dist  
 7.  $\sim T \vee \sim N$  6, Com, Simp  
 8.  $T \supset \sim N$  7, Impl  
 9.  $N \vee T$  2, Com  
 10.  $\sim N \supset T$  9, DN, Impl  
 11.  $(T \supset \sim N) \cdot (\sim N \supset T)$  8, 10, Conj  
 12.  $T \equiv \sim N$  11, Equiv
- (32) 1.  $(D \supset E) \supset (E \supset D)$   
 2.  $(D \equiv E) \supset \sim(G \cdot \sim H)$   
 3.  $E \cdot G$  /  $G \cdot H$   
 4.  $E$  3, Simp  
 5.  $E \vee \sim D$  4, Add  
 6.  $\sim D \vee E$  5, Com  
 7.  $D \supset E$  6, Impl  
 8.  $E \supset D$  1, 7, MP  
 9.  $(D \supset E) \cdot (E \supset D)$  7, 8, Conj  
 10.  $D \equiv E$  9, Equiv  
 11.  $\sim(G \cdot \sim H)$  2, 10, MP  
 12.  $\sim G \vee H$  11, DM, DN  
 13.  $G$  3, Com, Simp  
 14.  $H$  12, 13, DN, DS  
 15.  $G \cdot H$  13, 14, Conj
- (33) 1.  $(O \supset R) \supset S$   
 2.  $(P \supset R) \supset \sim S$  /  $\sim R$   
 3.  $S \supset \sim(P \supset R)$  2, Trans  
 4.  $(O \supset R) \supset \sim(P \supset R)$  1, 3, HS  
 5.  $\sim(O \supset R) \vee \sim(P \supset R)$  4, Impl  
 6.  $\sim(\sim O \vee R) \vee \sim(\sim P \vee R)$  5, Impl  
 7.  $(O \cdot \sim R) \vee (P \cdot \sim R)$  6, DM, DN  
 8.  $(O \vee P) \cdot \sim R$  7, Com, Dist, Com  
 9.  $\sim R$  8, Com, Simp
- (34) 1.  $(L \vee P) \supset U$   
 2.  $(M \supset U) \supset I$   
 3.  $P$  /  $I$   
 4.  $P \vee L$  3, Add  
 5.  $L \vee P$  4, Com



- |                    |          |
|--------------------|----------|
| 6. $U$             | 1, 5, MP |
| 7. $U \vee \sim M$ | 6, Add   |
| 8. $\sim M \vee U$ | 7, Com   |
| 9. $M \supset U$   | 8, Impl  |
| 10. $I$            | 2, 9, MP |

- (35)
- |                                             |                    |
|---------------------------------------------|--------------------|
| 1. $A \equiv W$                             |                    |
| 2. $\sim A \vee \sim W$                     |                    |
| 3. $R \supset A$                            | $/ \sim(W \vee R)$ |
| 4. $(A \cdot W) \vee (\sim A \cdot \sim W)$ | 1, Equiv           |
| 5. $\sim(A \cdot W)$                        | 2, DM              |
| 6. $\sim A \cdot \sim W$                    | 4, 5, DS           |
| 7. $\sim A$                                 | 6, Simp            |
| 8. $\sim R$                                 | 3, 7, MT           |
| 9. $\sim W$                                 | 6, Com, Simp       |
| 10. $\sim W \cdot \sim R$                   | 8, 9, Conj         |
| 11. $\sim(W \vee R)$                        | 10, DM             |

- (36)
- |                                               |              |
|-----------------------------------------------|--------------|
| 1. $(S \vee T) \supset (S \supset \sim T)$    |              |
| 2. $(S \supset \sim T) \supset (T \supset K)$ |              |
| 3. $S \vee T$                                 | $/ S \vee K$ |
| 4. $S \supset \sim T$                         | 1, 3, MP     |
| 5. $T \supset K$                              | 2, 4, MP     |
| 6. $\sim S \supset T$                         | 3, DN, Impl  |
| 7. $\sim S \supset K$                         | 5, 6, HS     |
| 8. $S \vee K$                                 | 7, Impl, DN  |

- (37)
- |                                             |               |
|---------------------------------------------|---------------|
| 1. $G \equiv M$                             |               |
| 2. $G \vee M$                               |               |
| 3. $G \supset (M \supset T)$                | $/ T$         |
| 4. $(G \cdot M) \vee (\sim G \cdot \sim M)$ | 1, Equiv      |
| 5. $\sim(\sim G \cdot \sim M)$              | 2, DN, DN, DM |
| 6. $G \cdot M$                              | 4, 5, Com, DS |
| 7. $(G \cdot M) \supset T$                  | 3, Exp        |
| 8. $T$                                      | 6, 7, MP      |

- (38)
- |                                            |                   |
|--------------------------------------------|-------------------|
| 1. $O \supset (Q \cdot N)$                 |                   |
| 2. $(N \vee E) \supset S$                  | $/ O \supset S$   |
| 3. $\sim O \vee (Q \cdot N)$               | 1, Impl           |
| 4. $(\sim O \vee Q) \cdot (\sim O \vee N)$ | 3, Dist           |
| 5. $\sim O \vee N$                         | 4, Com, Simp      |
| 6. $O \supset N$                           | 5, Impl           |
| 7. $\sim(N \vee E) \vee S$                 | 2, Impl           |
| 8. $(\sim N \cdot \sim E) \vee S$          | 7, DM             |
| 9. $(\sim N \vee S) \cdot (\sim E \vee S)$ | 8, Com, Dist, Com |

## Exercise 7.4

- |                     |           |
|---------------------|-----------|
| 10. $\sim N \vee S$ | 9, Simp   |
| 11. $N \supset S$   | 10, Impl  |
| 12. $O \supset S$   | 6, 11, HS |

- (39)
- |                                                                      |            |
|----------------------------------------------------------------------|------------|
| 1. $H \equiv I$                                                      |            |
| 2. $H \supset (I \supset F)$                                         |            |
| 3. $\sim(H \vee I) \supset F$                                        | / F        |
| 4. $(H \cdot I) \vee (\sim H \cdot \sim I)$                          | 1, Equiv   |
| 5. $(H \cdot I) \supset F$                                           | 2, Exp     |
| 6. $(\sim H \cdot \sim I) \supset F$                                 | 3, DM      |
| 7. $[(H \cdot I) \supset F] \cdot [(\sim H \cdot \sim I) \supset F]$ | 5, 6, Conj |
| 8. $F \vee F$                                                        | 4, 7, CD   |
| 9. F                                                                 | 8, Taut    |

- (40)
- |                                                              |                                   |
|--------------------------------------------------------------|-----------------------------------|
| 1. $P \supset A$                                             |                                   |
| 2. $Q \supset B$                                             | / $(P \vee Q) \supset (A \vee B)$ |
| 3. $\sim P \vee A$                                           | 1, Impl                           |
| 4. $\sim Q \vee B$                                           | 2, Impl                           |
| 5. $(\sim P \vee A) \vee B$                                  | 3, Add                            |
| 6. $(\sim Q \vee B) \vee A$                                  | 4, Add                            |
| 7. $\sim P \vee (A \vee B)$                                  | 5, Assoc                          |
| 8. $\sim Q \vee (A \vee B)$                                  | 6, Assoc, Com                     |
| 9. $[\sim P \vee (A \vee B)] \cdot [\sim Q \vee (A \vee B)]$ | 7, 8, Conj                        |
| 10. $(\sim P \cdot \sim Q) \vee (A \vee B)$                  | 9, Com, Dist                      |
| 11. $\sim(P \vee Q) \vee (A \vee B)$                         | 10, DM                            |
| 12. $(P \vee Q) \supset (A \vee B)$                          | 11, Impl                          |

## Part IV

- (1)
- |                              |                 |
|------------------------------|-----------------|
| 1. $D \supset C$             |                 |
| 2. $\sim(C \cdot \sim S)$    | / $D \supset S$ |
| 3. $\sim C \vee \sim \sim S$ | 2, DM           |
| 4. $C \supset \sim \sim S$   | 3, Impl         |
| 5. $C \supset S$             | 4, DN           |
| 6. $D \supset S$             | 1, 5, HS        |
- (2)
- |                            |            |
|----------------------------|------------|
| 1. $C \supset (D \cdot M)$ |            |
| 2. $\sim M$                | / $\sim C$ |
| 3. $\sim M \vee \sim D$    | 2, Add     |
| 4. $\sim D \vee \sim M$    | 3, Com     |
| 5. $\sim(D \cdot M)$       | 4, DM      |
| 6. $\sim C$                | 1, 5, MP   |

- (3) 1.  $\sim(M \vee S)$  /  $S \supset U$   
 2.  $\sim M \cdot \sim S$  1, DM  
 3.  $\sim S \cdot \sim M$  2, Com  
 4.  $\sim S$  3, Simp  
 5.  $\sim S \vee U$  4, Add  
 6.  $S \supset U$  5, Impl
- (4) 1.  $D \supset P$  /  $(I \cdot D) \supset P$   
 2.  $\sim D \vee P$  1, Impl  
 3.  $(\sim D \vee P) \vee \sim I$  2, Add  
 4.  $\sim I \vee (\sim D \vee P)$  3, Com  
 5.  $(\sim I \vee \sim D) \vee P$  4, Assoc  
 6.  $\sim(I \cdot D) \vee P$  5, DM  
 7.  $(I \cdot D) \supset P$  6, Impl
- (5) 1.  $(E \vee P) \supset U$  /  $E \supset U$   
 2.  $\sim(E \vee P) \vee U$  1, Impl  
 3.  $(\sim E \cdot \sim P) \vee U$  2, DM  
 4.  $U \vee (\sim E \cdot \sim P)$  3, Com  
 5.  $(U \vee \sim E) \cdot (U \vee \sim P)$  4, Dist  
 6.  $U \vee \sim E$  5, Simp  
 7.  $\sim E \vee U$  6, Com  
 8.  $E \supset U$  7, Impl
- (6) 1.  $(W \cdot C) \supset \sim S$   
 2.  $W$  /  $C \supset \sim S$   
 3.  $W \supset (C \supset \sim S)$  1, Exp  
 4.  $C \supset \sim S$  2, 3, MP
- (7) 1.  $G \supset A$   
 2.  $G \supset L$  /  $G \supset (A \cdot L)$   
 3.  $\sim G \vee A$  1, Impl  
 4.  $\sim G \vee L$  2, Impl  
 5.  $(\sim G \vee A) \cdot (\sim G \vee L)$  3, 4, Conj  
 6.  $\sim G \vee (A \cdot L)$  5, Dist  
 7.  $G \supset (A \cdot L)$  6, Impl
- (8) 1.  $S \supset (D \cdot I)$   
 2.  $S \vee (U \cdot D)$  /  $D$   
 3.  $\sim S \vee (D \cdot I)$  1, Impl  
 4.  $(\sim S \vee D) \cdot (\sim S \vee I)$  3, Dist  
 5.  $\sim S \vee D$  4, Simp  
 6.  $S \supset D$  5, Impl  
 7.  $(S \vee U) \cdot (S \vee D)$  2, Dist  
 8.  $(S \vee D) \cdot (S \vee U)$  7, Com

## Exercise 7.5

|                             |              |
|-----------------------------|--------------|
| 9. $S \vee D$               | 8, Simp      |
| 10. $\sim S \supset D$      | 9, DN, Impl  |
| 11. $\sim D \supset \sim S$ | 6, Trans     |
| 12. $\sim D \supset D$      | 10, 11, HS   |
| 13. $D \vee D$              | 12, Impl, DN |
| 14. $D$                     | 13, Taut     |

|                                          |                                       |
|------------------------------------------|---------------------------------------|
| (9) 1. $T \supset (G \supset R)$         | $/ \sim R \supset (G \supset \sim T)$ |
| 2. $\sim T \vee (G \supset R)$           | 1, Impl                               |
| 3. $\sim T \vee (\sim G \vee R)$         | 2, Impl                               |
| 4. $(\sim T \vee \sim G) \vee R$         | 3, Assoc                              |
| 5. $R \vee (\sim T \vee \sim G)$         | 4, Com                                |
| 6. $R \vee (\sim G \vee \sim T)$         | 5, Com                                |
| 7. $\sim R \supset (\sim G \vee \sim T)$ | 6, DN, Impl                           |
| 8. $\sim R \supset (G \supset \sim T)$   | 7, Impl                               |

|                                                                      |            |
|----------------------------------------------------------------------|------------|
| (10) 1. $(A \cdot U) \equiv \sim R$                                  |            |
| 2. $\sim(\sim R \vee \sim A)$                                        | $/ \sim U$ |
| 3. $[(A \cdot U) \supset \sim R] \cdot [\sim R \supset (A \cdot U)]$ | 1, Equiv   |
| 4. $(A \cdot U) \supset \sim R$                                      | 3, Simp    |
| 5. $\sim \sim R \cdot \sim \sim A$                                   | 2, DM      |
| 6. $\sim \sim R$                                                     | 5, Simp    |
| 7. $\sim(A \cdot U)$                                                 | 4, 6, MT   |
| 8. $\sim A \vee \sim U$                                              | 7, DM      |
| 9. $\sim \sim A \cdot \sim \sim R$                                   | 5, Com     |
| 10. $\sim \sim A$                                                    | 9, Simp    |
| 11. $\sim U$                                                         | 8, 10, DS  |

## Exercise 7.5

### Part I

|                            |                           |
|----------------------------|---------------------------|
| (1) 1. $N \supset O$       |                           |
| 2. $N \supset P$           | $/ N \supset (O \cdot P)$ |
| 3. $N$                     | ACP                       |
| 4. $O$                     | 1, 3, MP                  |
| 5. $P$                     | 2, 3, MP                  |
| 6. $O \cdot P$             | 4, 5, Conj                |
| 7. $N \supset (O \cdot P)$ | 3-6, CP                   |

|                            |                 |
|----------------------------|-----------------|
| (2) 1. $F \supset E$       |                 |
| 2. $(F \cdot E) \supset R$ | $/ F \supset R$ |
| 3. $F$                     | ACP             |
| 4. $E$                     | 1, 3, MP        |
| 5. $F \cdot E$             | 3, 4, Conj      |
| 6. $R$                     | 2, 5, MP        |
| 7. $F \supset R$           | 3-6, CP         |

- (3) 1.  $G \supset T$   
 2.  $(T \vee S) \supset K$  /  $G \supset K$   
 3.  $G$  ACP  
 4.  $T$  1, 3, MP  
 5.  $T \vee S$  4, Add  
 6.  $K$  2, 5, MP  
 7.  $G \supset K$  3-6, CP
- (4) 1.  $(G \vee H) \supset (S \cdot T)$   
 2.  $(T \vee U) \supset (C \cdot D)$  /  $G \supset C$   
 3.  $G$  ACP  
 4.  $G \vee H$  3, Add  
 5.  $S \cdot T$  1, 4, MP  
 6.  $T$  5, Com, Simp  
 7.  $T \vee U$  6, Add  
 8.  $C \cdot D$  2, 7, MP  
 9.  $C$  8, Simp  
 10.  $G \supset C$  3-9, CP
- (5) 1.  $A \supset \sim(A \vee E)$  /  $A \supset F$   
 2.  $A$  ACP  
 3.  $\sim(A \vee E)$  1, 2, MP  
 4.  $\sim A \cdot \sim E$  3, DM  
 5.  $\sim A$  4, Simp  
 6.  $A \vee F$  2, Add  
 7.  $F$  5, 6, DS  
 8.  $A \supset F$  2-7, CP
- (6) 1.  $J \supset (K \supset L)$   
 2.  $J \supset (M \supset L)$   
 3.  $\sim L / J \supset \sim(K \vee M)$   
 4.  $J$  ACP  
 5.  $K \supset L$  1, 4, MP  
 6.  $M \supset L$  2, 4, MP  
 7.  $\sim K$  3, 5, MT  
 8.  $\sim M$  3, 6, MT  
 9.  $\sim K \cdot \sim M$  7, 8, Conj  
 10.  $\sim(K \vee M)$  9, DM  
 11.  $J \supset \sim(K \vee M)$  4-10, CP
- (7) 1.  $M \vee (N \cdot O)$  /  $\sim N \supset M$   
 2.  $\sim M$  ACP  
 3.  $N \cdot O$  1, 2, DS  
 4.  $N$  3, Simp  
 5.  $\sim M \supset N$  2-4, CP  
 6.  $\sim N \supset M$  5, Trans, DN

# Exercise 7.5

- (8)
- |                                        |               |
|----------------------------------------|---------------|
| 1. $P \supset (Q \vee R)$              |               |
| 2. $(P \supset R) \supset (S \cdot T)$ |               |
| 3. $Q \supset R$                       | / T           |
| 4. P                                   | ACP           |
| 5. $Q \vee R$                          | 1, 4, MP      |
| 6. $R \vee Q$                          | 5, Com        |
| 7. $\sim R \supset Q$                  | 6, DN, Impl   |
| 8. $\sim R \supset R$                  | 3, 7, HS      |
| 9. $R \vee R$                          | 8, Impl, DN   |
| 10. R                                  | 9, Taut       |
| 11. $P \supset R$                      | 4-10, CP      |
| 12. $S \cdot T$                        | 2, 11, MP     |
| 13. T                                  | 12, Com, Simp |

- (9)
- |                                            |               |
|--------------------------------------------|---------------|
| 1. $H \supset (I \supset N)$               |               |
| 2. $(H \supset \sim I) \supset (M \vee N)$ |               |
| 3. $\sim N$                                | / M           |
| 4. H                                       | ACP           |
| 5. $I \supset N$                           | 1, 4, MP      |
| 6. $\sim I$                                | 3, 5, MT      |
| 7. $H \supset \sim I$                      | 4-6, CP       |
| 8. $M \vee N$                              | 2, 7, MP      |
| 9. M                                       | 3, 8, Com, DS |

- (10)
- |                                                            |  |                         |
|------------------------------------------------------------|--|-------------------------|
| 1. $C \supset (A \cdot D)$                                 |  |                         |
| 2. $B \supset (A \cdot E)$                                 |  | $/(C \vee B) \supset A$ |
| 3. $C \vee B$                                              |  | ACP                     |
| 4. $[C \supset (A \cdot D)] \cdot [B \supset (A \cdot E)]$ |  | 1, 2, Conj              |
| 5. $(A \cdot D) \vee (A \cdot E)$                          |  | 3, 4, CD                |
| 6. $A \cdot (D \vee E)$                                    |  | 5, Dist                 |
| 7. A                                                       |  | 6, Simp                 |
| 8. $(C \vee B) \supset A$                                  |  | 3-7, CP                 |

- (11)
- |                               |                             |
|-------------------------------|-----------------------------|
| 1. $M \supset (K \supset L)$  |                             |
| 2. $(L \vee N) \supset J$     | / $M \supset (K \supset J)$ |
| 3. M                          | ACP                         |
| 4. K                          | ACP                         |
| 5. $K \supset L$              | 1, 3, MP                    |
| 6. L                          | 4, 5, MP                    |
| 7. $L \vee N$                 | 6, Add                      |
| 8. J                          | 2, 7, MP                    |
| 9. $K \supset J$              | 4-8, CP                     |
| 10. $M \supset (K \supset J)$ | 3-9, CP                     |

- (12) 1.  $F \supset (G \cdot H)$  /  $(A \supset F) \supset (A \supset H)$   
       2.  $A \supset F$  ACP  
           3.  $A$  ACP  
           4.  $F$  2, 3, MP  
           5.  $G \cdot H$  1, 4, MP  
           6.  $H$  5, Com, Simp  
           7.  $A \supset H$  3-6, CP  
       8.  $(A \supset F) \supset (A \supset H)$  2-7, CP

- (13) 1.  $R \supset B$   
       2.  $R \supset (B \supset F)$   
       3.  $B \supset (F \supset H)$  /  $R \supset H$   
           4.  $R$  ACP  
           5.  $B$  1, 4, MP  
           6.  $B \supset F$  2, 4, MP  
           7.  $F$  5, 6, MP  
           8.  $F \supset H$  3, 5, MP  
           9.  $H$  7, 8, MP  
       10.  $R \supset H$  4-9, CP

- (14) 1.  $(F \cdot G) \equiv H$   
       2.  $F \supset G$  /  $F \equiv H$   
       3.  $[(F \cdot G) \supset H] \cdot [H \supset (F \cdot G)]$  1, Equiv  
           4.  $F$  ACP  
           5.  $G$  2, 4, MP  
           6.  $F \cdot G$  4, 5, Conj  
           7.  $(F \cdot G) \supset H$  3, Simp  
           8.  $H$  6, 7, MP  
       9.  $F \supset H$  4-8, CP  
           10.  $H$  ACP  
           11.  $H \supset (F \cdot G)$  3, Com, Simp  
           12.  $F \cdot G$  10, 11, MP  
           13.  $F$  12, Simp  
       14.  $H \supset F$  10-13, CP  
       15.  $(F \supset H) \cdot (H \supset F)$  9, 14, Conj  
       16.  $F \equiv H$  15, Equiv

- (15) 1.  $C \supset (D \vee \sim E)$   
       2.  $E \supset (D \supset F)$  /  $C \supset (E \supset F)$   
           3.  $C$  ACP  
           4.  $E$  ACP  
           5.  $D \vee \sim E$  1, 3, MP  
           6.  $D$  4, 5, Com, DN, DS  
           7.  $D \supset F$  2, 4, MP  
           8.  $F$  6, 7, MP  
           9.  $E \supset F$  4-8, CP  
       10.  $C \supset (E \supset F)$  3-9, CP

# Exercise 7.5

- (16) 1.  $Q \supset (R \supset S)$   
 2.  $Q \supset (T \supset \sim U)$   
 3.  $U \supset (R \vee T)$  /  $Q \supset (U \supset S)$   
 4.  $Q$  ACP  
 5.  $U$  ACP  
 6.  $R \supset S$  1, 4, MP  
 7.  $T \supset \sim U$  2, 4, MP  
 8.  $\sim T$  5, 7, DN, MT  
 9.  $R \vee T$  3, 5, MP  
 10.  $R$  8, 9, Com, DS  
 11.  $S$  6, 10, MP  
 12.  $U \supset S$  5-11, CP  
 13.  $Q \supset (U \supset S)$  4-12, CP
- (17) 1.  $N \supset (O \cdot P)$   
 2.  $Q \supset (R \cdot S)$  /  $(P \supset Q) \supset (N \supset S)$   
 3.  $P \supset Q$  ACP  
 4.  $N$  ACP  
 5.  $O \cdot P$  1, 4, MP  
 6.  $P$  5, Com, Simp  
 7.  $Q$  3, 6, MP  
 8.  $R \cdot S$  2, 7, MP  
 9.  $S$  8, Com, Simp  
 10.  $N \supset S$  4-9, CP  
 11.  $(P \supset Q) \supset (N \supset S)$  3-10, CP
- (18) 1.  $E \supset (F \supset G)$   
 2.  $H \supset (G \supset I)$   
 3.  $(F \supset I) \supset (J \vee \sim H)$  /  $(E \cdot H) \supset J$   
 4.  $E \cdot H$  ACP  
 5.  $E$  4, Simp  
 6.  $F \supset G$  1, 5, MP  
 7.  $H$  4, Com, Simp  
 8.  $G \supset I$  2, 7, MP  
 9.  $F \supset I$  6, 8, HS  
 10.  $J \vee \sim H$  3, 9, MP  
 11.  $J$  7, 10, DN, Com, DS  
 12.  $(E \cdot H) \supset J$  4-11, CP
- (19) 1.  $P \supset [(L \vee M) \supset (N \cdot O)]$   
 2.  $(O \vee T) \supset W$  /  $P \supset (M \supset W)$   
 3.  $P$  ACP  
 4.  $M$  ACP  
 5.  $(L \vee M) \supset (N \cdot O)$  1, 3, MP  
 6.  $M \vee L$  4, Add



|                               |              |
|-------------------------------|--------------|
| 7. $L \vee M$                 | 6, Com       |
| 8. $N \cdot O$                | 5, 7, MP     |
| 9. $O$                        | 8, Com, Simp |
| 10. $O \vee T$                | 9, Add       |
| 11. $W$                       | 2, 10, MP    |
| 12. $M \supset W$             | 4-11, CP     |
| 13. $P \supset (M \supset W)$ | 3-12, CP     |

|                                                  |                                    |
|--------------------------------------------------|------------------------------------|
| (20) 1. $A \supset [B \supset (C \cdot \sim D)]$ |                                    |
| 2. $(B \vee E) \supset (D \vee E)$               | $/(A \cdot B) \supset (C \cdot E)$ |
| 3. $A \cdot B$                                   | ACP                                |
| 4. $A$                                           | 3, Simp                            |
| 5. $B \supset (C \cdot D)$                       | 1, 4, MP                           |
| 6. $B$                                           | 3, Com, Simp                       |
| 7. $C \cdot \sim D$                              | 5, 6, MP                           |
| 8. $C$                                           | 7, Simp                            |
| 9. $\sim D$                                      | 7, Com, Simp                       |
| 10. $B \vee E$                                   | 6, Add                             |
| 11. $D \vee E$                                   | 2, 10, MP                          |
| 12. $E$                                          | 9, 11, DS                          |
| 13. $C \cdot E$                                  | 8, 12, Conj                        |
| 14. $(A \cdot B) \supset (C \cdot E)$            | 3-13, CP                           |

## Part II

|                                      |                                    |
|--------------------------------------|------------------------------------|
| (1) 1. $H \supset D$                 |                                    |
| 2. $U \supset S$                     | $/(H \cdot U) \supset (S \cdot D)$ |
| 3. $H \cdot U$                       | ACP                                |
| 4. $H$                               | 3, Simp                            |
| 5. $D$                               | 1, 4, MP                           |
| 6. $U$                               | 3, Com, Simp                       |
| 7. $S$                               | 2, 6, MP                           |
| 8. $S \cdot D$                       | 5, 7, Conj                         |
| 9. $(H \cdot U) \supset (S \cdot D)$ | 3-8, CP                            |

  

|                                    |                                 |
|------------------------------------|---------------------------------|
| (2) 1. $P \supset (I \supset W)$   |                                 |
| 2. $I \supset (W \supset \sim S)$  | $/P \supset (I \supset \sim S)$ |
| 3. $P$                             | ACP                             |
| 4. $I$                             | ACP                             |
| 5. $I \supset W$                   | 1, 3, MP                        |
| 6. $W \supset \sim S$              | 2, 4, MP                        |
| 7. $I \supset \sim S$              | 5, 6, HS                        |
| 8. $\sim S$                        | 4, 7, MP                        |
| 9. $I \supset \sim S$              | 4-8, CP                         |
| 10. $P \supset (I \supset \sim S)$ | 3-9, CP                         |

## Exercise 7.6

- (3)
- |                                                                |                 |
|----------------------------------------------------------------|-----------------|
| 1. $D \equiv (S \supset F)$                                    |                 |
| 2. $R \supset (D \cdot P)$                                     |                 |
| 3. S                                                           | $/ R \supset F$ |
| 4. R                                                           | ACP             |
| 5. $D \cdot P$                                                 | 2, 4, MP        |
| 6. D                                                           | 5, Simp         |
| 7. $[D \supset (S \supset F)] \cdot [(S \supset F) \supset D]$ | 1, Equiv        |
| 8. $D \supset (S \supset F)$                                   | 7, Simp         |
| 9. $S \supset F$                                               | 6, 8, MP        |
| 10. F                                                          | 3, 9, MP        |
| 11. $R \supset F$                                              | 4-10, CP        |

- (4)
- |                               |                             |
|-------------------------------|-----------------------------|
| 1. $J \supset D$              |                             |
| 2. $(J \cdot D) \supset C$    |                             |
| 3. $(N \cdot C) \supset I$    | $/ J \supset (N \supset I)$ |
| 4. J                          | ACP                         |
| 5. N                          | ACP                         |
| 6. D                          | 1, 4, MP                    |
| 7. $J \cdot D$                | 4, 6, Conj                  |
| 8. C                          | 2, 7, MP                    |
| 9. $N \cdot C$                | 5, 8, Conj                  |
| 10. I                         | 3, 9, MP                    |
| 11. $N \supset I$             | 5-10, CP                    |
| 12. $J \supset (N \supset I)$ | 4-11, CP                    |

- (5)
- |                                           |                                         |
|-------------------------------------------|-----------------------------------------|
| 1. $A \supset H$                          |                                         |
| 2. $(F \vee W) \supset L$                 | $/ (H \supset F) \supset (A \supset L)$ |
| 3. $H \supset F$                          | ACP                                     |
| 4. A                                      | ACP                                     |
| 5. H                                      | 1, 4, MP                                |
| 6. F                                      | 3, 5, MP                                |
| 7. $F \vee W$                             | 6, Add                                  |
| 8. L                                      | 2, 7, MP                                |
| 9. $A \supset L$                          | 4-8, CP                                 |
| 10. $(H \supset F) \supset (A \supset L)$ | 3-9, CP                                 |

## Exercise 7.6

### Part I

- (1)
- |                                |            |
|--------------------------------|------------|
| 1. $(S \vee T) \supset \sim S$ | $/ \sim S$ |
| 2. S                           | AIP        |
| 3. $S \vee T$                  | 2, Add     |
| 4. $\sim S$                    | 1, 3, MP   |
| 5. $S \cdot \sim S$            | 2, 4, Conj |
| 6. $\sim S$                    | 2-5, IP    |

- (2) 1.  $(K \supset K) \supset R$   
 2.  $(R \vee M) \supset N$  / N  
 3.  $\sim N$  AIP  
 4.  $\sim(R \vee M)$  2, 3, MT  
 5.  $\sim R \cdot \sim M$  4, DM  
 6.  $\sim R$  5, Simp  
 7.  $\sim(K \supset K)$  1, 6, MT  
 8.  $\sim(\sim K \vee K)$  7, Impl  
 9.  $K \cdot \sim K$  8, DM, DN  
 10. N 3-9, IP, DN
- (3) 1.  $(C \cdot D) \supset E$   
 2.  $(D \cdot E) \supset F$  /  $(C \cdot D) \supset F$   
 3.  $C \cdot D$  ACP  
 4. E 1, 3, MP  
 5. D 3, Com, Simp  
 6.  $D \cdot E$  4, 5, Conj  
 7. F 2, 6, MP  
 8.  $(C \cdot D) \supset F$  3-7, CP
- (4) 1.  $H \supset (L \supset K)$   
 2.  $L \supset (K \supset \sim L)$  /  $\sim H \vee \sim L$   
 3.  $H \cdot L$  AIP  
 4. H 3 Simp  
 5.  $L \supset K$  1, 4, MP  
 6. L 3, Com, Simp  
 7.  $K \supset \sim L$  2, 6, MP  
 8. K 5, 6, MP  
 9.  $\sim L$  7, 8, MP  
 10.  $L \cdot \sim L$  6, 9, Conj  
 11.  $\sim(H \cdot L)$  3-10, IP  
 12.  $\sim H \vee \sim L$  11, DM
- (5) 1.  $S \supset (T \vee \sim U)$   
 2.  $U \supset (\sim T \vee R)$   
 3.  $(S \cdot U) \supset \sim R$  /  $\sim S \vee \sim U$   
 4.  $\sim(\sim S \vee \sim U)$  AIP  
 5.  $S \cdot U$  4, DM, DN  
 6.  $\sim R$  3, 5, MP  
 7. S 5, Simp  
 8. U 6, Com, Simp  
 9.  $T \vee \sim U$  1, 7, MP  
 10.  $\sim T \vee R$  2, 8, MP  
 11.  $\sim T$  6, 10, Com, DS  
 12.  $\sim U$  9, 11, DS

# Exercise 7.6

|                                           |                 |
|-------------------------------------------|-----------------|
| 13. $U \bullet \sim U$                    | 8, 12, Conj     |
| 14. $\sim(\sim S \vee \sim U)$            | 4-13, IP        |
| 15. $\sim S \vee \sim U$                  | 14, DN          |
| (6) 1. $\sim A \supset (B \bullet C)$     |                 |
| 2. $D \supset \sim C$                     | / $D \supset A$ |
| 3. $D$                                    | ACP             |
| 4. $\sim C$                               | 2, 3, MP        |
| 5. $\sim C \vee \sim B$                   | 4, Add          |
| 6. $\sim B \vee \sim C$                   | 5, Com          |
| 7. $\sim(B \bullet C)$                    | 6, DM           |
| 8. $A$                                    | 1, 7, MT, DN    |
| 9. $D \supset A$                          | 3-8, CP         |
| (7) 1. $(E \vee F) \supset (C \bullet D)$ |                 |
| 2. $(D \vee G) \supset H$                 |                 |
| 3. $E \vee G$                             | / $H$           |
| 4. $\sim H$                               | AIP             |
| 5. $\sim(D \vee G)$                       | 2, 4, MT        |
| 6. $\sim D \bullet \sim G$                | 5, DM           |
| 7. $\sim D$                               | 6, Simp         |
| 8. $\sim D \vee \sim C$                   | 7, Add          |
| 9. $\sim C \vee \sim D$                   | 8, Com          |
| 10. $\sim(C \bullet D)$                   | 9, DM           |
| 11. $\sim(E \vee F)$                      | 1, 10, MT       |
| 12. $\sim E \bullet \sim F$               | 11, DM          |
| 13. $\sim E$                              | 12, Simp        |
| 14. $G$                                   | 3, 13, DS       |
| 15. $\sim G$                              | 6, Com, Simp    |
| 16. $G \bullet \sim G$                    | 14, 15, Conj    |
| 17. $H$                                   | 4-16, IP, DN    |
| (8) 1. $\sim M \supset (N \bullet O)$     |                 |
| 2. $N \supset P$                          |                 |
| 3. $O \supset \sim P$                     | / $M$           |
| 4. $\sim M$                               | AIP             |
| 5. $N \bullet O$                          | 1, 4, MP        |
| 6. $N$                                    | 5, Simp         |
| 7. $P$                                    | 2, 6, MP        |
| 8. $\sim O$                               | 3, 7, DN, MT    |
| 9. $O$                                    | 5, Com, Simp    |
| 10. $O \bullet \sim O$                    | 8, 9, Conj      |
| 11. $M$                                   | 4-10, IP, DN    |

- (9)
- |                                         |             |
|-----------------------------------------|-------------|
| 1. $(R \vee S) \supset T$               |             |
| 2. $(P \vee Q) \supset T$               |             |
| 3. $R \vee P$                           | / T         |
| 4. R                                    | ACP         |
| 5. $R \vee S$                           | 4, Add      |
| 6. T                                    | 1, 5, MP    |
| 7. $R \supset T$                        | 4-6, CP     |
| 8. P                                    | ACP         |
| 9. $P \vee Q$                           | 8, Add      |
| 10. T                                   | 2, 9, MP    |
| 11. $P \supset T$                       | 8-10, CP    |
| 12. $(R \supset T) \cdot (P \supset T)$ | 7, 11, Conj |
| 13. $T \vee T$                          | 3, 12, CD   |
| 14. T                                   | 13, Taut    |
- (10)
- |                              |                             |
|------------------------------|-----------------------------|
| 1. K                         | / $S \supset (T \supset S)$ |
| 2. S                         | ACP                         |
| 3. $S \vee \sim T$           | 2, Add                      |
| 4. $\sim T \vee S$           | 3, Com                      |
| 5. $T \supset S$             | 4, Impl                     |
| 6. $S \supset (T \supset S)$ | 2-5, CP                     |
- (11)
- |                                |              |
|--------------------------------|--------------|
| 1. $(A \vee B) \supset C$      |              |
| 2. $(\sim A \vee D) \supset E$ | / $C \vee E$ |
| 3. $\sim C$                    | ACP          |
| 4. $\sim(A \vee B)$            | 1, 3, MT     |
| 5. $\sim A \cdot \sim B$       | 4, DM        |
| 6. $\sim A$                    | 5, Simp      |
| 7. $\sim A \vee D$             | 6, Add       |
| 8. E                           | 2, 7, MP     |
| 9. $\sim C \supset E$          | 3-8, CP      |
| 10. $C \vee E$                 | 9, Impl, DN  |
- (12)
- |                                          |              |
|------------------------------------------|--------------|
| 1. $(K \vee L) \supset (M \cdot N)$      |              |
| 2. $(N \vee O) \supset (P \cdot \sim K)$ | / $\sim K$   |
| 3. K                                     | AIP          |
| 4. $K \vee L$                            | 3, Add       |
| 5. $M \cdot N$                           | 1, 4, MP     |
| 6. N                                     | 5, Com, Simp |
| 7. $N \vee O$                            | 6, Add       |
| 8. $P \cdot \sim K$                      | 2, 7, MP     |
| 9. $\sim K$                              | 8, Com, Simp |
| 10. $K \cdot \sim K$                     | 3, 9, Conj   |
| 11. $\sim K$                             | 3-10, IP     |

# Exercise 7.6

- (13) 1.  $[C \supset (D \supset C)] \supset E$  / E  
 2. C ACP  
 3.  $C \vee \sim D$  2, Add  
 4.  $\sim D \vee C$  3, Com  
 5.  $D \supset C$  4, Impl  
 6.  $C \supset (D \supset C)$  2-5, CP  
 7. E 1, 6, MP

- (14) 1.  $F / (G \supset H) \vee (\sim G \supset J)$   
 2.  $G \cdot \sim H$  ACP  
 3. G 2, Simp  
 4.  $G \vee J$  3, Add  
 5.  $\sim G \supset J$  4, DN, Impl  
 6.  $(G \cdot \sim H) \supset (\sim G \supset J)$  2-5, CP  
 7.  $\sim(G \cdot \sim H) \vee (\sim G \supset J)$  6, Impl  
 8.  $(\sim G \vee H) \vee (\sim G \supset J)$  7, DM, DN  
 9.  $(G \supset H) \vee (\sim G \supset J)$  8, Impl

- (15) 1.  $B \supset (K \cdot M)$   
 2.  $(B \cdot M) \supset (P \equiv \sim P)$  /  $\sim B$   
 3. B AIP  
 4.  $K \cdot M$  1, 3, MP  
 5. M 4, Com, Simp  
 6.  $B \cdot M$  3, 5, Conj  
 7.  $P \equiv \sim P$  2, 6, MP  
 8.  $(P \cdot \sim P) \vee (\sim P \cdot \sim \sim P)$  7, Equiv  
 9.  $(P \cdot \sim P) \vee (P \cdot \sim P)$  8, Com, DN  
 10.  $P \cdot \sim P$  9, Taut  
 11.  $\sim B$  3-10, IP

- (16) 1.  $(N \vee O) \supset (C \cdot D)$   
 2.  $(D \vee K) \supset (P \vee \sim C)$   
 3.  $(P \vee G) \supset \sim(N \cdot D)$  /  $\sim N$   
 4. N AIP  
 5.  $N \vee O$  4, Add  
 6.  $C \cdot D$  1, 5, MP  
 7. D 6, Com, Simp  
 8.  $D \vee K$  7, Add  
 9.  $P \vee \sim C$  2, 8, MP  
 10. C 6, Simp  
 11. P 9, 10, Com, DN, DS  
 12.  $P \vee G$  11, Add  
 13.  $\sim(N \cdot D)$  3, 12, MP  
 14.  $\sim N \vee \sim D$  13, DM  
 15.  $\sim D$  4, 14, DN, DS

- |                      |             |
|----------------------|-------------|
| 16. $D \cdot \sim D$ | 7, 15, Conj |
| 17. $\sim N$         | 4-16, IP    |
- 
- (17)
- |                                                                                |                |
|--------------------------------------------------------------------------------|----------------|
| 1. $(R \cdot S) \equiv (G \cdot H)$                                            |                |
| 2. $R \supset S$                                                               |                |
| 3. $H \supset G$                                                               | $/ R \equiv H$ |
| 4. $[(R \cdot S) \supset (G \cdot H)] \cdot [(G \cdot H) \supset (R \cdot S)]$ | 1, Equiv       |
| 5. $R$                                                                         | ACP            |
| 6. $S$                                                                         | 2, 5, MP       |
| 7. $R \cdot S$                                                                 | 5, 6, Conj     |
| 8. $(R \cdot S) \supset (G \cdot H)$                                           | 4, Simp        |
| 9. $G \cdot H$                                                                 | 7, 8, MP       |
| 10. $H$                                                                        | 9, Com, Simp   |
| 11. $R \supset H$                                                              | 5-10, CP       |
| 12. $H$                                                                        | ACP            |
| 13. $G$                                                                        | 3, 12, MP      |
| 14. $G \cdot H$                                                                | 12, 13, Conj   |
| 15. $(G \cdot H) \supset (R \cdot S)$                                          | 4, Com, Simp   |
| 16. $R \cdot S$                                                                | 14, 15, MP     |
| 17. $R$                                                                        | 16, Simp       |
| 18. $H \supset R$                                                              | 12-17, CP      |
| 19. $(R \supset H) \cdot (H \supset R)$                                        | 11, 18, Conj   |
| 20. $R \equiv H$                                                               | 19, Equiv      |
- 
- (18)
- |                                                      |                                |
|------------------------------------------------------|--------------------------------|
| 1. $K \supset [(M \vee N) \supset (P \cdot Q)]$      |                                |
| 2. $L \supset [(Q \vee R) \supset (S \cdot \sim N)]$ | $/ (K \cdot L) \supset \sim N$ |
| 3. $K \cdot L$                                       | ACP                            |
| 4. $K$                                               | 3, Simp                        |
| 5. $(M \vee N) \supset (P \cdot Q)$                  | 1, 4, MP                       |
| 6. $L$                                               | 3, Com, Simp                   |
| 7. $(Q \vee R) \supset (S \cdot \sim N)$             | 2, 6, MP                       |
| 8. $N$                                               | AIP                            |
| 9. $N \vee M$                                        | 8, Add                         |
| 10. $M \vee N$                                       | 9, Com                         |
| 11. $P \cdot Q$                                      | 5, 10, MP                      |
| 12. $Q$                                              | 11, Com, Simp                  |
| 13. $Q \vee R$                                       | 12, Add                        |
| 14. $S \cdot \sim N$                                 | 7, 13, MP                      |
| 15. $\sim N$                                         | 14, Com, Simp                  |
| 16. $N \cdot \sim N$                                 | 8, 15, Conj                    |
| 17. $\sim N$                                         | 8-16, IP                       |
| 18. $(K \cdot L) \supset \sim N$                     | 3-17, CP                       |

## Exercise 7.6

- (19) 1.  $A \supset [(N \vee \sim N) \supset (S \vee T)]$   
 2.  $T \supset \sim(F \vee \sim F)$  /  $A \supset S$   
 3.  $A \cdot \sim S$  AIP  
 4.  $A$  3, Simp  
 5.  $(N \vee \sim N) \supset (S \vee T)$  1, 4, MP  
 6.  $N$  ACP  
 7.  $N \vee N$  6, Add  
 8.  $N$  7, Taut  
 9.  $N \supset N$  6-8, CP  
 10.  $\sim N \vee N$  9, Impl  
 11.  $N \vee \sim N$  10, Com  
 12.  $S \vee T$  5, 11, MP  
 13.  $\sim S$  3, Com, Simp  
 14.  $T$  12, 13, DS  
 15.  $\sim(F \vee \sim F)$  2, 14, MP  
 16.  $\sim F \cdot F$  15, DM, DN  
 17.  $\sim(A \cdot \sim S)$  3-16, IP  
 18.  $\sim A \vee S$  17, DM, DN  
 19.  $A \supset S$  18, Impl

- (20) 1.  $F \supset [(C \supset C) \supset G]$   
 2.  $G \supset \{[H \supset (E \supset H)] \supset (K \cdot \sim K)\}$  /  $\sim F$   
 3.  $F$  AIP  
 4.  $(C \supset C) \supset G$  1, 3, MP  
 5.  $C$  ACP  
 6.  $C \vee C$  5, Add  
 7.  $C$  6, Taut  
 8.  $C \supset C$  5-7, CP  
 9.  $G$  4, 8, MP  
 10.  $[H \supset (E \supset H)] \supset (K \cdot \sim K)$  2, 9, MP  
 11.  $H$  ACP  
 12.  $H \vee \sim E$  11, Add  
 13.  $\sim E \vee H$  12, Com  
 14.  $E \supset H$  13, Impl  
 15.  $H \supset (E \supset H)$  11-14, CP  
 16.  $K \cdot \sim K$  10, 15, MP  
 17.  $\sim F$  3-16, IP

## Part II

- (1) 1.  $(C \cdot R) \supset (I \cdot D)$   
 2.  $R \supset \sim D$  /  $\sim C \vee \sim R$   
 3.  $C \cdot R$  AIP  
 4.  $I \cdot D$  1, 3, MP  
 5.  $D \cdot I$  4, Com



|                          |            |
|--------------------------|------------|
| 6. D                     | 5, Simp    |
| 7. $R \cdot C$           | 3, Com     |
| 8. R                     | 7, Simp    |
| 9. $\sim D$              | 2, 8, MP   |
| 10. $D \cdot \sim D$     | 6, 9, Conj |
| 11. $\sim(C \cdot R)$    | 3-10, IP   |
| 12. $\sim C \vee \sim R$ | 11, DM     |

|                                            |              |
|--------------------------------------------|--------------|
| (2) 1. $(C \vee R) \supset (N \cdot I)$    |              |
| 2. $(N \vee P) \supset (I \supset \sim C)$ | / $\sim C$   |
| 3. C                                       | AIP          |
| 4. $C \vee R$                              | 3, Add       |
| 5. $N \cdot I$                             | 1, 4, MP     |
| 6. N                                       | 5, Simp      |
| 7. $N \vee P$                              | 6, Add       |
| 8. $I \supset \sim C$                      | 2, 7, MP     |
| 9. I                                       | 5, Com, Simp |
| 10. $\sim C$                               | 8, 9, MP     |
| 11. $C \cdot \sim C$                       | 3, 10, Conj  |
| 12. $\sim C$                               | 3-11, IP     |

|                                  |                   |
|----------------------------------|-------------------|
| (3) 1. $S \supset (P \supset B)$ |                   |
| 2. $S \supset P$                 |                   |
| 3. $C \supset (S \vee B)$        | / $\sim C \vee B$ |
| 4. $C \cdot \sim B$              | AIP               |
| 5. C                             | 4, Simp           |
| 6. $S \vee B$                    | 3, 5, MP          |
| 7. $\sim B$                      | 4, Com, Simp      |
| 8. S                             | 6, 7, Com, DS     |
| 9. $P \supset B$                 | 1, 8, MP          |
| 10. $S \supset B$                | 2, 9, HS          |
| 11. B                            | 8, 10, MP         |
| 12. $B \cdot \sim B$             | 7, 11, Conj       |
| 13. $\sim(C \cdot \sim B)$       | 4-12, IP          |
| 14. $\sim C \vee \sim \sim B$    | 13, DM            |
| 15. $\sim C \vee B$              | 14, DN            |

|                                  |          |
|----------------------------------|----------|
| (4) 1. $(Z \supset C) \supset B$ |          |
| 2. $(V \supset Z) \supset B$     | / B      |
| 3. $\sim B$                      | AIP      |
| 4. $\sim(Z \supset C)$           | 1, 3, MT |
| 5. $\sim(\sim Z \vee C)$         | 4, Impl  |
| 6. $\sim \sim Z \cdot \sim C$    | 5, DM    |
| 7. $\sim \sim Z$                 | 6, Simp  |
| 8. $\sim(V \supset Z)$           | 2, 3, MT |

# Exercise 7.7

|                               |             |
|-------------------------------|-------------|
| 9. $\sim(\sim V \vee Z)$      | 8, Impl     |
| 10. $\sim\sim V \cdot \sim Z$ | 9, DM       |
| 11. $\sim Z \cdot \sim\sim V$ | 10, Com     |
| 12. $\sim Z$                  | 11, Simp    |
| 13. $\sim Z \cdot \sim\sim Z$ | 7, 12, Conj |
| 14. $\sim\sim B$              | 3-13, IP    |
| 15. B                         | 14, DN      |

|                                       |               |
|---------------------------------------|---------------|
| (5) 1. $(B \cdot S) \vee (H \cdot C)$ |               |
| 2. $(P \vee H) \supset S$             | / S           |
| 3. $\sim S$                           | AIP           |
| 4. $\sim(P \vee H)$                   | 2, 3, MT      |
| 5. $\sim P \cdot \sim H$              | 4, DM         |
| 6. $\sim H$                           | 5, Com, Simp  |
| 7. $\sim H \vee \sim C$               | 6, Add        |
| 8. $\sim(H \cdot C)$                  | 7, DM         |
| 9. $(H \cdot C) \vee (B \cdot S)$     | 1, Com        |
| 10. $B \cdot S$                       | 8, 9, DS      |
| 11. S                                 | 10, Com, Simp |
| 12. $S \cdot \sim S$                  | 3, 11, Conj   |
| 13. $\sim\sim S$                      | 3-12 IP       |
| 14. S                                 | 13, DN        |

## Exercise 7.7

|                                                     |                                           |
|-----------------------------------------------------|-------------------------------------------|
| (1)                                                 | / $P \supset [(P \supset Q) \supset Q]$   |
| 1. P                                                | ACP                                       |
| 2. $P \supset Q$                                    | ACP                                       |
| 3. Q                                                | 1, 2, MP                                  |
| 4. $(P \supset Q) \supset Q$                        | 2-3, CP                                   |
| 5. $P \supset [(P \supset Q) \supset Q]$            | 1-4, CP                                   |
| (2)                                                 | / $(\sim P \supset Q) \vee (P \supset R)$ |
| 1. $\sim[(\sim P \supset Q) \vee (P \supset R)]$    | AIP                                       |
| 2. $\sim(\sim P \supset Q) \cdot \sim(P \supset R)$ | 1, DM                                     |
| 3. $\sim(P \vee Q) \cdot \sim(\sim P \vee R)$       | 2, Impl, DN                               |
| 4. $(\sim P \cdot \sim Q) \cdot (P \cdot \sim R)$   | 3, DM, DN                                 |
| 5. $\sim P \cdot \sim Q$                            | 4, Simp                                   |
| 6. $P \cdot \sim R$                                 | 4, Com, Simp                              |
| 7. $\sim P$                                         | 5, Simp                                   |
| 8. P                                                | 6, Simp                                   |
| 9. $P \cdot \sim P$                                 | 7, 8, Conj                                |
| 10. $(\sim P \supset Q) \vee (P \supset R)$         | 1-9, IP, DN                               |

- (3)  $/ P \equiv [P \vee (Q \cdot P)]$   
 1. P ACP  
 2.  $P \vee (Q \cdot P)$  1, Add  
 3.  $P \supset [P \vee (Q \cdot P)]$  1-2, CP  
 4.  $P \vee (Q \cdot P)$  ACP  
 5.  $(P \vee Q) \cdot (P \vee P)$  4, Dist  
 6.  $P \vee P$  5, Com, Simp  
 7. P 6, Taut  
 8.  $[P \vee (Q \cdot P)] \supset P$  4-7, CP  
 9. {line 3}  $\cdot$  {line 8} 3, 8, Conj  
 10.  $P \equiv [P \vee (Q \cdot P)]$  9, Equiv
- (4)  $/ (P \supset Q) \supset [(P \cdot R) \supset (Q \cdot R)]$   
 1.  $P \supset Q$  ACP  
 2.  $P \cdot R$  ACP  
 3. P 2, Simp  
 4. Q 1, 3, MP  
 5. R 2, Com, Simp  
 6.  $Q \cdot R$  4, 5, Conj  
 7.  $(P \cdot R) \supset (Q \cdot R)$  2-6, CP  
 8.  $(P \supset Q) \supset [(P \cdot R) \supset (Q \cdot R)]$  1-7, CP
- (5)  $/ (P \vee \sim Q) \supset [(\sim P \vee R) \supset (Q \supset R)]$   
 1.  $P \vee \sim Q$  ACP  
 2.  $\sim P \vee R$  ACP  
 3.  $\sim Q \vee P$  1, Com  
 4.  $Q \supset P$  3, Impl  
 5.  $P \supset R$  2, Impl  
 6.  $Q \supset R$  4, 5, HS  
 7.  $(\sim P \vee R) \supset (Q \supset R)$  2-6, CP  
 8.  $(P \vee \sim Q) \supset [(\sim P \vee R) \supset (Q \supset R)]$  1-7, CP
- (6)  $/ P \equiv [P \cdot (Q \vee \sim Q)]$   
 1. P ACP  
 2. Q ACP  
 3.  $Q \vee Q$  2, Add  
 4. Q 3, Taut  
 5.  $Q \supset Q$  2-4, CP  
 6.  $\sim Q \vee Q$  5, Impl  
 7.  $Q \vee \sim Q$  6, Com  
 8.  $P \cdot (Q \vee \sim Q)$  1, 7, Conj  
 9.  $P \supset [P \cdot (Q \vee \sim Q)]$  1-8, CP  
 10.  $P \cdot (Q \vee \sim Q)$  ACP  
 11. P 10, Simp  
 12.  $[P \cdot (Q \vee \sim Q)] \supset P$  10-11, CP  
 13. {line 9}  $\cdot$  {line 12} 9, 12, Conj  
 14.  $P \equiv [P \cdot (Q \vee \sim Q)]$  13, Equiv

# Exercise 7.7

(7)

1.  $\sim[(P \supset Q) \vee (\sim Q \supset P)]$
2.  $\sim(P \supset Q) \cdot \sim(\sim Q \supset P)$
3.  $\sim(\sim P \vee Q) \cdot \sim(Q \vee P)$
4.  $(P \cdot \sim Q) \cdot (\sim Q \cdot \sim P)$
5.  $P \cdot \sim Q$
6.  $\sim Q \cdot \sim P$
7.  $P$
8.  $\sim P$
9.  $P \cdot \sim P$
10.  $(P \supset Q) \vee (\sim Q \supset P)$

$/(P \supset Q) \vee (\sim Q \supset P)]$

- AIP  
1, DM  
2, Impl, DN  
3, DM, DN  
4, Simp  
4, Com, Simp  
5, Simp  
6, Com, Simp  
7, 8, Conj  
1-9, IP, DN

(8)

1.  $P \supset Q$
2.  $P$
3.  $Q$
4.  $P \cdot Q$
5.  $P \supset (P \cdot Q)$
6.  $(P \supset Q) \supset [P \supset (P \cdot Q)]$
7.  $P \supset (P \cdot Q)$
8.  $P$
9.  $P \cdot Q$
10.  $Q$
11.  $P \supset Q$
12.  $[P \supset (P \cdot Q)] \supset (P \supset Q)$
13. {line 6}  $\cdot$  {line 12}
14.  $(P \supset Q) \equiv [P \supset (P \cdot Q)]$

$/(P \supset Q) \equiv [P \supset (P \cdot Q)]$

- ACP  
ACP  
1, 2, MP  
2, 3, Conj  
2-4, CP  
1-5, CP  
ACP  
ACP  
7, 8, MP  
9, Com, Simp  
8-10, CP  
7-11, CP  
6, 12, Conj  
13, Equiv

(9)

1.  $(P \supset Q) \cdot (P \supset R)$
2.  $P$
3.  $P \supset Q$
4.  $Q$
5.  $P \supset R$
6.  $R$
7.  $Q \cdot R$
8.  $P \supset (Q \cdot R)$
9.  $[(P \supset Q) \cdot (P \supset R)] \supset [P \supset (Q \cdot R)]$

$/[ (P \supset Q) \cdot (P \supset R) ] \supset [P \supset (Q \cdot R)]$

- ACP  
ACP  
1, Simp  
2, 3, MP  
1, Com, Simp  
2, 5, MP  
4, 6, Conj  
2-7, CP  
1-8, CP

(10)

1.  $\sim(P \cdot \sim Q) \cdot \sim Q$
2.  $\sim(P \cdot \sim Q)$
3.  $\sim P \vee Q$
4.  $\sim Q$
5.  $\sim P$
6.  $[\sim(P \cdot \sim Q) \cdot \sim Q] \supset \sim P$

$/[ \sim(P \cdot \sim Q) \cdot \sim Q ] \supset \sim P$

- ACP  
1, simp  
2, DM, DN  
1, Com, Simp  
3, 4, Com, DS  
1-5, CP

(11)

1.  $\sim[(P \supset Q) \vee (Q \supset P)]$
2.  $\sim(P \supset Q) \cdot \sim(Q \supset P)$
3.  $\sim(\sim P \vee Q) \cdot \sim(\sim Q \vee P)$
4.  $(P \cdot \sim Q) \cdot (Q \cdot \sim P)$
5.  $P \cdot \sim Q$
6.  $Q \cdot \sim P$
7.  $P$
8.  $\sim P$
9.  $P \cdot \sim P$
10.  $(P \supset Q) \vee (Q \supset P)$

 $/(P \supset Q) \vee (Q \supset P)$ 

AIP  
 1, DM  
 2, Impl  
 3, DM, DN  
 4, Simp  
 4, Com, Simp  
 5, Simp  
 6, Com, Simp  
 7, 8, Conj  
 1-9, IP, DN

(12)

1.  $P \supset (Q \supset R)$
2.  $Q$
3.  $P$
4.  $Q \supset R$
5.  $R$
6.  $P \supset R$
7.  $Q \supset (P \supset R)$
8.  $[P \supset (Q \supset R)] \supset [Q \supset (P \supset R)]$
9.  $Q \supset (P \supset R)$
10.  $P$
11.  $Q$
12.  $P \supset R$
13.  $R$
14.  $Q \supset R$
15.  $P \supset (Q \supset R)$
16.  $[Q \supset (P \supset R)] \supset [P \supset (Q \supset R)]$
17. {line 8}  $\cdot$  {line 16}
18.  $[P \supset (Q \supset R)] \equiv [Q \supset (P \supset R)]$

 $/[P \supset (Q \supset R)] \equiv [Q \supset (P \supset R)]$ 

ACP  
 ACP  
 ACP  
 1, 3, MP  
 2, 4, MP  
 3-5, CP  
 2-6, CP  
 1-7, CP  
 ACP  
 ACP  
 ACP  
 9, 11, MP  
 10, 12, MP  
 11-13, CP  
 10-14, CP  
 9-15, CP  
 8, 16, Conj  
 17, Equiv

(13)

1.  $P \supset Q$
2.  $P \supset \sim Q$
3.  $Q \supset \sim P$
4.  $P \supset \sim P$
5.  $\sim P \vee \sim P$
6.  $\sim P$
7.  $(P \supset \sim Q) \supset \sim P$
8.  $(P \supset Q) \supset [(P \supset \sim Q) \supset \sim P]$

 $/(P \supset Q) \supset [(P \supset \sim Q) \supset \sim P]$ 

ACP  
 ACP  
 2, Trans, DN  
 1, 3, HS  
 4, Impl  
 5, Taut  
 2-6, CP  
 1-7, CP

# Exercise 7.7

|                                                                        |  |                                                                      |
|------------------------------------------------------------------------|--|----------------------------------------------------------------------|
| (14)                                                                   |  | $/ [(P \supset Q) \supset R] \supset [(R \supset \sim R) \supset P]$ |
| 1. $(P \supset Q) \supset R$                                           |  | ACP                                                                  |
| 2. $R \supset \sim R$                                                  |  | ACP                                                                  |
| 3. $\sim R \vee \sim R$                                                |  | 2, Impl                                                              |
| 4. $\sim R$                                                            |  | 3, Taut                                                              |
| 5. $\sim(P \supset Q)$                                                 |  | 1, 4, MT                                                             |
| 6. $\sim(\sim P \vee Q)$                                               |  | 5, Impl                                                              |
| 7. $P \cdot \sim Q$                                                    |  | 6, DM, DN                                                            |
| 8. $P$                                                                 |  | 7, Simp                                                              |
| 9. $(R \supset \sim R) \supset P$                                      |  | 2-8, CP                                                              |
| 10. $[(P \supset Q) \supset R] \supset [(R \supset \sim R) \supset P]$ |  | 1-9, CP                                                              |

|                                                                     |  |                                                                    |
|---------------------------------------------------------------------|--|--------------------------------------------------------------------|
| (15)                                                                |  | $/ (\sim P \vee Q) \supset [(P \vee \sim Q) \supset (P \equiv Q)]$ |
| 1. $\sim P \vee Q$                                                  |  | ACP                                                                |
| 2. $P \vee \sim Q$                                                  |  | ACP                                                                |
| 3. $P \supset Q$                                                    |  | 1, Impl                                                            |
| 4. $\sim Q \vee P$                                                  |  | 2, Com                                                             |
| 5. $Q \supset P$                                                    |  | 4, Impl                                                            |
| 6. $(P \supset Q) \cdot (Q \supset P)$                              |  | 3, 5, Conj                                                         |
| 7. $P \equiv Q$                                                     |  | 6, Equiv                                                           |
| 8. $(P \vee \sim Q) \supset (P \equiv Q)$                           |  | 2-7, CP                                                            |
| 9. $(\sim P \vee Q) \supset [(P \vee \sim Q) \supset (P \equiv Q)]$ |  | 1-8, CP                                                            |

|                                                        |  |                                                       |
|--------------------------------------------------------|--|-------------------------------------------------------|
| (16)                                                   |  | $/ \sim[(P \supset \sim P) \cdot (\sim P \supset P)]$ |
| 1. $(P \supset \sim P) \cdot (\sim P \supset P)$       |  | AIP                                                   |
| 2. $(\sim P \vee \sim P) \cdot (P \vee P)$             |  | 1, Impl, Impl, DN                                     |
| 3. $\sim P \cdot P$                                    |  | 2, Taut, Taut                                         |
| 4. $P \cdot \sim P$                                    |  | 3, Com                                                |
| 5. $\sim[(P \supset \sim P) \cdot (\sim P \supset P)]$ |  | 1-4, IP                                               |

|                                             |  |                                            |
|---------------------------------------------|--|--------------------------------------------|
| (17)                                        |  | $/ P \supset [(Q \cdot \sim Q) \supset R]$ |
| 1. $P$                                      |  | ACP                                        |
| 2. $Q \cdot \sim Q$                         |  | ACP                                        |
| 3. $Q$                                      |  | 2, Simp                                    |
| 4. $Q \vee R$                               |  | 3, Add                                     |
| 5. $\sim Q$                                 |  | 2, Com, Simp                               |
| 6. $R$                                      |  | 4, 5, DS                                   |
| 7. $(Q \cdot \sim Q) \supset R$             |  | 2-6, CP                                    |
| 8. $P \supset [(Q \cdot \sim Q) \supset R]$ |  | 1-7, CP                                    |

|                                  |  |                                                                          |
|----------------------------------|--|--------------------------------------------------------------------------|
| (18)                             |  | $/ [(P \cdot Q) \vee R] \supset [(\sim R \vee Q) \supset (P \supset Q)]$ |
| 1. $(P \cdot Q) \vee R$          |  | ACP                                                                      |
| 2. $\sim R \vee Q$               |  | ACP                                                                      |
| 3. $P$                           |  | ACP                                                                      |
| 4. $(P \vee R) \cdot (Q \vee R)$ |  | 1, Com, Dist, Com                                                        |

5.  $Q \vee R$   
 6.  $\sim Q \supset R$   
 7.  $R \supset Q$   
 8.  $\sim Q \supset Q$   
 9.  $Q \vee Q$   
 10.  $Q$   
 11.  $P \supset Q$   
 12.  $(\sim R \vee Q) \supset (P \supset Q)$   
 13.  $(P \cdot Q) \vee R] \supset [(\sim R \vee Q) \supset (P \supset Q)]$
- 4, Com, Simp  
 5, DN, Impl  
 2, Impl  
 6, 7, HS  
 8, Impl, DN  
 9, Taut  
 3-10, CP  
 2-11, CP  
 1-12, CP

(19)  $/P \equiv [P \vee (Q \cdot \sim Q)]$ 

1.  $P$   
 2.  $P \vee (Q \cdot \sim Q)$   
 3.  $P \supset [P \vee (Q \cdot \sim Q)]$   
 4.  $P \vee (Q \cdot \sim Q)$   
 5.  $\sim P$   
 6.  $Q \cdot \sim Q$   
 7.  $P$   
 8.  $[P \vee (Q \cdot \sim Q)] \supset P$   
 9. {line 3}  $\cdot$  {line 8}  
 10.  $P \equiv [P \vee (Q \cdot \sim Q)]$
- 1, Add  
 1-2, CP  
 ACP  
 1, Add  
 4, 5, DS  
 5-6, IP, DN  
 4-7, CP  
 3, 8, Conj  
 9, Equiv

(20)

1.  $P$   
 2.  $Q$   
 3.  $Q \vee \sim P$   
 4.  $\sim P \vee Q$   
 5.  $P \supset Q$   
 6.  $Q \supset (P \supset Q)$   
 7.  $P \supset Q$   
 8.  $Q$   
 9.  $(P \supset Q) \supset Q$   
 10.  $[Q \supset (P \supset Q)] \cdot [(P \supset Q) \supset Q]$   
 11.  $Q \equiv (P \supset Q)$   
 12.  $P \supset [Q \equiv (P \supset Q)]$
- ACP  
 ACP  
 2, Add  
 3, Com  
 4, Impl  
 2-5, CP  
 ACP  
 1, 7, MP  
 7-8, CP  
 6, 9, Conj  
 10, Equiv  
 1-11, CP

Exercise 8.1

1. Ce

2.  $\sim Sn$ 3.  $\sim (Iw \vee Is)$ 4.  $Jr \vee Nr$ 5.  $Di \supset Dm$