

Solutions to Proofs to Review for Test on Chapter 7

I. Do without CP or IP

- 1) 1. $A \supset B$
 2. $(E \vee D) \supset \sim B$ $/A \supset (\sim E \cdot \sim D)$
 3. $B \supset \sim(E \vee D)$ 2, Trans, DN
 4. $A \supset \sim(E \vee D)$ 1, 3, HS
 5. $A \supset (\sim E \cdot \sim D)$ 4, DM

QED

- 2) 1. $\sim(G \cdot H) \supset I$
 2. $\sim(I \vee J)$
 3. $\sim J \supset F$ $/(F \cdot G) \cdot H$
 4. $\sim I \cdot \sim J$ 2, DM
 5. $\sim I$ 4, Simp
 6. $G \cdot H$ 1, 5, MT, DN
 7. $\sim J$ 4, Com, Simp
 8. F 3, 7, MP
 9. $F \cdot (G \cdot H)$ 8, 6, Conj
 10. $(F \cdot G) \cdot H$ 9, Assoc

QED

- 3) 1. $P \vee O$
 2. $Q \supset \sim O$ $/\sim Q \vee P$
 3. $\sim O \supset P$ 1, Com, DN, Impl
 4. $Q \supset P$ 2, 3, HS
 5. $\sim Q \supset P$ 4, Impl

QED

- 4) 1. $T \equiv V$
 2. $\sim T \vee \sim V$ $/\sim T$
 3. $(T \cdot V) \vee (\sim T \cdot \sim V)$ 1, Equiv
 4. $\sim(T \cdot V)$ 2, DM
 5. $\sim T \cdot \sim V$ 3, 4, DS
 6. $\sim T$ 5, Simp

QED

- 5) 1. $(K \supset \sim M) \cdot [(L \cdot N) \supset \sim M]$
 2. $(K \vee L) \cdot (K \vee N)$ $/\sim M$
 3. $K \vee (L \cdot N)$ 2, Dist
 4. $\sim M \vee \sim M$ 1, 3, CD
 5. $\sim M$ 4, Taut

QED

- 6) 1. $(X \vee Y) \cdot (Z \supset W)$
 2. $\sim(Y \cdot W)$ $/\sim(\sim X \cdot Z)$
 3. $\sim Y \vee \sim W$ 2, DM
 4. $(Y \vee X) \cdot (Z \supset W)$ 1, Com
 5. $(\sim Y \supset X) \cdot (Z \supset W)$ 4, DN, Impl
 6. $(\sim Y \supset X) \cdot (\sim W \supset \sim Z)$ 5, Trans
 7. $X \vee \sim Z$ 6, 3, CD
 8. $\sim\sim(X \vee \sim Z)$ 7, DN
 9. $\sim(\sim X \cdot Z)$ 8, DM, DN

QED

- 7) 1. $X \vee Z$
 2. $Z \supset \sim Y$
 3. $X \supset Y$ $/\sim X \equiv Z$
 4. $\sim Y \supset \sim X$ 3, Trans
 5. $Z \supset \sim X$ 2, 4, HS
 6. $\sim X \supset Z$ 1, DN, Impl
 7. $(\sim X \supset Z) \cdot (Z \supset \sim X)$ 6, 5, Conj
 8. $\sim X \equiv Z$ 7, Equiv

QED

- 8) 1. $A \supset B$
 2. $B \supset \sim A$
 3. $(A \vee D) \vee E$
 4. $(D \vee E) \supset F$ $/F$
 5. $A \supset \sim A$ 1, 2, HS
 6. $\sim A \vee \sim A$ 5, Impl
 7. $\sim A$ 6, Taut
 8. $A \vee (D \vee E)$ 3, Assoc
 9. $\sim A \supset (D \vee E)$ 8, Dn, Impl
 10. $D \vee E$ 9, 7, MP
 11. F 4, 10, MP

QED

II. You may use CP or IP.

- 1) 1. $G \supset H$
 2. $G \supset I$ $/G \supset (H \cdot I)$
 | 3. G ACP
 | 4. H 1, 3, MP
 | 5. I 2, 3, MP
 | 6. $H \cdot I$ 4, 5, Conj
 7. $G \supset (H \cdot I)$ 3-6, CP

QED

- 2) 1. $J \equiv K$
 2. $K \equiv L$ $/J \equiv L$
 3. $(J \supset K) \cdot (K \supset J)$ 1, Equiv

4. $(K \supset L) \cdot (L \supset K)$	2, Equiv
5. $J \supset K$	3, Simp
6. $K \supset L$	4, Simp
7. $J \supset L$	5, 6, HS
8. $L \supset K$	4, Com, Simp
9. $K \supset J$	3, Com, Simp
10. $L \supset J$	8, 9, HS
11. $(J \supset L) \cdot (L \supset J)$	7, 10, Conj
12. $J \equiv L$	11, Equiv

QED

3)	1. $M \supset \{N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$	
	2. N	ACP
	3. O	ACP
	4. P	ACP
	5. $M \cdot N$	1, 2, Conj
	6. $O \cdot P$	3, 4, Conj
	7. $(M \cdot N) \cdot (O \cdot P)$	5, 6, Conj
	8. $P \supset [(M \cdot N) \cdot (O \cdot P)]$	4-7, CP
	9. $O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}$	3-8, CP
	10. $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$	2-9, CP

QED

4)	$\supset [R \supset (R \supset S) \supset S]$	
	1. R	ACP
	2. $R \supset S$	ACP
	3. S	2, 1, MP
	4. $(R \supset S) \supset S$	2-3, CP
	5. $R \supset [(R \supset S) \supset S]$	1-4, CP

QED

5)	1. $D \supset E$	
	2. $E \supset (F \cdot G)$	
	3. $\sim F \vee \sim G$	$\sim D$
	4. D	AIP
	5. E	1, 4, MP
	6. $F \cdot G$	2, 5, MP
	7. F	6, Simp
	8. $\sim G$	3, 7, DN, DS
	9. G	6, Com, Simp
	10. $G \cdot \sim G$	9, 8, Conj
	11. $\sim D$	4-10, IP

QED

6)	1. $(P \vee Q) \vee (\sim R \vee S)$	
	2. $\sim Q \cdot (\sim S \cdot \sim P)$	$\sim R$
	3. $\sim Q$	2, Simp
	4. $(Q \vee P) \vee (\sim R \vee S)$	1, Com
	5. $Q \vee [P \vee (\sim R \vee S)]$	4, Assoc

- | | |
|-----------------------------|----------------|
| 6. $P \vee (\sim R \vee S)$ | 5, 3, DS |
| 7. $\sim S \cdot \sim P$ | 2, Com, Simp |
| 8. $\sim P$ | 7, Com, Simp |
| 9. $\sim R \vee S$ | 6, 8, DS |
| 10. $\sim S$ | 7, Simp |
| 11. $\sim R$ | 9, 10, Com, DS |

QED

7) $\neg(A \vee \neg A)$

- | | |
|-------------------------------|-------------|
| 1. $\neg(A \vee \neg A)$ | AIP |
| 2. $\neg A \cdot \neg \neg A$ | 1, DM |
| 3. $A \cdot \neg A$ | 2, DN, Com |
| 4. $A \vee \neg A$ | 1-3, IP, DN |

QED

8) $\neg A \vee [(\neg A \vee B) \cdot (\neg A \vee C)]$

- | | |
|---|--------------|
| 1. $\neg\{A \vee [(\neg A \vee B) \cdot (\neg A \vee C)]\}$ | AIP |
| 2. $\neg A \cdot \neg[(\neg A \vee B) \cdot (\neg A \vee C)]$ | 1, DM |
| 3. $\neg A$ | 2, Simp |
| 4. $\neg[(\neg A \vee B) \cdot (\neg A \vee C)]$ | 2, Com, Simp |
| 5. $\neg[\neg A \vee (B \cdot C)]$ | 4, Dist |
| 6. $A \cdot \neg(B \cdot C)$ | 5, DM, DN |
| 7. A | 6, Simp |
| 8. $A \cdot \neg A$ | 7, 3, Conj |
| 9. $A \vee [(\neg A \vee B) \cdot (\neg A \vee C)]$ | 1-8, IP, DN |

QED