

Philosophy 109, Introduction to Modern Logic, Queens College
 Russell Marcus, Instructor
 email: philosophy@thatmarcusfamily.org
 website: <http://philosophy.thatmarcusfamily.org>
 Office phone: (718) 997-5287

Solutions to Problems to Review for Final

WARNING: These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

I. Translations (no relational predicates or identity theory)

1. $(x)[(Sx \cdot Cx) \supset Ex]$
2. $(x)[(Mx \vee Wx) \supset Ax]$
3. $(x)[Lx \supset (Ix \cdot Sx)]$
4. $\sim(x)(Dx \supset Gx)$ or $(\exists x)(Dx \cdot \sim Gx)$
5. $(x)[(Dx \cdot Ex) \supset \sim Gx]$ or $\sim(\exists x)[(Ex \cdot Dx) \cdot Gx]$
6. $(x)[(Px \cdot Ax) \supset \sim Ix]$ or $\sim(\exists x)[(Px \cdot Ax) \cdot Ix]$
7. $(x)[Ex \supset (Gx \cdot Px)]$
8. $(x)(Px \supset \sim Ex) \vee (\exists x)(Px \cdot Nx)$
9. $(\exists x)(Px \cdot Lx) \cdot (\exists x)(Px \cdot Cx)$
10. $(x)(Px \supset Mx) \vee (x)(Px \supset \sim Mx)$

II. Derivations

- 1)
 1. $Ab \vee Bc$
 2. $(x)\sim Bx$ / $(\exists x)Ax$
 3. $\sim Bc$ 2, UI
 4. Ab 1,3 Com, DS
 5. $(\exists x)Ax$ 4, EG

QED

- 2)
 1. $(x)(Rx \supset Ox)$
 2. $(\exists x)\sim Ox$
 3. $(x)(\sim Rx \supset Px)$ / $(\exists x)Px$
 4. $\sim Oa$ 2, EI
 5. $Ra \supset Oa$ 1, UI
 6. $\sim Ra$ 5, 4, MT
 7. $\sim Ra \supset Pa$ 3, UI
 8. Pa 7, 6, MP
 9. $(\exists x)Px$ 8, EG

QED

- 3)
 1. $(x)(Fx \supset Gx)$
 2. $(y)(Gy \supset Hy)$ / $(z)(\sim Hz \supset \sim Fz)$
 3. $Fx \supset Gx$ 1, UI
 4. $Gx \supset Hx$ 2, UI
 5. $Fx \supset Hx$ 3, 4, HS
 6. $\sim Hx \supset \sim Fx$ 5, Trans
 7. $(z)(\sim Hz \supset \sim Fz)$ 6, UG

QED

- 4)
 1. $(\exists x)(Ax \cdot Bx) \supset (x) Dx$
 2. $\sim Da$ / $(x)(Ax \supset \sim Bx)$
 3. $(\exists x)\sim Dx$ 2, EG

- | | |
|-----------------------------------|----------|
| 4. $\sim(x)Dx$ | 3, CQ |
| 5. $\sim(\exists x)(Ax \cdot Bx)$ | 1, 4, MT |
| 6. $(x)\sim(Ax \cdot Bx)$ | 5, CQ |
| 7. $(x)(\sim Ax \vee \sim Bx)$ | 6, DM |
| 8. $(x)(Ax \supset \sim Bx)$ | 7, Impl |

QED

- 5)
- | | |
|---|-------------------|
| 1. $(y)[Ay \supset (\sim By \supset Dy)]$ | |
| 2. $\sim Ba$ | / $Aa \supset Da$ |
| 3. Aa | ACP |
| 4. $Aa \supset (\sim Ba \supset Da)$ | 1, UI |
| 5. $\sim Ba \supset Da$ | 4, 3, MP |
| 6. Da | 5, 2, MP |
| 7. $Aa \supset Da$ | 3-6, CP |

QED

- 6)
- | | |
|--------------------------------------|-------------------------------------|
| 1. $(x)(Qx \supset \sim Px)$ | / $(\exists x)Px \supset \sim(x)Qx$ |
| 2. $(\exists x)Px$ | ACP |
| 3. Pa | 2, EI |
| 4. $Qa \supset \sim Pa$ | 1, UI |
| 5. $\sim Qa$ | 4, 3, DN, MT |
| 6. $(\exists x)\sim Qx$ | 5, EG |
| 7. $\sim(x)Qx$ | 6, CQ |
| 8. $(\exists x)Px \supset \sim(x)Qx$ | 2-7, CP |

QED

- 7)
- | | |
|------------------------------------|--------------|
| 1. $(x)[Ax \supset (Bx \cdot Dx)]$ | |
| 2. $(x)[(Ax \cdot Dx) \supset Ex]$ | |
| 3. $(x)(Ex \supset \sim Dx)$ | / $\sim Aa$ |
| 4. Aa | AIP |
| 5. $Aa \supset (Ba \cdot Da)$ | 1, UI |
| 6. $Ba \cdot Da$ | 5, 4, MP |
| 7. Da | 6, Com, Simp |
| 8. $Aa \cdot Da$ | 4, 7, Conj |
| 9. $(Aa \cdot Da) \supset Ea$ | 2, UI |
| 10. Ea | 9, 8, MP |
| 11. $Ea \supset \sim Da$ | 3, UI |
| 12. $\sim Da$ | 11, 10, MP |
| 13. $Da \cdot \sim Da$ | 7, 12, Conj |
| 14. $\sim Aa$ | 4-13, CP |

QED

- 8)
- | | |
|---|---------------------------------------|
| 1. $(x)(Ax \supset Bx)$ | |
| 2. $(x)[Bx \supset (Ax \supset \sim Fx)]$ | |
| 3. $(x)[(\sim Cx \cdot Dx) \supset Fx]$ | / $(x)[Ax \supset (Cx \vee \sim Dx)]$ |
| 4. Ax | ACP |
| 5. $Ax \supset Bx$ | 1, UI |
| 6. Bx | 5, 4, MP |
| 7. $Bx \supset (Ax \supset \sim Fx)$ | 2, UI |
| 8. $Ax \supset \sim Fx$ | 7, 6, MP |
| 9. $\sim Fx$ | 8, 4, MP |
| 10. $(\sim Cx \cdot Dx) \supset Fx$ | 3, UI |
| 11. $\sim(\sim Cx \cdot Dx)$ | 10, 9, MT |
| 12. $Cx \vee \sim Dx$ | 11, DM, DN |
| 13. $Ax \supset (Cx \vee \sim Dx)$ | 4-12, CP |
| 14. $(x)[Ax \supset (Cx \vee \sim Dx)]$ | 13, UG |

QED

- 9) 1. $(\exists x)Gx \supset (x)(Fx \supset Dx)$
 2. $(\exists x)(Gx \cdot \sim Dx)$ / $\sim(x)Fx$
 3. $Ga \cdot \sim Da$ 2, EI
 4. Ga 3, Simp
 5. $(\exists x)Gx$ 4, EG
 6. $(x)(Fx \supset Dx)$ 1, 5, MP
 7. $Fa \supset Da$ 6, UI
 8. $\sim Da$ 3, Com, Simp
 9. $\sim Fa$ 7, 8, MT
 10. $(\exists x)\sim Fx$ 9, EG
 11. $\sim(x)Fx$ 10, CQ

QED

- 10) 1. $(\exists x)Qx \supset (x)(Rx \supset Sx)$
 2. $(x)\sim Qx \supset (\exists x)Sx$
 3. $(x) Rx$ / $(\exists x)Sx$
 4. $\sim(\exists x)Sx$ AIP
 5. $\sim(x)\sim Qx$ 2, 4, MT
 6. $(\exists x)Qx$ 5, CQ
 7. $(x)(Rx \supset Sx)$ 1, 6, MP
 8. $Rx \supset Sx$ 7, UI
 9. Rx 3, UI
 10. Sx 8, 9, MP
 11. $(\exists x)Sx$ 10, EG
 12. $(\exists x)Sx \cdot \sim(\exists x)Sx$ 11, 4, Conj
 13. $(\exists x)Sx$ 4-12, IP, DN

QED

III. Translations (including relational predicates and identity theory)

1. $(x)(Fx \supset Px)$
2. $(x)[(Gx \cdot Fx) \supset Px]$
3. $(x)(Fx \supset Mnx)$
4. $(x)(Wpx \supset Mnx)$
5. $(x)(Sxn \supset Mnx)$
6. $(x)(\sim Mxx \supset Mnx)$
7. $(x)(y)(Sxy \supset \sim Syx)$
8. $(x)\{ (Fx \cdot Px) \supset (\exists y)[(Gy \cdot Py) \cdot Rxy] \} \supset (\exists x)[Gx \cdot (y)(Fy \supset Sxy)]$
9. $Bac \cdot (x)(Bxc \supset x=a)$
 $(\exists x)(Nx \cdot Wex)$
 $\sim(\exists x)(Nx \cdot Wax)$ / $\sim Bec$
10. $(x)(y)(Rxy \supset \sim x=y)$ / $(x)\sim Rxx$
11. $(x)(y)(z)(x=y \vee y=z \vee x=z)$
 $(\exists x)(\sim x=c \cdot Hx)$ / $(\exists x)(\exists y)[\sim x=y \cdot (z)(z=x \vee z=y)]$
12. $(\exists x)[Bxc \cdot (y)(Byc \supset y=x) \cdot Hx]$ / $(\exists x)(Bxc)$
13. $(x)[(\sim x=c \cdot \sim x=b) \supset Hx]$
 $\sim Ha$ / $a=c \vee a=b$

IV. Derivations

- 1) 1. $(x)(\exists y)(\sim Fx \vee Gy)$ / $(x)Fx \supset (\exists y)Gy$
 2. $(x)Fx$ ACP
 3. $(\exists y)(\sim Fx \vee Gy)$ 1, UI
 4. $\sim Fx \vee Ga$ 3, EI
 5. Fx 2, UI
 6. Ga 4, 5, DN, DS

- | | |
|----------------------------------|--------|
| 7. $(\exists y)Gy$ | 6, EG |
| 8. $(x)Fx \supset (\exists y)Gy$ | 2-7 CP |

QED

- | | | |
|----|--|----------------------------|
| 2) | 1. $(x)(\exists y)Fxy \supset (x)(\exists y)Gxy$ | |
| | 2. $(\exists x)(y)\sim Gxy$ | / $(\exists x)(y)\sim Fxy$ |
| | 3. $\sim (x)(\exists y)Gxy$ | 2, CQ, CQ |
| | 4. $\sim (x)(\exists y)Fxy$ | 1, 3, MT |
| | 5. $(\exists x)(y)\sim Fxy$ | 4, CQ, CQ |

QED

- | | | |
|----|---|-------------------------------------|
| 3) | 1. $(x)[(Fx \vee Gx) \supset (Hx \cdot Kx)]$ | |
| | 2. $(x)\{(Hx \vee Lx) \supset [(Hx \cdot Nx) \supset Px]\}$ | / $(x)[Fx \supset (Nx \supset Px)]$ |
| | 3. Fx | ACP |
| | 4. $Fx \vee Gx$ | 3, Add |
| | 5. $(Fx \vee Gx) \supset (Hx \cdot Kx)$ | 1, UI |
| | 6. $Hx \cdot Kx$ | 5, 4, MP |
| | 7. Hx | 6, Simp |
| | 8. $Hx \vee Lx$ | 7, Add |
| | 9. $(Hx \vee Lx) \supset [(Hx \cdot Nx) \supset Px]$ | 2, UI |
| | 10. $(Hx \cdot Nx) \supset Px$ | 9, 8, MP |
| | 11. Nx | ACP |
| | 12. $Hx \cdot Nx$ | 7, 11, Conj |
| | 13. Px | 10, 12, MP |
| | 14. $Nx \supset Px$ | 11-13, CP |
| | 15. $Fx \supset (Nx \supset Px)$ | 3-14, CP |
| | 16. $(x)[Fx \supset (Nx \supset Px)]$ | 15, UG |

- | | | |
|----|---|--------------------------|
| 4) | 1. $\sim (\exists x)(Axa \cdot \sim Bxb)$ | |
| | 2. $\sim (\exists x)(Dxd \cdot Dbx)$ | |
| | 3. $(x)(Bex \supset Dxd)$ | / $\sim (Aea \cdot Dgd)$ |
| | 4. $Aea \cdot Dgd$ | AIP |
| | 5. $(x)\sim (Axa \cdot \sim Bxb)$ | 1, CQ |
| | 6. $(x)(\sim Axa \vee Bxb)$ | 5, DM, DN |
| | 7. $\sim Aea \vee Beb$ | 6, UI |
| | 8. Aea | 4, Simp |
| | 9. Beb | 7, 8, DN, DS |
| | 10. $(x)\sim (Dxd \cdot Dbx)$ | 2, CQ |
| | 11. $(x)(\sim Dxd \vee \sim Dbx)$ | 10, DM |
| | 12. $\sim Dgd \vee \sim Dbg$ | 11, UI |
| | 13. Dgd | 4, Com, Simp |
| | 14. $\sim Dbg$ | 12, 13, DN, DS |
| | 15. $Beb \supset Dbg$ | 3, UI |
| | 16. Dbg | 15, 9, MP |
| | 17. $Dbg \cdot \sim Dbg$ | 16, 14, Conj |
| | 18. $\sim (Aea \cdot Dgd)$ | 4-17, IP |

QED

- | | | |
|----|--------------------------------|--|
| 5) | 1. $(x)(Ax \supset Bx)$ | / $(x)[(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)]$ |
| | 2. $(\exists y)(Ay \cdot Cxy)$ | ACP |
| | 3. $Aa \cdot Cxa$ | 2, EI |
| | 4. Aa | 3, Simp |
| | 5. $Aa \supset Ba$ | 1, UI |
| | 6. Ba | 5, 4, MP |
| | 7. Cxa | 3, Com, Simp |
| | 8. $Ba \cdot Cxa$ | 6, 7, Conj |

- | | |
|--|---------|
| 9. $(\exists z)(Bz \cdot Cxz)$ | 8, EG |
| 10. $(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)$ | 2-9, CP |
| 11. $(x)[(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)]$ | 10, UG |

QED

- 6)
- | | |
|---|--------------|
| 1. $(\exists x)(Nx \cdot Wjx \cdot Ix)$ | |
| 2. $Nc \cdot Wjc \cdot (x)[(Nx \cdot Wjx) \supset x=c]$ | / Ic |
| 3. $Na \cdot Wja \cdot Ia$ | 1, EI |
| 4. $(x)[(Nx \cdot Wjx) \supset x=c]$ | 2, Com, Simp |
| 5. $(Na \cdot Wja) \supset a=c$ | 4, UI |
| 6. $Na \cdot Wja$ | 3, Simp |
| 7. $a=c$ | 5, 6, MP |
| 8. Ia | 3, Com, Simp |
| 9. Ic | 8, 7, ID |

QED

- 7)
- | | |
|---|--------------|
| 1. $Pa \cdot Oa \cdot (y)[(Py \cdot Oy) \supset y=a]$ | |
| 2. $Pw \cdot Sw \cdot (y)[(Py \cdot Sy) \supset y=w]$ | |
| 3. $(\exists x)(Px \cdot Sx \cdot Ox)$ | / a=w |
| 4. $Pb \cdot Sb \cdot Ob$ | 3, EI |
| 5. $(y)[(Py \cdot Oy) \supset y=a]$ | 1, Com, Simp |
| 6. $Pb \cdot Ob$ | 4, Simp |
| 7. $(Pb \cdot Ob) \supset b=a$ | 5, UI |
| 8. $b=a$ | 7, 6, MP |
| 9. $(y)[(Py \cdot Sy) \supset y=w]$ | 2, Com, Simp |
| 10. $Pb \cdot Sb$ | 4, Simp |
| 11. $(Pb \cdot Sb) \supset b=w$ | 9, UI |
| 12. $b=w$ | 11, 10, MP |
| 13. $a=w$ | 12, 8, ID |

QED

- 8)
- | | |
|--|--|
| 1. $(\exists x)\{Mx \cdot Tx \cdot (y)[(My \cdot y \neq x) \supset Hxy]\}$ | / $(\exists x)\{Mx \cdot Tx \cdot (y)[(My \cdot \sim Ty) \supset Hxy]\}$ |
| 2. $Ma \cdot Ta \cdot (y)[(My \cdot \sim y=a) \supset Hay]$ | 1, EI |
| 3. $My \cdot \sim Ty$ | ACP |
| 4. $(y)[(My \cdot \sim y=a) \supset Hay]$ | 2, Com, Simp |
| 5. $(My \cdot \sim y=a) \supset Hay$ | 4, UI |
| 6. $y=a$ | AIP |
| 7. Ta | 2, Simp |
| 8. $\sim Ty$ | 3, Com, Simp |
| 9. Ty | 7, 6, ID |
| 10. $Ty \cdot \sim Ty$ | 9, 8, Conj |
| 11. $\sim y=a$ | 6-10, IP |
| 12. My | 3, Simp |
| 13. $My \cdot \sim y=a$ | 12, 11, Conj |
| 14. Hay | 5, 13, MP |
| 15. $(My \cdot \sim Ty) \supset Hay$ | 3-14, CP |
| 16. $(y)[(My \cdot \sim Ty) \supset Hay]$ | 15, UG |
| 17. $Ma \cdot Ta$ | 2, Simp |
| 18. $Ma \cdot Ta \cdot (y)[(My \cdot \sim Ty) \supset Hay]$ | 17, 16, Conj |
| 19. $(\exists x)\{Mx \cdot Tx \cdot (y)[(My \cdot \sim Ty) \supset Hxy]\}$ | 18, EG |

QED

- 9)
- | |
|---|
| 1. $(x)(y)(z)[(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (x=y \vee y=z \vee x=z)]$ |
| 2. $(\exists x)(\exists y)(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Rx \cdot Ry \cdot x \neq y)$ |

3. $(x)(Rx \supset \sim Cx)$	$/ (Sa \cdot Ca) \supset \sim La$	
4. $Sa \cdot Ca$		ACP
5. La		AIP
6. $(\exists y)(Sb \cdot Lb \cdot Sy \cdot Ly \cdot Rb \cdot Ry \cdot b \neq y)$		3, EI
7. $Sb \cdot Lb \cdot Sc \cdot Lc \cdot Rb \cdot Rc \cdot b \neq c$		6, EI
8. $Sb \cdot Lb \cdot Sc \cdot Lc$		7, Simp
9. Sa		4, Simp
10. $Sa \cdot La$		9, 5, Conj
11. $Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc$		10, 8, Conj
12. $(y)(z)[(Sa \cdot La \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (a=y \vee y=z \vee a=z)]$		1, UI
13. $(z)[(Sa \cdot La \cdot Sb \cdot Lb \cdot Sz \cdot Lz) \supset (a=b \vee b=z \vee a=z)]$		12, UI
14. $(Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc) \supset (a=b \vee b=c \vee a=c)$		13, UI
15. $a=b \vee b=c \vee a=c$		14, 11, MP
16. $\sim b=c$		7, Simp
17. $a=b \vee a=c$		17, 16, Com, DS
18. $Ra \supset \sim Ca$		3, UI
19. Ca		4, Com, Simp
20. $\sim Ra$		18, 19, DN, MT
21. Rb		7, Simp
22. $a=b$		AIP
23. $\sim Rb$		20, 22, ID
24. $Rb \cdot \sim Rb$		21, 24, Conj
25. $\sim a=b$		22-24, IP
26. $a=c$		17, 25, DS
27. Rc		7, Simp
28. $Rc \supset \sim Cc$		3, UI
29. $\sim Cc$		28, 27, MP
30. Cc		19, 26, ID
31. $Cc \cdot \sim Cc$		30, 29, Conj
33. $\sim La$		5-31, IP
34. $(Sa \cdot Ca) \supset \sim La$		4-33, CP

QED

V. Invalidity

- 1) Invalid in a 1-member universe, where: Aa: True; Ba: False; Ca: False.
- 2) Invalid in a 2-member universe, where Ea: True; Fa: False; Ga: False; Eb: True or False; Fb: True; Gb: True
- 3) Invalid in a 3-member universe, where Pa: True; Qa: False; Ra: False
Pb: False; Qb: True; Rb: False
Pc: False; Qc: False; Rc: True or False