Philosophy 240
Symbolic Logic

Russell Marcus
Hamilton College
Fall 2015

Class #32 - Translation Using Relational Predicates

Business

- Test #5 is next Friday
  - Invalidity in M
  - Derivations in F
  - (Someday I'll add invalidity in F, but not this year)
  - We'll start with derivations on Monday
    - Not much to do except practice
- Paper proposals are due on Monday
  - Proposals should be two-three paragraphs, describing your topic and citing, properly, sources. Failure to submit a satisfactory proposal on time will result in a deduction of your paper grade.
  - On paper, even if we've talked
- Friday: final Philosophy Friday, on the philosophical importance of ∃
- Today: Relation Predicates and their semantics

Limits of Monadic Predicates

Consider:

1. Andrew is taller than Bob.
2. Bob is taller than Charles.
3. For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z.

So, Andrew is taller than Charles.

1. Ta
2. Yb
3. ???

/ Ya

Relational (Polyadic) Predicates

- Dyadic:
    - Txy: x is taller than y
    - Kxy: x knows y
    - Bxy: x believes y
    - Dxy: x does y
- Triadic:
    - Gxyz: x gives y to z
    - Kxyz: x kisses y in z
    - Bxyz: x is between y and z
- We can construct four-place and higher-place predicates, too.

Choosing Your Predicates

- Andrés loves Beatriz
  - La
  - Lab
- Camila gave David the earring.
  - Gc
  - Gcde
- There is something blue over there now.
  - (∃x)Bxabct
- By using a relational predicate, we reveal more logical structure.
- The more logical structure we reveal, the more we can facilitate inferences.

Full First-Order Logic

- We are now using **F**, Full First-Order Predicate Logic, rather than **M**.
- For the purposes of this course, the differences between **F** and **M** are minor.
- Beyond this course, the differences between **M** and **F** are significant; we have breached a barrier.
- **M** admits of a decision procedure: there is a way of deciding, for any given formula, whether it is a theorem or not.
- **F** is not decidable.
- There are formulas for which there are no effective methods for deciding whether they are theorems or not.
  - There is no effective procedure for churning out all the theorems and all the non-theorems.

Syntax for **M** and **F**

Vocabulary for **M** and **F**

Capital letters A...Z used as predicates
Lower case letters (singular terms)
  a, b, c,...u are constants.
  v, w, x, y, z are variables.
Five connectives: ~, •, ∨, ⊃ ≡
Quantifier Symbols: ∃, ∀
Punctuation: ( ), [ ], { }

Formation Rules for Wffs of **M**

1. A predicate (capital letter) followed by a singular term (lower-case letter) is a wff.
2. For any variable β, if α is a wff that does not contain either ‘(∃β)’ or ‘(∀β)’, then ‘(∃β)α’ and ‘(∀β)α’ are wffs.
3. If α is a wff, so is ~α.
4. If α and β are wffs, then so are:
   (α · β)
   (α ∨ β)
   (α ⊃ β)
   (α ≡ β)
5. These are the only ways to make wffs.

Formation Rules for Wffs of **F**

1. **A predicate followed by any number of singular terms is a wff.**
2. For any variable β, if α is a wff that does not contain either ‘(∃β)’ or ‘(∀β)’, then ‘(∃β)α’ and ‘(∀β)α’ are wffs.
3. If α is a wff, so is ~α.
4. If α and β are wffs, then so are:
   (α · β)
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   (α ⊃ β)
   (α ≡ β)
5. These are the only ways to make wffs.

Semantics for F

- Recall that there are four steps for providing a semantics for **M**.
  - Step 1. Specify a set to serve as a domain of interpretation.
  - Step 2. Assign a member of the domain to each constant.
  - Step 3. Assign some set of objects in the domain to each predicate.
  - Step 4. Use the customary truth tables for the interpretation of the connectives.
- The introduction of relational predicates for **F** requires adjustment only to Step 3.
- We assign sets of ordered n-tuples to each relational predicate.
  - **New** Step 3. Assign a set of ordered n-tuples of objects in the domain to each n-place predicate.
    - Taking a 1-tuple (single) of objects to be just an object

N-Tuples

- An n-tuple is an n-place relation.
  - an ordered sequence of objects
  - Singles are objects themselves
  - doubles, triples, quadruples...
  - a set with structure
- Sets are not ordered.
  - {1, 2} = {2, 1}
- N-tuples are ordered
  - <1, 2, 5> ≠ <2, 1, 5> ≠ <5, 2, 1>
- For the semantics for **F**, an n-place predicate is assigned sets of ordered n-tuples
- Domain = {1, 2, 3, 4, 5}
  - Nx: {1, 2, 3, 4, 5}
  - Ex: {2, 4}
  - Ox: {1, 3, 5}
  - Gxy: {<2,1>, <3,1>, <4,1>, <5,1>, <3,2>, <4,2>, <5,2>, <4,3>, <5,3>, <5,4>}
  - Lxy: {<1,2>, <1,3>, <1,4>, <1,5>, <2,3>, <2,4>, <2,5>, <3,4>, <3,5>, <4,5>}

Satisfaction and Truth

- Objects in the domain (still) can satisfy one-place predicates.
- Ordered n-tuples satisfy (or don’t) relational predicates.
- A wff will be satisfiable if there are objects in the domain of quantification which stand in the relations indicated in the wff.
- A wff will be true for an interpretation if all objects in the domain of quantification stand in the relations indicated in the wff.
- And, still, a wff will be logically true if it is true for every interpretation.

A Sample Theory and Interpretation

1. $Pa \bullet Pb$
2. $Wa \bullet {\sim}Wb$
3. Oab
4. Obc
5. $(\exists x)(Px \bullet Oxb)$
6. $(\exists x)(Px \bullet Obx)$
7. $(\forall x)[Wx \supset (\exists y)(Py \bullet Oxy)]$

- Domain: {Bob Simon, Rick Werner, Katheryn Doran, Todd Franklin, Marianne Janack, Russell Marcus, Theresa Lopez, Alex Plakias, Doug Edwards}
- Constants
  - a: Katheryn Doran
  - b: Bob Simon
  - c: Russell Marcus
- Predicates
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- 1 and 2 are true

A Sample Theory and Interpretation

1. Pa • Pb
2. Wa • ~Wb
3. Oab
4. Obc
5. (∃x)(Px • Oxb)
6. (∃x)(Px • Obx)
7. (∀x)[Wx ⊃ (∃y)(Py • Oxy)]

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- 3 is false.

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- 4 is true

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- 5 is false.

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- 6 is true.

A Sample Theory and Interpretation

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7. $(\forall x)[Wx \supset (\exists y)(Py \bullet Oxy)]$

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- 7 is false

Some Translations

1. John loves Mary (Lxy: x loves y)
- Ljm

2. Tokyo isn’t smaller than New York. (Sxy: x is smaller than y)
- ~Stn

3. Marco was introduced to Paco by Erika. (Ixyz: x introduced y to z)
- Iemp

4. America took California from Mexico. (Txyz: x was taken by y from z)
- Tcam

Our Original Argument

Consider:

1. Bob is taller than Charles.
2. Andrew is taller than Bob.
3. For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z.

So, Andrew is taller than Charles.

1. Tbc
2. Tab
3. ???

/ Tac

Quantifiers and Relational Predicates

Bxy: x is bigger than y

- Joe is bigger than some thing.
  $(\exists x)Bjx$
- Something is bigger than Joe.
  $(\exists x)Bxj$
- Joe is bigger than everything.
  $(\forall x)Bjx$
- Everything is bigger than Joe.
  $(\forall x)Bxj$

Overlapping Quantifiers

Lxy: x loves y

- Everything loves something.
  $(\forall x)(\exists y)Lxy$
- Something loves everything.
  $(\exists x)(\forall y)Lxy$
- $(\forall x)(\exists y)Lyx$
  Everything is loved by something.
- $(\exists x)(\forall y)Lyx$
  Something is loved by everything.

Our Original Argument

Finally Translated

Consider:

1. Bob is taller than Charles.
2. Andrew is taller than Bob.
3. For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z.

So, Andrew is taller than Charles.

1. Tbc
2. Tab
3. $(\forall x)(\forall y)(\forall z)[(Txy \bullet Tyz) \supset Txz]$ / Tac

Derivation on Wednesday!

More Examples

Something teaches Plato. (Txy: x teaches y)

- $(\exists x)Txp$

Someone teaches Plato. (Px: x is a person)

- $(\exists x)(Px \bullet Txp)$

Plato teaches everyone.

- $(\forall x)(Px \supset Tpx)$

Everyone teaches something.

- $(\forall x)[Px \supset (\exists y)Txy]$

Some people teach themselves.

- $(\exists x)(Px \bullet Txx)$

There are teachers.

- $(\exists x)(\exists y)Txy$

There are students.

- $(\exists x)(\exists y)Tyx$

Skilled teachers are interesting.

- $(\forall x)[(\exists y)Txy \supset (Sx \supset Ix)]$

Skilled teachers are better than unskilled teachers.

- $(\forall x)\{[(\exists y)Txy \bullet Sx] \supset \{(\forall z)[(\exists w)Tzw \bullet {\sim}Sz] \supset Bxz\}\}$

Wide and Narrow Scope

- Wide: $(\exists x)(\exists y)[(Px \bullet Py) \bullet Lxy]$
- Narrow: $(\exists x)[Px \bullet (\exists y)(Py \bullet Lxy)]$
- Give your quantifiers as narrow a scope as possible.
- Not equivalent:
  - $(\forall x)[Px \supset (\exists y)(Py \bullet Qxy)]$
    - ‘all people love someone’
  - $(\exists y)(\forall x)[Px \supset (Py \bullet Qxy)]$
    - ‘there is someone everyone loves’
- $(\forall x)(\forall y)(\forall z)[(Txy \bullet Tyz) \supset Txz]$
  - $(\forall x)(\forall y)(\forall z)[Txy \supset (Tyz \supset Txz)]$
  - $(\forall x)(\forall y)[Txy \supset (\forall z) (Tyz \supset Txz)]$

Moving Quantifiers

Of the same type

- In some cases, we can move quantifiers around without much worry.
  - If quantifiers are of the same type, we can push them in or pull them out.
  - Be careful not to accidentally bind any variables!
- Everyone loves everyone
  - $(\forall x)[Px \supset (\forall y)(Py \supset Lxy)]$
  - $(\forall x)(\forall y)[(Px \cdot Py) \supset Lxy]$
  - $(\forall y)(\forall x)[(Px \cdot Py) \supset Lxy]$
- Someone loves someone
  - $(\exists x)[Px \cdot (\exists y)(Py \cdot Lxy)]$
  - $(\exists x)(\exists y)[(Px \cdot Py) \cdot Lxy]$
  - $(\exists y)(\exists x)[(Px \cdot Py) \cdot Lxy]$

Mixing Quantifiers

- None of the following examples are equivalent:
  - Everyone loves someone: $(\forall x)(\exists y)[Px \supset (Py \cdot Lxy)]$
  - Everyone is loved by someone: $(\forall x)(\exists y)[Px \supset (Py \cdot Lyx)]$
  - Someone loves everyone: $(\exists x)(\forall y)[Px \cdot (Py \supset Lxy)]$
  - Someone is loved by everyone: $(\exists x)(\forall y)[Px \cdot (Py \supset Lyx)]$
- The first word in each translation above corresponds to the leading quantifier.
- The connectives which directly follow the ‘Px’ and the ‘Py’ are determined by the quantifier binding that variable.

Using Narrow Scope

- Everyone loves someone.
  - (∀x)[Px ⊃ (∃y)(Py · Lxy)]
- Everyone is loved by someone.
  - (∀x)[Px ⊃ (∃y)(Py • Lyx)]
- Someone loves everyone.
  - (∃x)[Px · (∀y)(Py ⊃ Lxy)]
- Someone is loved by everyone.
  - (∃x)[Px · (∀y)(Py ⊃ Lyx)]

Moving Mixed Quantifiers: A Problem

- The following sentences are *not* equivalent
  - $(\forall x)[(\exists y)Lxy \supset Hx]$
    - For any x, if there is a y that x loves, then x is happy.
    - All lovers are happy.
  - $(\forall x)(\exists y)(Lxy \supset Hx)$
    - For any x, there is a y such that if x loves y then x is happy.
- The first does not commit to the existence of something that, by being loved, makes a person happy.
- The second does.

Prenex Normal Form (PNF)

- Some metalogical proofs require all statements of **F** to be written with all quantifiers having wide scope.
- A sentence is in Prenex Normal Form (PNF) if all of its quantifiers are in the front, having wide scope.
- Rules of Passage allow us to transform all statements of **F** into PNF.
- They are rules of replacement.
- I will not require that you use them in proofs.
- They may be useful in learning how to translate.

Rules of Passage

- For all variables α and all formulas Γ and Δ:
  - RP1: $(\exists\alpha)(\Gamma \vee \Delta) \rightleftarrows (\exists\alpha)\Gamma \vee (\exists\alpha)\Delta$
  - RP2: $(\forall\alpha)(\Gamma \bullet \Delta) \rightleftarrows (\forall\alpha)\Gamma \bullet (\forall\alpha)\Delta$
- For all variables α, all formulas Γ containing α, and all formulas Δ not containing α:
  - RP3: $(\exists\alpha)(\Delta \bullet \Gamma\alpha) \rightleftarrows \Delta \bullet (\exists\alpha)\Gamma\alpha$
  - RP4: $(\forall\alpha)(\Delta \bullet \Gamma\alpha) \rightleftarrows \Delta \bullet (\forall\alpha)\Gamma\alpha$
  - RP5: $(\exists\alpha)(\Delta \vee \Gamma\alpha) \rightleftarrows \Delta \vee (\exists\alpha)\Gamma\alpha$
  - RP6: $(\forall\alpha)(\Delta \vee \Gamma\alpha) \rightleftarrows \Delta \vee (\forall\alpha)\Gamma\alpha$
  - RP7: $(\exists\alpha)(\Delta \supset \Gamma\alpha) \rightleftarrows \Delta \supset (\exists\alpha)\Gamma\alpha$
  - RP8: $(\forall\alpha)(\Delta \supset \Gamma\alpha) \rightleftarrows \Delta \supset (\forall\alpha)\Gamma\alpha$
  - RP9: $(\exists\alpha)(\Gamma\alpha \supset \Delta) \rightleftarrows (\forall\alpha)\Gamma\alpha \supset \Delta$
  - RP10: $(\forall\alpha)(\Gamma\alpha \supset \Delta) \rightleftarrows (\exists\alpha)\Gamma\alpha \supset \Delta$
- These are rules of replacement and may be used on parts of lines.
- Rules of Passage are **not required**, but see §4.12.

More Translations

1. Everyone loves something. (Px, Lxy: x loves y)
   - $(\forall x)[Px \supset (\exists y)Lxy]$
2. No one knows everything. (Px, Kxy: x knows y)
   - $(\forall x)[Px \supset (\exists y){\sim}Kxy]$
3. No one knows everyone.
   - $(\forall x)[Px \supset (\exists y)(Py \bullet {\sim}Kxy)]$
4. Every woman is stronger than some man. (Wx, Mx, Sxy: x is stronger than y)
   - $(\forall x)[Wx \supset (\exists x)(Mx \bullet Sxy)]$
5. No cat is smarter than any horse. (Cx, Hx, Sxy: x is smarter than y)
   - $(\forall x)[Cx \supset {\sim}(\exists y)(Hy \bullet Sxy)]$
   - $(\forall x)[Cx \supset (\forall y)(Hy \supset {\sim}Sxy)]$

Even More Translations

6. Dead men tell no tales. (Dx, Mx, Tx, Txy: x tells y)

- (∀x)[(Dx • Mx) ⊃ (∀y)(Ty ⊃ ~Txy)]

7. There is a city between New York and Washington. (Cx, Bxyz: y is between x and z)

- (∃x)(Cx • Bnxw)

8. Everyone gives something to someone. (Px, Gxyz: y gives x to z)

- (∀x)[Px ⊃ (∃y)(∃z)(Pz • Gyxz)]

9. A dead lion is more dangerous than a live dog. (Ax: x is alive, Lx, Dx, Dxy: x is more dangerous than y)

- (∀x){(Lx • ~Ax) ⊃ (∀y)[(Dy • Ay) ⊃ Dxy]}

10. A lawyer who pleads his own case has a fool for a client. (Lx, Fx, Pxy: x pleads y's case; Cxy: y is a client of x)

- (∀x)[(Lx • Pxx) ⊃ (∃y)(Fy • Cxy)]
- (∀x)[(Lx • Pxx) ⊃ Fx]