Philosophy 240
Symbolic Logic

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Class #23 - Translation into Predicate Logic II (§3.2)

Names of Languages We Will Study

- **PL**: Propositional Logic
- **M**: Monadic (First-Order) Predicate Logic
- **F**: Full (First-Order) Predicate Logic
- **FF**: Full (First-Order) Predicate Logic with functors
- **S**: Second-Order Predicate Logic

How to Make a Logic

- Start with a language
  - Syntax
  - Vocabulary
  - Formation rules for wffs
  - We'll do this today.
- Specify a semantics
  - Which wffs are privileged (true)?
    - Axioms, theorems
  - Truth tables for **PL**
  - The semantics are more complicated for the languages of predicate logic (**M**, **F**, **FF**, **S**) as we'll see.
    - Reference (singular terms)
    - Satisfaction (predicates)
- Proof Theory
  - Rules for derivations are usually justified semantically.
  - Definition of validity
  - We start with proof theory for **M** on Monday.

Languages and Systems of Deduction

- With **PL**, we used one language and one set of inference rules.
- But we can use the same language in different deductive systems and we can use the same deductive system with different languages.
- We will use **M** and **F** with the same deductive system.
  - We'll tweak one rule just a bit to accommodate the relational predicates.
- We will introduce new rules for the language **F**, covering a special identity predicate.
  - We'll call this a new deductive system, though it's again just a small extension.

Vocabulary of M

- Capital letters A...Z used as one-place predicates
- Lower case letters for singular terms
  - a, b, c,...u are used as constants.
  - v, w, x, y, z are used as variables.
- Five connectives: $\sim$, $\bullet$, $\vee$, $\supset$ $\equiv$
- Quantifier symbols: $\exists$, $\forall$
- Punctuation: ( ), [ ], { }

Toward the Formation Rules for M

- Formation rules for **PL** were pretty easy:
  1. A single capital English letter is a wff.
  2. If α is a wff, so is $\sim\alpha$.
  3. If α and β are wffs, then so are:
     $(\alpha \cdot \beta)$
     $(\alpha \vee \beta)$
     $(\alpha \supset \beta)$
     $(\alpha \equiv \beta)$
  4. These are the only ways to make wffs.
- For **M**, we need some further concepts:
  Scope
  Binding
  Open and Closed formulas

On Scope

- $(\forall x)(Px \supset Qx)$
  Every P is Q
- $(\forall x)Px \supset Qx$
  If everything is P, then x is Q
- The difference between these two expressions is the scope of the quantifier.

Scope of a Negation

(in propositional logic)

The **scope of a negation** (in **PL**) is whatever directly follows the tilde.

- If what follows the tilde is a single propositional variable, then the scope of the negation is just that propositional variable.
- If what follows the tilde is another tilde, then the scope of the first (outside) negation is the scope of the second (inside) negation plus that inside tilde.
- If what follows the tilde is a bracket, then the entire formula which occurs between the opening and closing of that bracket is in the scope of the negation.

$\sim\{(P \bullet Q) \supset [\sim R \vee \sim\sim(S \equiv T)]\}$

Scope of a Quantifier

If what follows the quantifier is a bracket, then any formulas that occur until that bracket is closed are in the scope of the quantifier.

If what follows the quantifier is a tilde, then the tilde and every formula in its scope is in the scope of the quantifier.

If what follows the quantifier is another quantifier, then the inside quantifier and every formula in the scope of the inside quantifier is in the scope of the outside quantifier.

Quantifier Scope Example

$(\forall w)\{Pw \supset (\exists x)(\forall y)[(Px \bullet Py) \supset (\exists z)\sim(Qz \vee Rz)]\}$

- $(\forall w)$
  - Widest scope
  - $\{Pw \supset (\exists x)(\forall y)[(Px \bullet Py) \supset (\exists z)\sim(Qz \vee Rz)]\}$
- $(\exists x)$
  - $(\forall y)[(Px \bullet Py) \supset (\exists z)\sim(Qz \vee Rz)]$
- $(\forall y)$
  - $[(Px \bullet Py) \supset (\exists z)\sim(Qz \vee Rz)]$
- $(\exists z)$
  - $\sim(Qz \vee Rz)$
  - Narrowest scope

Binding

- Quantifiers bind every instance of their variable in their scope.
- A **bound variable** is attached to the quantifier which binds it.
  1. $(\forall x)(Px \supset Qx)$
  2. $(\forall x)Px \supset Qx$
  - In 1, the ‘x’ in ‘Qx’ is bound.
  - In 2, the ‘x’ in ‘Qx’ is not bound.
- An unbound variable is called a **free variable**.
  3. $(\forall x)Px \vee Qx$
  4. $(\exists x)(Px \vee Qy)$
  - In 3, ‘Qx’ is not in the scope of the quantifier, so that ‘x’ is unbound.
  - In 4, ‘Qy’ is in the scope of the quantifier, but ‘y’ is not the quantifier variable, so is unbound.
- Free variables lack character.
- Sentences with free variables are meaningless.

Open and Closed Sentences

- Wffs that contain at least one unbound variable are called **open sentences**.
  - Ax
  - (∀x)Px ∨ Qx
  - (∃x)(Px ∨ Qy)
  - (∀x)(Px ⊃ Qx) ⊃ Rz
- If a wff has no free variables, it is a **closed sentence**, and expresses a **proposition**.
  - (∀y)[(Py • Qy) ⊃ (Ra ∨ Sa)]
  - (∃x)(Px • Qx) ∨ (∀y)(Ay ⊃ By)
- Both closed and open sentences may be wffs.
- Translations from English into **M** should ordinarily yield closed sentences.
- We will use open sentences during proofs.
- We will remember the character (existential or universal) of each variable.

Formation Rules for Wffs of M

1. A predicate (capital letter) followed by a singular term (lower-case letter) is a wff.

2. For any variable β, if α is a wff that does not contain either ‘(∃β)’ or ‘(∀β)’ then ‘(∃β)α’ and ‘(∀β)α’ are wffs.

3. If α is a wff, so is ~α.

4. If α and β are wffs, then so are:

   (α • β)
   (α ∨ β)
   (α ⊃ β)
   (α ≡ β)

5. These are the only ways to make wffs.

Atomic Formulas Subformulas

- A wff constructed only using rule 1 is called an **atomic formula**.
  - Pa
  - Qt
  - Ax
- A wff that is part of another wff is called a **subformula**.
  - A **proper subformula** of α is a subformula of α not identical to α.
- In '(Pa • Qb) ⊃ (∃x)Rx', the following are all proper subformulas:
  - Pa
  - Qb
  - Rx
  - (∃x)Rx
  - Pa • Qb

Main Operators

- Quantifiers and connectives are called **operators**, or logical operators.
  - Atomic formulas lack operators.
  - The last operator added according to the formation rules is called the **main operator**.

Overlapping Quantifiers

- $(\exists x)[Px \bullet (\forall x)(Qx \supset Rx)]$
- Disappointingly ambiguous
- ill-formed
  - Formation rule 2: For any variable β, if α is a wff that does not contain either ‘(∃β)’ or ‘(∀β)’ then ‘(∃β)α’ and ‘(∀β)α’ are wffs.

HW for Friday

- More translation, and to English
- Some work on scope, binding, open and closed sentences, and main operators
- Philosophy Friday is a new topic, infinity, so I'll be very interested in feedback.