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Reference Sheeet for *What Follows*

Names of Languages

PL: Propositional Logic
M: Monadic (First-Order) Predicate Logic
F: Full (First-Order) Predicate Logic
FF: Full (First-Order) Predicate Logic with functors
S: Second-Order Predicate Logic

Basic Truth Tables

~	α
0	1
1	0

α	·	β
1	1	1
1	0	0
0	0	1
0	0	0

α	∨	β
1	1	1
1	1	0
0	1	1
0	0	0

α	⊃	β
1	1	1
1	0	0
0	1	1
0	1	0

α	≡	β
1	1	1
1	0	0
0	0	1
0	1	0

Translating Conditionals

If A then B	A ⊃ B
If B then A	B ⊃ A
A only if (only when) B	A ⊃ B
A is necessary for B	B ⊃ A
A is sufficient for B	A ⊃ B
B is necessary for A	A ⊃ B
B is sufficient for A	B ⊃ A
A entails (implies, means) B	A ⊃ B
A given (provided, on the condition) that B	B ⊃ A

Rules of Inference

Modus Ponens (MP)

$\alpha \supset \beta$
$\alpha \quad / \beta$

Modus Tollens (MT)

$\alpha \supset \beta$
$\sim\beta \quad / \sim\alpha$

Disjunctive Syllogism (DS)

$\alpha \vee \beta$
$\sim\alpha \quad / \beta$

Hypothetical Syllogism (HS)

$\alpha \supset \beta$
$\beta \supset \gamma \quad / \alpha \supset \gamma$

Conjunction (Conj)

α
$\beta \quad / \alpha \cdot \beta$

Addition (Add)

$\alpha \quad / \alpha \vee \beta$

Simplification (Simp)

$\alpha \cdot \beta \quad / \alpha$

Constructive Dilemma (CD)

$(\alpha \supset \beta)$
$(\gamma \supset \delta)$
$\alpha \vee \gamma \quad / \beta \vee \delta$

Rules of Equivalence

DeMorgan's Laws (DM)

$\sim(\alpha \cdot \beta) \rightleftarrows \sim\alpha \vee \sim\beta$
$\sim(\alpha \vee \beta) \rightleftarrows \sim\alpha \cdot \sim\beta$

Association (Assoc)

$\alpha \vee (\beta \vee \gamma) \rightleftarrows (\alpha \vee \beta) \vee \gamma$
$\alpha \cdot (\beta \cdot \gamma) \rightleftarrows (\alpha \cdot \beta) \cdot \gamma$

Distribution (Dist)

$\alpha \cdot (\beta \vee \gamma) \rightleftarrows (\alpha \cdot \beta) \vee (\alpha \cdot \gamma)$
$\alpha \vee (\beta \cdot \gamma) \rightleftarrows (\alpha \vee \beta) \cdot (\alpha \vee \gamma)$

Commutativity (Com)

$\alpha \vee \beta \rightleftarrows \beta \vee \alpha$
$\alpha \cdot \beta \rightleftarrows \beta \cdot \alpha$

Double Negation (DN)

$\alpha \rightleftarrows \sim\sim\alpha$

Contraposition (Cont)

$\alpha \supset \beta \rightleftarrows \sim\beta \supset \sim\alpha$

Material Implication (Impl)

$\alpha \supset \beta \rightleftarrows \sim\alpha \vee \beta$

Material Equivalence (Equiv)

$\alpha \equiv \beta \rightleftarrows (\alpha \supset \beta) \cdot (\beta \supset \alpha)$
$\alpha \equiv \beta \rightleftarrows (\alpha \cdot \beta) \vee (\sim\alpha \cdot \sim\beta)$

Exportation (Exp)

$\alpha \supset (\beta \supset \gamma) \rightleftarrows (\alpha \cdot \beta) \supset \gamma$

Tautology (Taut)

$\alpha \rightleftarrows \alpha \cdot \alpha$
$\alpha \rightleftarrows \alpha \vee \alpha$

Seven Derived Rules for the Biconditional

Rules of Inference

Biconditional Modus Ponens (BMP)

$\alpha \equiv \beta$
$\alpha \quad / \beta$

Biconditional Modus Tollens (BMT)

$\alpha \equiv \beta$
$\sim\alpha \quad / \sim\beta$

Biconditional Hypothetical Syllogism (BHS)

$\alpha \equiv \beta$
$\beta \equiv \gamma \quad / \alpha \equiv \gamma$

Rules of Equivalence

Biconditional DeMorgan's Law (BDM)

$\sim(\alpha \equiv \beta) \rightleftarrows \sim\alpha \equiv \beta$

Biconditional Commutativity (BCom)

$\alpha \equiv \beta \rightleftarrows \beta \equiv \alpha$

Biconditional Contraposition (BCont)

$\alpha \equiv \beta \rightleftarrows \sim\alpha \equiv \sim\beta$

Biconditional Associativity (BAssoc)

$\alpha \equiv (\beta \equiv \gamma) \rightleftarrows (\alpha \equiv \beta) \equiv \gamma$

Rules for Quantifier Instantiation and Generalization

Universal Instantiation (UI)

$\frac{(\forall\alpha)\mathscr{F}\alpha}{\mathscr{F}\beta}$ for any variable α, any formula $\mathscr{F}$ containing α, and any singular term β

Universal Generalization (UG)

$\frac{\mathscr{F}\beta}{(\forall\alpha)\mathscr{F}\alpha}$ for any variable β, any formula $\mathscr{F}$ containing β, and for any variable α

Never UG within the scope of an assumption for conditional or indirect proof on a variable that is free in the first line of the assumption.
Never UG on a variable when there is a constant present, and the variable was free when the constant was introduced.

Existential Generalization (EG)

$\frac{\mathscr{F}\beta}{(\exists\alpha)\mathscr{F}\alpha}$ for any singular term β, any formula $\mathscr{F}$ containing β, and for any variable α

Existential Instantiation (EI)

$\frac{(\exists\alpha)\mathscr{F}\alpha}{\mathscr{F}\beta}$ for any variable α, any formula $\mathscr{F}$ containing α, and any *new* constant β

Quantifier Exchange (QE)

$(\forall\alpha)\mathscr{F}\alpha$	$\rightleftarrows$	$\sim(\exists\alpha)\sim\mathscr{F}\alpha$
$(\exists\alpha)\mathscr{F}\alpha$	$\rightleftarrows$	$\sim(\forall\alpha)\sim\mathscr{F}\alpha$
$(\forall\alpha)\sim\mathscr{F}\alpha$	$\rightleftarrows$	$\sim(\exists\alpha)\mathscr{F}\alpha$
$(\exists\alpha)\sim\mathscr{F}\alpha$	$\rightleftarrows$	$\sim(\forall\alpha)\mathscr{F}\alpha$

Rules Governing the Identity Predicate (ID)

For any singular terms, α and β:

IDr $\alpha=\alpha$

IDs $\alpha=\beta \rightleftarrows \beta=\alpha$

IDi $\mathscr{F}\alpha$
$\alpha=\beta$ / $\mathscr{F}\beta$

Rules of Passage

For all variables α and all formulas Γ and Δ:

RP1:	$(\exists\alpha)(\Gamma \vee \Delta)$	$\rightleftarrows$	$(\exists\alpha)\Gamma \vee (\exists\alpha)\Delta$
RP2:	$(\forall\alpha)(\Gamma \bullet \Delta)$	$\rightleftarrows$	$(\forall\alpha)\Gamma \bullet (\forall\alpha)\Delta$

For all variables α, all formulas Γ containing α, and all formulas Δ not containing α:

RP3:	$(\exists\alpha)(\Delta \bullet \Gamma\alpha)$	$\rightleftarrows$	$\Delta \bullet (\exists\alpha)\Gamma\alpha$
RP4:	$(\forall\alpha)(\Delta \bullet \Gamma\alpha)$	$\rightleftarrows$	$\Delta \bullet (\forall\alpha)\Gamma\alpha$
RP5:	$(\exists\alpha)(\Delta \vee \Gamma\alpha)$	$\rightleftarrows$	$\Delta \vee (\exists\alpha)\Gamma\alpha$
RP6:	$(\forall\alpha)(\Delta \vee \Gamma\alpha)$	$\rightleftarrows$	$\Delta \vee (\forall\alpha)\Gamma\alpha$
RP7:	$(\exists\alpha)(\Delta \supset \Gamma\alpha)$	$\rightleftarrows$	$\Delta \supset (\exists\alpha)\Gamma\alpha$
RP8:	$(\forall\alpha)(\Delta \supset \Gamma\alpha)$	$\rightleftarrows$	$\Delta \supset (\forall\alpha)\Gamma\alpha$
RP9:	$(\exists\alpha)(\Gamma\alpha \supset \Delta)$	$\rightleftarrows$	$(\forall\alpha)\Gamma\alpha \supset \Delta$
RP10:	$(\forall\alpha)(\Gamma\alpha \supset \Delta)$	$\rightleftarrows$	$(\exists\alpha)\Gamma\alpha \supset \Delta$

Here are versions of each of the rules that are less-meta-linguistic and maybe easier to read:

RP1:	$(\exists x)(Px \vee Qx)$	$\rightleftarrows$	$(\exists x)Px \vee (\exists x)Qx$
RP2:	$(\forall x)(Px \bullet Qx)$	$\rightleftarrows$	$(\forall x)Px \bullet (\forall x)Qx$
RP3:	$(\exists x)(\mathscr{F} \bullet Px)$	$\rightleftarrows$	$\mathscr{F} \bullet (\exists x)Px$
RP4:	$(\forall x)(\mathscr{F} \bullet Px)$	$\rightleftarrows$	$\mathscr{F} \bullet (\forall x)Px$
RP5:	$(\exists x)(\mathscr{F} \vee Px)$	$\rightleftarrows$	$\mathscr{F} \vee (\exists x)Px$
RP6:	$(\forall x)(\mathscr{F} \vee Px)$	$\rightleftarrows$	$\mathscr{F} \vee (\forall x)Px$
RP7:	$(\exists x)(\mathscr{F} \supset Px)$	$\rightleftarrows$	$\mathscr{F} \supset (\exists x)Px$
RP8:	$(\forall x)(\mathscr{F} \supset Px)$	$\rightleftarrows$	$\mathscr{F} \supset (\forall x)Px$
RP9:	$(\exists x)(Px \supset \mathscr{F})$	$\rightleftarrows$	$(\forall x)Px \supset \mathscr{F}$
RP10:	$(\forall x)(Px \supset \mathscr{F})$	$\rightleftarrows$	$(\exists x)Px \supset \mathscr{F}$