Practice Problems for Test #6

I. Translations.

Use the following legend to translate the sentences below. For questions 1-9, do not use any functions.

a: Al
b: Bud
c: Cindy
e: Ed
m: Megha
n: Nietzsche
p: Plato

f(x): the father of x
g(x): the mother of x
f(x,y): the only son of x and y

Ax: x is an altruist
Jx: x is joyful
Nx: x is a novel
Px: x is a philosopher
Rx: x is Russian
Tx: x is thoughtful

Bxy: x is a brother of y
Mxy: x mocks y
Pxy: x produces y
Rxy: x is richer than y
Sxy: x is smarter than y

1. Megha's only brother is Al. Ed produces novels. Al doesn't. So, Ed isn't Megha's brother.
2. If one thing is richer than a second, then the two aren't identical. So, nothing is richer than itself.
3. There are at most two things. Something other than Cindy is joyful. So, there are exactly two things.
4. The brother of Cindy is joyful. So, Cindy has a brother.
5. Everything is joyful, except Megha and Bud. Al is not joyful. So, Al is either Megha or Bud.
6. The smartest Russian mocks Al and Bud.
7. The richest philosopher is smarter than any of Ed's brothers.
8. All thoughtful philosophers except Nietzsche are altruists.
9. Exactly three philosophers mock Plato and Nietzsche.
10. Bud's father is an altruist, but Cindy's mother is not.
11. The only son of Cindy and Ed has no brother.
12. If Cindy is thoughtful, then her mother is a joyful Russian and her father is an altruist who produces novels.
13. There are properties that Nietzsche has that Plato lacks.
14. All Russians have something in common.
15. Some transitive relations are asymmetric.
16. Everything is self-identical. Therefore, there is some relation that everything has to itself.

II. Derivations. Derive the conclusions of each of the following arguments.

1.
1. $(\exists x)(Nx \cdot Pjx \cdot Ix)$
2. $Nc \cdot Pjc \cdot (\forall x)[(Nx \cdot Pjx) \supset x=c]$ / Ic

2.
1. $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot y\neq x) \supset Dxy]$ / $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot {\sim}Ty) \supset Dxy]\}$

3.
1. $(\forall x)(\forall y)(\forall z)[(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (x=y \vee y=z \vee x=z)]$
2. $(\exists x)(\exists y)(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Rx \cdot Ry \cdot x\neq y)$
3. $(\forall x)(Rx \supset {\sim}Cx)$ / $(Sa \cdot Ca) \supset {\sim}La$

4.
1. $(\forall x)(\forall y)f(x,y)=f(y,x)$
2. $(\forall x)f(x,o)=o$ / $(\forall x)f(o,x)=o$

5.
1. $(\forall x)(\forall y)(Bxy \equiv Lyx)$
2. $(\forall x)Bf(x)x$ / $(\forall x)Lxf(x)$

6.
1. $(\forall x)(\forall y)(\exists z)Sf(x)yz$
2. $(\forall x)(\forall y)(\forall z)[Sxyz \supset {\sim}(Cxyz \vee Mzyx)]$ / $(\exists x)(\exists y)(\exists z){\sim}Mzg(y)f(g(x))$

There will be no derivations in second-order logic on the test.