

Solutions to Practice Problems for Test #6

I. Translations

1.  $Bam \cdot (\forall x)(Bxm \supset x=a)$   
 $(\exists x)(Nx \cdot Pex)$   
 $\sim(\exists x)(Nx \cdot Pax)$  /  $\sim Bem$
2.  $(\forall x)(\forall y)(Rxy \supset \sim x=y)$  /  $(\forall x)\sim Rxx$
3.  $(\forall x)(\forall y)(\forall z)(x=y \vee y=z \vee x=z)$   
 $(\exists x)(\sim x=c \cdot Jx)$  /  $(\exists x)(\exists y)[\sim x=y \cdot (\forall z)(z=x \vee z=y)]$
4.  $(\exists x)[Bxc \cdot (\forall y)(Byc \supset y=x) \cdot Jx]$  /  $(\exists x)(Bxc)$
5.  $\sim Jm \cdot \sim Jb \cdot (\forall x)[(\sim x=m \cdot \sim x=b) \supset Jx]$   
 $\sim Ja$  /  $a=m \vee a=b$
6.  $(\exists x)\{Rx \cdot (\forall y)[(Ry \cdot y \neq x) \supset Sxy] \cdot Mxa \cdot Mxb\}$
7.  $(\exists x)\{Px \cdot (\forall y)[(Py \cdot y \neq x) \supset Rxy] \cdot (\forall y)(Bye \supset Sxy)\}$
8.  $(\forall x)[(Px \cdot Tx \cdot x \neq n) \supset Ax]$
9.  $(\exists x)(\exists y)(\exists z)\{Px \cdot Mxp \cdot Mxn \cdot Py \cdot Myp \cdot Myn \cdot Pz \cdot Mzp \cdot Mzn \cdot x \neq y \cdot x \neq z \cdot y \neq z \cdot$   
 $(\forall w)[(Pw \cdot Mwp \cdot Mwn) \supset (w=x \vee w=y \vee w=z)]\}$
10.  $Af(b) \cdot \sim Ag(c)$
11.  $\sim(\exists x)Bxf(c,e)$
12.  $Tc \supset \{(Jg(c) \cdot Rg(c)) \cdot [Af(c) \cdot (\exists x)(Nx \cdot Pf(c)x)]\}$
13.  $(\exists X)(Xn \cdot \sim Xp)$
14.  $(\forall x)(\forall y)[(Rx \cdot Ry) \supset (\exists X)(Xx \cdot Xy)]$
15.  $(\exists X)\{(\forall x)(\forall y)(\forall z)[(Xxy \cdot Xyz) \supset Xxz] \cdot (\forall x)(\forall y)(Xxy \equiv \sim Xyx)\}$
16.  $(\forall x)x=x$  /  $(\exists X)(\forall x)Xxx$

II. Derivations

Note: These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

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|----|--|--------------|
| 1. | 1. $(\exists x)(Nx \cdot Px \cdot Ix)$                         |              |
|    | 2. $Nc \cdot Pjc \cdot (\forall x)[(Nx \cdot Px) \supset x=c]$ | / Ic         |
|    | 3. $Na \cdot Pja \cdot Ia$                                     | 1, EI        |
|    | 4. $(\forall x)[(Nx \cdot Px) \supset x=c]$                    | 2, Com, Simp |
|    | 5. $(Na \cdot Pja) \supset a=c$                                | 4, UI        |
|    | 6. $Na \cdot Pja$  | 3, Simp      |
|    | 7. $a=c$   | 5, 6, MP     |
|    | 8. $Ia$  | 3, Com, Simp |
|    | 9. $Ic$  | 8, 7, IDi    |

QED

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|----|--|--|
| 2. | 1. $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot y \neq x) \supset Dxy]\}$ | / $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot \sim Ty) \supset Dxy]\}$ |
|    | 2. $Ma \cdot Ta \cdot (\forall y)[(My \cdot \sim y=a) \supset Day]$                | 1, EI  |
|    | 3. $My \cdot \sim Ty$  | ACP  |
|    | 4. $(\forall y)[(My \cdot \sim y=a) \supset Day]$                                  | 2, Com, Simp   |
|    | 5. $(My \cdot \sim y=a) \supset Day$   | 4, UI  |
|    | 6. $y=a$   | AIP  |
|    | 7. $Ta$  | 2, Simp  |
|    | 8. $\sim Ty$   | 3, Com, Simp   |
|    | 9. $Ty$  | 7, 6, IDi  |
|    | 10. $Ty \cdot \sim Ty$   | 9, 8, Conj   |
|    | 11. $\sim y=a$   | 6-10, IP   |
|    | 12. $My$   | 3, Simp  |
|    | 13. $My \cdot \sim y=a$  | 12, 11, Conj   |
|    | 14. $Day$  | 5, 13, MP  |
|    | 15. $(My \cdot \sim Ty) \supset Day$   | 3-14, CP   |
|    | 16. $(\forall y)[(My \cdot \sim Ty) \supset Day]$                                  | 15, UG   |
|    | 17. $Ma \cdot Ta$  | 2, Simp  |
|    | 18. $Ma \cdot Ta \cdot (\forall y)[(My \cdot \sim Ty) \supset Day]$                | 17, 16, Conj   |
|    | 19. $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot \sim Ty) \supset Dxy]\}$ | 18, EG   |

QED

3. 1.  $(\forall x)(\forall y)(\forall z)[(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (x=y \vee y=z \vee x=z)]$   
 2.  $(\exists x)(\exists y)(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Rx \cdot Ry \cdot x \neq y)$   
 3.  $(\forall x)(Rx \supset \sim Cx)$  /  $(Sa \cdot Ca) \supset \sim La$   
 4.  $Sa \cdot Ca$  ACP  
     5.  $La$  AIP  
     6.  $(\exists y)(Sb \cdot Lb \cdot Sy \cdot Ly \cdot Rb \cdot Ry \cdot b \neq y)$  3, EI  
     7.  $Sb \cdot Lb \cdot Sc \cdot Lc \cdot Rb \cdot Rc \cdot b \neq c$  6, EI  
     8.  $Sb \cdot Lb \cdot Sc \cdot Lc$  7, Simp  
     9.  $Sa$  4, Simp  
     10.  $Sa \cdot La$  9, 5, Conj  
     11.  $Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc$  10, 8, Conj  
     12.  $(\forall y)(\forall z)[(Sa \cdot La \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (a=y \vee y=z \vee a=z)]$  1, UI  
     13.  $(\forall z)[(Sa \cdot La \cdot Sb \cdot Lb \cdot Sz \cdot Lz) \supset (a=b \vee b=z \vee a=z)]$  12, UI  
     14.  $(Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc) \supset (a=b \vee b=c \vee a=c)$  13, UI  
     15.  $a=b \vee b=c \vee a=c$  14, 11, MP  
     16.  $\sim b=c$  7, Simp  
     17.  $a=b \vee a=c$  17, 16, Com, DS  
     18.  $Ra \supset \sim Ca$  3, UI  
     19.  $Ca$  4, Com, Simp  
     20.  $\sim Ra$  18, 19, DN, MT  
     21.  $Rb$  7, Simp  
         22.  $a=b$  AIP  
         23.  $\sim Rb$  20, 22, IDi  
         24.  $Rb \cdot \sim Rb$  21, 24, Conj  
     25.  $\sim a=b$  22-24, IP  
     26.  $a=c$  17, 25, DS  
     27.  $Rc$  7, Simp  
     28.  $Rc \supset \sim Cc$  3, UI  
     29.  $\sim Cc$  28, 27, MP  
     30.  $Cc$  19, 26, IDi  
     31.  $Cc \cdot \sim Cc$  30, 29, Conj  
     33.  $\sim La$  5-31, IP  
 34.  $(Sa \cdot Ca) \supset \sim La$  4-33, CP

QED

4. 1.  $(\forall x)(\forall y)f(x,y)=f(y,x)$   
 2.  $(\forall x)f(x,o)=o$  /  $(\forall x)f(o,x)=o$   
 3.  $f(x,o)=o$  2, UI  
 4.  $(\forall y)f(o,y)=f(y,o)$  1, UI  
 5.  $f(o,x)=f(x,o)$  4, UI  
 6.  $f(o,x)=o$  5, 3, IDi  
 7.  $(\forall x)f(o,x)=o$  6, UG

QED

- 5.
- |   |                       |
|---|-----------------------|
| 1. $(\forall x)(\forall y)(Bxy \equiv Lyx)$               |                       |
| 2. $(\forall x)Bf(x)x$                                    | $/ (\forall x)Lxf(x)$ |
| 3. $(\forall y)(Bf(x)y \equiv Lyf(x))$                    | 1, UI                 |
| 4. $Bf(x)x \equiv Lxf(x)$                                 | 3, UI                 |
| 5. $Bf(x)x$   | 2, UI                 |
| 6. $[Bf(x)x \supset Lxf(x)] \cdot [Lxf(x) \equiv Bf(x)x]$ | 4, Equiv              |
| 7. $Bf(x)x \supset Lxf(x)$                                | 6, Simp               |
| 8. $Lxf(x)$   | 7, 5, MP              |
| 9. $(\forall x)Lxf(x)$                                    | 8, UG                 |

QED

- 6.
- |  |   |
|--|---|
| 1. $(\forall x)(\forall y)(\exists z)Sf(x)yz$                                    |   |
| 2. $(\forall x)(\forall y)(\forall z)[Sxyz \supset \sim(Cxyz \vee Mzyx)]$        | $/ (\exists x)(\exists y)(\exists z)\sim Mzg(y)f(g(x))$ |
| 3. $(\forall y)(\exists z)Sf(g(x))yz$  | 1, UI   |
| 4. $(\exists z)Sf(g(x))g(y)z$  | 3, UI   |
| 5. $Sf(g(x))g(y)a$   | 4, EI   |
| 6. $(\forall y)(\forall z)[Sf(g(x))yz \supset \sim(Cf(g(x))yz \vee Mzyf(g(x)))]$ | 2, UI   |
| 7. $(\forall z)[Sf(g(x))g(y)z \supset \sim(Cf(g(x))g(y)z \vee Mzg(y)f(g(x)))]$   | 6, UI   |
| 8. $Sf(g(x))g(y)a \supset \sim(Cf(g(x))g(y)a \vee Mag(y)f(g(x)))$                | 7, UI   |
| 9. $\sim(Cf(g(x))g(y)a \vee Mag(y)f(g(x)))$                                      | 8, 5, MP  |
| 10. $\sim Cf(g(x))g(y)a \cdot \sim Mag(y)f(g(x))$                                | 9, DM   |
| 11. $\sim Mag(y)f(g(x))$   | 10, Com, Simp   |
| 12. $(\exists z)\sim Mzg(y)f(g(x))$  | 11, EG  |
| 13. $(\exists y)(\exists z)\sim Mzg(y)f(g(x))$                                   | 12, EG  |
| 14. $(\exists x)(\exists y)(\exists z)\sim Mzg(y)f(g(x))$                        | 13, EG  |

QED