

Practice Problems for Test #5

I. Invalidity in **M**. Demonstrate the invalidity of each of the following arguments. Provide a counterexample.

1. 1. $(\exists x)(Ax \cdot \sim Bx)$
 2. $(\forall x)(Bx \supset Cx)$ / $(\exists x)(Ax \cdot Cx)$

2. 1. $(\forall x)(Fx \supset Gx)$
 2. $(\exists x)Fx$ / $(\forall x)(\sim Gx \supset \sim Ex)$

3. 1. $(\forall x)[(Px \cdot Qx) \supset Rx]$
 2. $(\exists x)(Qx \cdot \sim Rx)$
 3. $(\exists x)(Px \cdot \sim Rx)$ / $(\exists x)(\sim Px \cdot \sim Qx)$

4. 1. $(\forall x)(Px \supset Qx) \supset (\exists x)(Px \cdot Rx)$
 2. $(\exists x)(Px \cdot Qx)$ / $(\exists x)Rx$

5. 1. $(\forall x)[(Ax \vee Bx) \supset Cx]$
 2. $(\exists x)(Bx \cdot \sim Ax)$
 3. $(\exists x)(Ax \cdot \sim Bx)$ / $(\forall x)Cx$

II. Translation in **F**. Use the following legend to translate the sentences below into **F**.

n: Nietzsche
p: Plato

Ax: x is an altruist
Bx: x is boneheaded
Px: x is a philosopher
Tx: x is thoughtful

Bxy: x is a brother of y
Mxy: x mocks y
Pxy: x produces y
Rxy: x is richer than y
Wxy: x is wiser than y

1. All altruists are philosophers.
2. All thoughtful altruists are philosophers.
3. All thoughtful altruists are wiser than Nietzsche
4. All thoughtful altruists are wiser than some philosopher.
5. All thoughtful altruists are wiser than some boneheaded philosopher.
6. No boneheaded altruists are richer than some thoughtful philosopher.
7. Some thoughtful philosophers are not richer than all boneheaded philosophers.
8. Nietzsche mocks all altruists.
9. Nietzsche mocks everything that Plato produces.
10. Nietzsche mocks everything wiser than him.
11. Nietzsche mocks a thing if it does not mock itself.
12. If one thing is wiser than a second, then the second is not wiser than the first.
13. If all altruist philosophers are richer than some thoughtful philosopher, then something thoughtful is wiser than all altruists.

III. Derivations in F.

1.
 1. $(\forall x)(\exists y)Axy \supset (\forall x)(\exists y)Bxy$
 2. $(\exists x)(\forall y)\sim Bxy$/ $(\exists x)(\forall y)\sim Axy$

2.
 1. $(\forall x)(Ax \supset Bx)$/ $(\forall x)[(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)]$

3.
 1. $\sim(\exists x)(Axa \cdot \sim Bxb)$
 2. $\sim(\exists x)(Dxd \cdot Dbx)$
 3. $(\forall x)(Bex \supset Dxd)$/ $\sim(Aea \cdot Dgd)$

4.
 1. $(\forall x)\{(Px \cdot Qx) \supset (\exists y)[(Py \cdot Qy) \cdot \sim Rxy]\}$
 2. $(\forall x)[Px \equiv (Qx \cdot Tx)]$
 3. $(\forall x)[Px \supset (\forall y)(Sy \supset Ryx)]$/ $(\forall x)(Px \supset \sim Sx)$

5.
 1. $(\forall x)(\forall y)(Axy \equiv Ayx)$
 2. $(\forall x)(\forall y)(\forall z)[(Axy \cdot Ayz) \supset Axz]$
 3. $(\exists x)(\exists y)Axy$/ $(\exists x)Axx$

6.
 1. $(\forall x)[(\exists y)Fxy \supset (\forall z)(Gz \supset Fxz)]$
 2. Fab
 3. Gc/ Fac

7.
 1. $(\forall x)(\forall y)[Nxy \equiv (Px \cdot Py)]$
 2. $(\forall x)(Ox \supset Px)$
 3. $(\exists x)(\exists y)[(Ox \cdot Oy) \cdot Myx]$/ $(\exists x)(\exists y)(Myx \cdot Nxy)$

8.
 1. $(\forall x)\{Px \supset (\exists y)[Qy \cdot (Rxy \cdot Ryx)]\}$
 2. $(\exists x)\{Px \cdot (\forall y)[Sy \supset (\sim Rxy \vee \sim Ryx)]\}$/ $(\exists x)(Qx \cdot \sim Sx)$