

Solutions to Practice Derivations for Test #4

These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

1. 1. $Ab \vee Bc$
2. $(\forall x)\sim Bx$ / $(\exists x)Ax$
3. $\sim Bc$ 2, UI
4. $Bc \vee Ab$ 1, Com
5. Ab 4, 3, DS
6. $(\exists x)Ax$ 5, EG

QED

2. 1. $(\forall x)(Rx \supset Ox)$
2. $(\exists x)\sim Ox$
3. $(\forall x)(\sim Rx \supset Px)$ / $(\exists x)Px$
4. $\sim Oa$ 2, EI
5. $Ra \supset Oa$ 1, UI
6. $\sim Ra$ 5, 4, MT
7. $\sim Ra \supset Pa$ 3, UI
8. Pa 7, 6, MP
9. $(\exists x)Px$ 8, EG

QED

3. 1. $(\forall x)(Fx \supset Gx)$
2. $(\forall y)(Gy \supset Hy)$ / $(\forall z)(\sim Hz \supset \sim Fz)$
3. $Fx \supset Gx$ 1, UI
4. $Gx \supset Hx$ 2, UI
5. $Fx \supset Hx$ 3, 4, HS
6. $\sim Hx \supset \sim Fx$ 5, Cont
7. $(\forall z)(\sim Hz \supset \sim Fz)$ 6, UG

QED

4. 1. $(\exists x)(Ax \cdot Bx) \supset (\forall x) Dx$
2. $\sim Da$ / $(\forall x)(Ax \supset \sim Bx)$
3. $(\exists x)\sim Dx$ 2, EG
4. $\sim(\forall x)Dx$ 3, QE
5. $\sim(\exists x)(Ax \cdot Bx)$ 1, 4, MT
6. $(\forall x)\sim(Ax \cdot Bx)$ 5, QE
7. $(\forall x)(\sim Ax \vee \sim Bx)$ 6, DM
8. $(\forall x)(Ax \supset \sim Bx)$ 7, Impl

QED

5. 1. $(\forall y)[Ay \supset (\sim By \supset Dy)]$
 2. $\sim Ba$ / $Aa \supset Da$
 | 3. Aa ACP
 | 4. $Aa \supset (\sim Ba \supset Da)$ 1, UI
 | 5. $\sim Ba \supset Da$ 4, 3, MP
 | 6. Da 5, 2, MP
 7. $Aa \supset Da$ 3-6, CP

QED

6. 1. $(\forall x)(Qx \supset \sim Px)$ / $(\exists x)Px \supset \sim(\forall x)Qx$
 | 2. $(\exists x)Px$ ACP
 | 3. Pa 2, EI
 | 4. $Qa \supset \sim Pa$ 1, UI
 | 5. $\sim Qa$ 4, 3, DN, MT
 | 6. $(\exists x)\sim Qx$ 5, EG
 | 7. $\sim(\forall x)Qx$ 6, QE
 8. $(\exists x)Px \supset \sim(\forall x)Qx$ 2-7, CP

QED

7. 1. $(\forall x)[Ax \supset (Bx \cdot Dx)]$
 2. $(\forall x)[(Ax \cdot Dx) \supset Ex]$
 3. $(\forall x)(Ex \supset \sim Dx)$ / $\sim Aa$
 | 4. Aa AIP
 | 5. $Aa \supset (Ba \cdot Da)$ 1, UI
 | 6. $Ba \cdot Da$ 5, 4, MP
 | 7. Da 6, Com, Simp
 | 8. $Aa \cdot Da$ 4, 7, Conj
 | 9. $(Aa \cdot Da) \supset Ea$ 2, UI
 | 10. Ea 9, 8, MP
 | 11. $Ea \supset \sim Da$ 3, UI
 | 12. $\sim Da$ 11, 10, MP
 | 13. $Da \cdot \sim Da$ 7, 12, Conj
 14. $\sim Aa$ 4-13, IP

QED

8. 1. $(\forall x)(Ax \supset Bx)$
 2. $(\forall x)[Bx \supset (Ax \supset \sim Fx)]$
 3. $(\forall x)[(\sim Cx \cdot Dx) \supset Fx]$ / $(\forall x)[Ax \supset (Cx \vee \sim Dx)]$
 4. Ax ACP
 5. $Ax \supset Bx$ 1, UI
 6. Bx 5, 4, MP
 7. $Bx \supset (Ax \supset \sim Fx)$ 2, UI
 8. $Ax \supset \sim Fx$ 7, 6, MP
 9. $\sim Fx$ 8, 4, MP
 10. $(\sim Cx \cdot Dx) \supset Fx$ 3, UI
 11. $\sim(\sim Cx \cdot Dx)$ 10, 9, MT
 12. $Cx \vee \sim Dx$ 11, DM, DN
 13. $Ax \supset (Cx \vee \sim Dx)$ 4-12, CP
 14. $(\forall x)[Ax \supset (Cx \vee \sim Dx)]$ 13, UG

QED

9. 1. $(\exists x)Gx \supset (\forall x)(Fx \supset Dx)$
 2. $(\exists x)(Gx \cdot \sim Dx)$ / $\sim(\forall x)Fx$
 3. $Ga \cdot \sim Da$ 2, EI
 4. Ga 3, Simp
 5. $(\exists x)Gx$ 4, EG
 6. $(\forall x)(Fx \supset Dx)$ 1, 5, MP
 7. $Fa \supset Da$ 6, UI
 8. $\sim Da$ 3, Com, Simp
 9. $\sim Fa$ 7, 8, MT
 10. $(\exists x)\sim Fx$ 9, EG
 11. $\sim(\forall x)Fx$ 10, QE

QED

10. 1. $(\exists x)Qx \supset (\forall x)(Rx \supset Sx)$
 2. $(\forall x)\sim Qx \supset (\exists x)Sx$
 3. $(\forall x) Rx$ / $(\exists x)Sx$
 4. $\sim(\exists x)Sx$ AIP
 5. $\sim(\forall x)\sim Qx$ 2, 4, MT
 6. $(\exists x)Qx$ 5, QE
 7. $(\forall x)(Rx \supset Sx)$ 1, 6, MP
 8. $Rx \supset Sx$ 7, UI
 9. Rx 3, UI
 10. Sx 8, 9, MP
 11. $(\exists x)Sx$ 10, EG
 12. $(\exists x)Sx \cdot \sim(\exists x)Sx$ 11, 4, Conj
 13. $(\exists x)Sx$ 4-12, IP, DN

QED