

Practice Proofs for Test #3

1.     1.  $G \supset H$   
       2.  $G \supset I$                      /  $G \supset (H \cdot I)$
  
2.     1.  $D \supset E$   
       2.  $E \supset (F \cdot G)$   
       3.  $\sim F \vee \sim G$                  /  $\sim D$
  
3.     1.  $(I \cdot E) \supset \sim F$   
       2.  $F \vee (G \cdot H)$   
       3.  $I \equiv E$                          /  $I \supset G$
  
4.     1.  $(P \vee Q) \vee (\sim R \vee S)$   
       2.  $\sim Q \cdot (\sim S \cdot \sim P)$          /  $\sim R$
  
5.     1.  $(A \cdot B) \supset C$   
       2.  $(F \cdot D) \supset E$   
       3.  $A \cdot D$   
       4.  $B \vee F$                          /  $C \vee E$
  
6.     Show that ' $R \supset \{(R \supset (S \cdot T)) \supset T\}$ ' is a logical truth.
  
7.     Show that ' $A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$ ' is a logical truth
  
8.     1.  $J \equiv K$   
       2.  $\sim K \equiv \sim L$   
       3.  $L \supset \sim M$   
       4.  $\sim L \supset M$                      /  $J \equiv \sim M$
  
9.     1.  $\sim F \supset (G \cdot H)$   
       2.  $F \supset (I \vee J)$   
       3.  $\sim[G \cdot (K \cdot \sim J)]$   
       4.  $K \equiv (L \cdot \sim J)$              /  $K \supset I$
  
10.    1.  $A \supset B$   
       2.  $B \supset D$   
       3.  $D \supset A$   
       4.  $A \supset \sim D$                      /  $\sim A \cdot \sim D$
  
11.    1.  $M$                              /  $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$

12.    1.  $(A \vee B) \supset \sim(F \cdot D)$   
       2.  $\sim(A \cdot \sim D)$   
       3.  $\sim F \supset \sim(C \cdot D)$   
       4.  $C \vee A$                     /  $A \equiv \sim C$
13.    Show that ' $(P \cdot Q) \supset [(P \vee R) \cdot (Q \vee R)]$ ' is a logical truth.
14.    Show that ' $(A \supset B) \vee (\sim A \supset C)$ ' is a logical truth.
15.    1.  $T \supset W$   
       2.  $T \supset \sim Y$   
       3.  $X \supset Y$   
       4.  $X \supset \sim W$   
       5.  $T \vee X$   
       6.  $(W \cdot \sim Y) \supset Z$   
       7.  $Y \supset (W \vee A)$             /  $Z \vee A$
16.    1.  $(\sim S \vee T) \cdot (\sim S \vee U)$   
       2.  $(T \vee S) \vee U$                     /  $T \vee (U \vee X)$

Sample Solutions to Practice Proofs for Test #3

1.     1.  $G \supset H$   
        2.  $G \supset I$                      /  $G \supset (H \cdot I)$   
            3.  $G$                      ACP  
            4.  $H$                      1, 3, MP  
            5.  $I$                      2, 3, MP  
            6.  $H \cdot I$                 4, 5, Conj  
        7.  $G \supset (H \cdot I)$            3-6, CP

QED

2.     1.  $D \supset E$   
        2.  $E \supset (F \cdot G)$   
        3.  $\sim F \vee \sim G$                /  $\sim D$   
            4.  $D$                      AIP  
            5.  $E$                      1, 4, MP  
            6.  $F \cdot G$                 2, 5, MP  
            7.  $\sim(F \cdot G)$             3, DM  
            8.  $(F \cdot G) \cdot \sim(F \cdot G)$  6, 7, Conj  
        9.  $\sim D$                      4-8, IP

QED

3.     1.  $(I \cdot E) \supset \sim F$   
        2.  $F \vee (G \cdot H)$   
        3.  $I \equiv E$                    /  $I \supset G$   
            4.  $I$                      ACP  
            5.  $E$                      3, 4, BMP  
            6.  $I \cdot E$                 4, 5, Conj  
            7.  $\sim F$                  1, 6, MP  
            8.  $G \cdot H$                2, 7, DS  
            9.  $G$                      8, Simp  
        10.  $I \supset G$                  5-9, CP

4.     1.  $(P \vee Q) \vee (\sim R \vee S)$   
        2.  $\sim Q \cdot (\sim S \cdot \sim P)$        /  $\sim R$   
        3.  $\sim Q$                      2, Simp  
        4.  $(Q \vee P) \vee (\sim R \vee S)$    1, Com  
        5.  $Q \vee [P \vee (\sim R \vee S)]$    4, Assoc  
        6.  $P \vee (\sim R \vee S)$        5, 3, DS  
        7.  $\sim S \cdot \sim P$              2, Com, Simp  
        8.  $\sim P$                      7, Com, Simp  
        9.  $\sim R \vee S$              6, 8, DS  
        10.  $\sim S$                     7, Simp  
        11.  $\sim R$                    9, 10, Com, DS

QED

- 5.
- |                            |               |
|----------------------------|---------------|
| 1. $(A \cdot B) \supset C$ |               |
| 2. $(F \cdot D) \supset E$ |               |
| 3. $A \cdot D$             |               |
| 4. $B \vee F$              | / $C \vee E$  |
| 5. $\sim(C \vee E)$        | AIP           |
| 6. $\sim C \cdot \sim E$   | 5, DM         |
| 7. $\sim C$                | 6, Simp       |
| 8. $\sim(A \cdot B)$       | 1, 7, MT      |
| 9. $\sim A \vee \sim B$    | 8, DM         |
| 10. $A$                    | 3, Simp       |
| 11. $\sim B$               | 9, 10, DN, DS |
| 12. $F$                    | 4, 11, DS     |
| 13. $D$                    | 3, Com, Simp  |
| 14. $F \cdot D$            | 12, 13, Conj  |
| 15. $E$                    | 2, 14, MP     |
| 16. $\sim E$               | 6, Com, Simp  |
| 17. $E \cdot \sim E$       | 15, 16, Conj  |
| 18. $C \vee E$             | 5-17, IP, DN  |

QED

- 6.
- |  |  |          |
|--|--|----------|
| 1. $R$   |  | ACP      |
| 2. $R \supset (S \cdot T)$                           |  | ACP      |
| 3. $S \cdot T$                                       |  | 2, 1, MP |
| 4. $T \cdot S$                                       |  | 3, Com   |
| 5. $T$   |  | 4, Simp  |
| 6. $[R \supset (S \cdot T)] \supset T$               |  | 2-5, CP  |
| 7. $R \supset \{[R \supset (S \cdot T)] \supset T\}$ |  | 1-6, CP  |

QED

- 7.
- |   |  |              |
|---|--|--------------|
| 1. $\sim\{A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]\}$   |  | AIP          |
| 2. $\sim A \cdot \sim[(\sim A \vee B) \cdot (\sim A \vee C)]$ |  | 1, DM        |
| 3. $\sim A$   |  | 2, Simp      |
| 4. $\sim[(\sim A \vee B) \cdot (\sim A \vee C)]$              |  | 2, Com, Simp |
| 5. $\sim[\sim A \vee (B \cdot C)]$                            |  | 4, Dist      |
| 6. $A \cdot \sim(B \cdot C)$                                  |  | 5, DM, DN    |
| 7. $A$  |  | 6, Simp      |
| 8. $A \cdot \sim A$   |  | 7, 3, Conj   |
| 9. $A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$           |  | 1-8, IP, DN  |

QED

- 8.
- |  |                     |
|--|---------------------|
| 1. $J \equiv K$                                  |                     |
| 2. $\sim K \equiv \sim L$                        |                     |
| 3. $L \supset \sim M$                            |                     |
| 4. $\sim L \supset M$                            | / $J \equiv \sim M$ |
| 5. $K \equiv L$                                  | 2, BCont            |
| 6. $J \equiv L$                                  | 1, 3, BHS           |
| 7. $\sim M \supset L$                            | 4, Cont, DN         |
| 8. $(L \supset \sim M) \cdot (\sim M \supset L)$ | 3, 7, Conj          |
| 9. $L \equiv \sim M$                             | 8, Equiv            |
| 10. $J \equiv \sim M$                            | 6, 9, BHS           |

QED

- 9.
- |     |  |                 |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|-----|--|-----------------|----------|-----|-----|-----------------------|-------------|-----|-------------------|--------|-----|----------|-----------|-----|-------------|-----------|-----|---|----------|-----|------------------|--------------|-----------|
| 1.  | $\sim F \supset (G \cdot H)$   |                 |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
| 2.  | $F \supset (I \vee J)$   |                 |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
| 3.  | $\sim [G \cdot (K \cdot \sim J)]$  |                 |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
| 4.  | $K \equiv (L \cdot \sim J)$  | / $K \supset I$ |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | 5. K   | ACP             |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | 6. $L \cdot \sim J$  | 4, 5, BMP       |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | 7. $\sim J \cdot L$  | 6, Com          |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | 8. $\sim J$  | 7, Simp         |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | 9. $K \cdot \sim J$  | 5, 8, Conj      |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | 10. $\sim G \vee \sim (K \cdot \sim J)$  | 3, DM           |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | 11. $\sim (K \cdot \sim J) \vee \sim G$  | 10, Com         |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | 12. $\sim \sim (K \cdot \sim J)$   | 9, DN           |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | 13. $\sim G$   | 11, 12, DS      |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | <table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 10%; vertical-align: top;">14.</td> <td><math>\sim I</math></td> <td style="vertical-align: top;">AIP</td> </tr> <tr> <td>15.</td> <td><math>\sim I \cdot \sim J</math></td> <td style="vertical-align: top;">14, 8, Conj</td> </tr> <tr> <td>16.</td> <td><math>\sim (I \vee J)</math></td> <td style="vertical-align: top;">15, DM</td> </tr> <tr> <td>17.</td> <td><math>\sim F</math></td> <td style="vertical-align: top;">2, 16, MT</td> </tr> <tr> <td>18.</td> <td><math>G \cdot H</math></td> <td style="vertical-align: top;">1, 17, MP</td> </tr> <tr> <td>19.</td> <td>G</td> <td style="vertical-align: top;">18, Simp</td> </tr> <tr> <td>20.</td> <td><math>G \cdot \sim G</math></td> <td style="vertical-align: top;">19, 13, Conj</td> </tr> </table> | 14.             | $\sim I$ | AIP | 15. | $\sim I \cdot \sim J$ | 14, 8, Conj | 16. | $\sim (I \vee J)$ | 15, DM | 17. | $\sim F$ | 2, 16, MT | 18. | $G \cdot H$ | 1, 17, MP | 19. | G | 18, Simp | 20. | $G \cdot \sim G$ | 19, 13, Conj | 14-20, IP |
| 14. | $\sim I$   | AIP             |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
| 15. | $\sim I \cdot \sim J$  | 14, 8, Conj     |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
| 16. | $\sim (I \vee J)$  | 15, DM          |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
| 17. | $\sim F$   | 2, 16, MT       |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
| 18. | $G \cdot H$  | 1, 17, MP       |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
| 19. | G  | 18, Simp        |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
| 20. | $G \cdot \sim G$   | 19, 13, Conj    |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | 21. I  | 5-21, CP        |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |
|     | 22. $K \supset I$  |                 |          |     |     |                       |             |     |                   |        |     |          |           |     |             |           |     |   |          |     |                  |              |           |

QED

- 10.
- |     |                         |                         |
|-----|-------------------------|-------------------------|
| 1.  | $A \supset B$           |                         |
| 2.  | $B \supset D$           |                         |
| 3.  | $D \supset A$           |                         |
| 4.  | $A \supset \sim D$      | / $\sim A \cdot \sim D$ |
| 5.  | $A \supset D$           | 1, 2, HS                |
| 6.  | $\sim D \supset \sim A$ | 5, Cont                 |
| 7.  | $A \supset \sim A$      | 4, 6, HS                |
| 8.  | $\sim A \vee \sim A$    | 7, Impl                 |
| 9.  | $\sim A$                | 8, Taut                 |
| 10. | $\sim D$                | 3, 9, MT                |
| 11. | $\sim A \cdot \sim D$   | 9, 10, Conj             |

QED

- 11.
- |    |   |   |     |  |  |      |     |  |                |     |  |                |            |  |                                    |            |  |  |            |         |
|----|---|---|-----|--|--|------|-----|--|----------------|-----|--|----------------|------------|--|------------------------------------|------------|--|--|------------|---------|
| 1. | $M$   |   |     |  |  |      |     |  |                |     |  |                |            |  |                                    |            |  |  |            |         |
|    | 2. N  | / $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$ |     |  |  |      |     |  |                |     |  |                |            |  |                                    |            |  |  |            |         |
|    | <table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 10%; vertical-align: top;">3.</td> <td style="width: 80%;"><math>O</math></td> <td style="width: 10%;"></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;">4. P</td> <td style="vertical-align: top;">ACP</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;">5. <math>M \cdot N</math></td> <td style="vertical-align: top;">ACP</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;">6. <math>O \cdot P</math></td> <td style="vertical-align: top;">1, 2, Conj</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;">7. <math>(M \cdot N) \cdot (O \cdot P)</math></td> <td style="vertical-align: top;">3, 4, Conj</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;">8. <math>P \supset [(M \cdot N) \cdot (O \cdot P)]</math></td> <td style="vertical-align: top;">5, 6, Conj</td> </tr> </table> | 3.  | $O$ |  |  | 4. P | ACP |  | 5. $M \cdot N$ | ACP |  | 6. $O \cdot P$ | 1, 2, Conj |  | 7. $(M \cdot N) \cdot (O \cdot P)$ | 3, 4, Conj |  | 8. $P \supset [(M \cdot N) \cdot (O \cdot P)]$ | 5, 6, Conj | 4-7, CP |
| 3. | $O$   |   |     |  |  |      |     |  |                |     |  |                |            |  |                                    |            |  |  |            |         |
|    | 4. P  | ACP   |     |  |  |      |     |  |                |     |  |                |            |  |                                    |            |  |  |            |         |
|    | 5. $M \cdot N$  | ACP   |     |  |  |      |     |  |                |     |  |                |            |  |                                    |            |  |  |            |         |
|    | 6. $O \cdot P$  | 1, 2, Conj  |     |  |  |      |     |  |                |     |  |                |            |  |                                    |            |  |  |            |         |
|    | 7. $(M \cdot N) \cdot (O \cdot P)$  | 3, 4, Conj  |     |  |  |      |     |  |                |     |  |                |            |  |                                    |            |  |  |            |         |
|    | 8. $P \supset [(M \cdot N) \cdot (O \cdot P)]$  | 5, 6, Conj  |     |  |  |      |     |  |                |     |  |                |            |  |                                    |            |  |  |            |         |
|    | 9. $O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}$  | 3-8, CP   |     |  |  |      |     |  |                |     |  |                |            |  |                                    |            |  |  |            |         |
|    | 10. $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$   | 2-9, CP   |     |  |  |      |     |  |                |     |  |                |            |  |                                    |            |  |  |            |         |

QED

12.	1. $(A \vee B) \supset \sim(F \cdot D)$	
	2. $\sim(A \cdot \sim D)$	
	3. $\sim F \supset \sim(C \cdot D)$	
	4. $C \vee A$	$/ A \equiv \sim C$
	5. A	ACP
	6. $A \vee B$	5, Add
	7. $\sim(F \cdot D)$	6, 1, 6, MP
	8. $\sim F \vee \sim D$	7, DM
	9. $\sim A \vee \sim \sim D$	2, DM
	10. $\sim \sim D$	9, 5, DN, DM
	11. $\sim F$	8, 10, Com, DS
	12. $\sim(C \cdot D)$	3, 11, MP
	13. $\sim C \vee \sim D$	12, DM
	14. $\sim C$	13, 10, Com, DS
	15. $A \supset \sim C$	5-14, CP
	16. $\sim C \supset A$	4, DN, Impl
	17. $(A \supset \sim C) \cdot (\sim C \supset A)$	15, 16, Conj
	18. $A \equiv \sim C$	17, Equiv

QED

13.	1. $P \cdot Q$	ACP
	2. P	1, Simp
	3. $P \vee R$	2, Add
	4. Q	1, Com, Simp
	5. $Q \vee R$	4, Add
	6. $(P \vee R) \cdot (Q \vee R)$	3, 5, Conj
	7. $(P \cdot Q) \supset [(P \vee R) \cdot (Q \vee R)]$	1-6, IP

QED

14.	1. $\sim[(A \supset B) \vee (\sim A \supset C)]$	AIP
	2. $\sim(A \supset B) \cdot \sim(\sim A \supset C)$	1, DM
	3. $\sim(A \supset B)$	2, Simp
	4. $\sim(\sim A \vee B)$	3, Impl
	5. $A \cdot \sim B$	4, DM, DN
	6. $\sim(\sim A \supset C)$	2, Com, Simp
	7. $\sim(A \vee C)$	6, Impl, DN
	8. $\sim A \cdot \sim C$	8, DM
	9. A	5, Simp
	10. $\sim A$	8, Simp
	11. $A \cdot \sim A$	9, 10, Conj
	12. $(A \supset B) \vee (\sim A \supset C)$	1-11, IP, DN

QED

- 15.
- |     |  |                 |
|-----|--|-----------------|
| 1.  | $T \supset W$                                      |                 |
| 2.  | $T \supset \sim Y$                                 |                 |
| 3.  | $X \supset Y$                                      |                 |
| 4.  | $X \supset \sim W$                                 |                 |
| 5.  | $T \vee X$   |                 |
| 6.  | $(W \bullet \sim Y) \supset Z$                     |                 |
| 7.  | $Y \supset (W \vee A)$                             | / $Z \vee A$    |
| 8.  | $W \vee Y$   | 1, 3, 5, CD     |
| 9.  | $\sim Y \vee \sim W$                               | 2, 4, 5, CD     |
|     | 10. $\sim(Z \vee A)$                               | AIP             |
|     | 11. $\sim Z \bullet \sim A$                        | 10, DM          |
|     | 12. $\sim Z$                                       | 11, Simp        |
|     | 13. $\sim(W \bullet \sim Y)$                       | 6, 12, MT       |
|     | 14. $\sim W \vee Y$                                | 13, DM, DN      |
|     | 15. $\sim W \vee \sim Y$                           | 9, Com          |
|     | 16. $(\sim W \vee Y) \bullet (\sim W \vee \sim Y)$ | 14, 15, Conj    |
|     | 17. $\sim W \vee (Y \bullet \sim Y)$               | 16, Dist        |
|     | 18. $Y \bullet \sim Y$                             | AIP             |
|     | 19. $\sim(Y \bullet \sim Y)$                       | 18, IP          |
|     | 20. $\sim W$                                       | 17, 19, Com, DS |
|     | 21. $Y$  | 8, 20, DS       |
|     | 22. $W \vee A$                                     | 7, 21, MP       |
|     | 23. $A$  | 22, 20, DS      |
|     | 24. $\sim A$                                       | 11, Com, Simp   |
|     | 25. $A \bullet \sim A$                             | 23, 24, Conj    |
| 26. | $Z \vee A$   | 10-25, IP, DN   |

QED

- 16.
- |     |   |                       |
|-----|---|-----------------------|
| 1.  | $(\sim S \vee T) \bullet (\sim S \vee U)$ |                       |
| 2.  | $(T \vee S) \vee U$                       | / $T \vee (U \vee X)$ |
|     | 3. $\sim(T \vee U)$                       | AIP                   |
|     | 4. $\sim T \bullet \sim U$                | 3, DM                 |
|     | 5. $\sim T$                               | 4, Simp               |
|     | 6. $T \vee (S \vee U)$                    | 2, Assoc              |
|     | 7. $S \vee U$                             | 6, 5, DS              |
|     | 8. $\sim U$                               | 4, Com, Simp          |
|     | 9. $S$                                    | 7, 8, Com, DS         |
|     | 10. $\sim S \vee T$                       | 1, Simp               |
|     | 11. $T$                                   | 10, 9, DN, DS         |
|     | 12. $T \bullet \sim T$                    | 11, 5, Conj           |
| 13. | $T \vee U$                                | 3-12, IP, DN          |
| 14. | $(T \vee U) \vee X$                       | 13, Add               |
| 15. | $T \vee (U \vee X)$                       | 14, Assoc             |

QED