

Practice with Proofs (§2.1 - §2.6)

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| 1. | $\begin{array}{l} 1. A \supset B \\ 2. \sim B \\ 3. \sim A \supset \sim C \end{array} \quad / \sim C$ | 11. | $\begin{array}{l} 1. X \vee Z \\ 2. Z \supset \sim Y \\ 3. X \supset Y \end{array} \quad / \sim X \equiv Z$ |
| 2. | $\begin{array}{l} 1. (A \vee B) \supset \sim C \\ 2. D \supset \sim E \\ 3. C \vee D \\ 4. E \end{array} \quad / \sim B$ | 12. | $\begin{array}{l} 1. A \supset B \\ 2. B \supset \sim A \\ 3. (A \vee D) \vee E \\ 4. (D \vee E) \supset F \end{array} \quad / F$ |
| 3. | $\begin{array}{l} 1. \sim(A \cdot B) \\ 2. \sim(\sim B \vee C) \end{array} \quad / \sim A \vee \sim D$ | 13. | $\begin{array}{l} 1. G \supset (H \supset I) \\ 2. I \supset (J \cdot K) \end{array} \quad / G \supset (H \supset J)$ |
| 4. | $\begin{array}{l} 1. A \supset B \\ 2. (E \vee D) \supset \sim B \end{array} \quad / A \supset (\sim E \cdot \sim D)$ | 14. | $\begin{array}{l} 1. (\sim L \vee M) \cdot (L \vee N) \\ 2. \sim O \supset \sim N \end{array} \quad / M \vee O$ |
| 5. | $\begin{array}{l} 1. \sim(G \cdot H) \supset I \\ 2. \sim(I \vee J) \\ 3. \sim J \supset F \end{array} \quad / (F \cdot G) \cdot H$ | 15. | $\begin{array}{l} 1. (P \supset Q) \cdot (R \supset S) \\ \quad \quad \quad / (P \cdot R) \supset (Q \cdot S) \end{array}$ |
| 6. | $\begin{array}{l} 1. P \vee O \\ 2. Q \supset \sim O \end{array} \quad / \sim Q \vee P$ | 16. | $\begin{array}{l} 1. T \supset W \\ 2. X \supset Y \\ 3. T \vee X \\ 4. T \supset \sim Y \\ 5. X \supset \sim W \\ 6. (W \cdot \sim Y) \supset Z \\ 7. Y \supset (W \vee A) \end{array} \quad / Z \vee A$ |
| 7. | $\begin{array}{l} 1. T \equiv V \\ 2. \sim T \vee \sim V \end{array} \quad / \sim T$ | 17. | $\begin{array}{l} 1. (B \cdot \sim D) \equiv (E \vee F) \\ 2. \sim(E \equiv \sim G) \\ 3. (G \vee \sim F) \cdot (G \vee B) \\ 4. \sim G \end{array} \quad / D$ |
| 8. | $\begin{array}{l} 1. K \supset \sim M \\ 2. (L \cdot N) \supset \sim M \\ 3. (K \vee L) \cdot (K \vee N) \end{array} \quad / \sim M$ | 18. | $\begin{array}{l} 1. P \equiv (Q \vee \sim R) \\ 2. \sim S \equiv \sim P \\ 3. (Q \supset S) \supset T \end{array} \quad / T$ |
| 9. | $\begin{array}{l} 1. X \vee Y \\ 2. Z \supset W \\ 3. \sim(Y \cdot W) \end{array} \quad / \sim(\sim X \cdot Z)$ | | |
| 10. | $\begin{array}{l} 1. \sim P \equiv Q \\ 2. \sim Q \equiv \sim S \end{array} \quad / \sim S \equiv P$ | | |

Sample Solutions

1. 1. $A \supset B$
 2. $\sim B$
 3. $\sim A \supset \sim C$ / $\sim C$
 4. $\sim A$ 1, 2, MT
 5. $\sim C$ 3, 4, MP

QED

2. 1. $(A \vee B) \supset \sim C$
 2. $D \supset \sim E$
 3. $C \vee D$
 4. E / $\sim B$
 5. $\sim \sim E$ 4, DN
 6. $\sim D$ 2, 5, MT
 7. $D \vee C$ 3, Com
 8. C 7, 6, DS
 9. $\sim \sim C$ 8, DN
 10. $\sim(A \vee B)$ 1, 9, MT
 11. $\sim A \cdot \sim B$ 10, DM
 12. $\sim B \cdot \sim A$ 11, Com
 13. $\sim B$ 12, Simp

QED

3. 1. $\sim(A \cdot B)$
 2. $\sim(\sim B \vee C)$ / $\sim A \vee \sim D$
 3. $\sim \sim B \cdot \sim C$ 2, DM
 4. $\sim \sim B$ 3, Simp
 5. $\sim A \vee \sim B$ 1, DM
 6. $\sim B \vee \sim A$ 5, Com
 7. $\sim A$ 6, 4, DS
 8. $\sim A \vee \sim D$ 7, Add

QED

4. 1. $A \supset B$
 2. $(E \vee D) \supset \sim B$ / $A \supset (\sim E \cdot \sim D)$
 3. $B \supset \sim(E \vee D)$ 2, Cont, DN
 4. $A \supset \sim(E \vee D)$ 1, 3, HS
 5. $A \supset (\sim E \cdot \sim D)$ 4, DM

QED

5. 1. $\sim(G \cdot H) \supset I$
 2. $\sim(I \vee J)$
 3. $\sim J \supset F$ / $(F \cdot G) \cdot H$
 4. $\sim I \cdot \sim J$ 2, DM
 5. $\sim I$ 4, Simp
 6. $G \cdot H$ 1, 5, MT, DN
 7. $\sim J$ 4, Com, Simp
 8. F 3, 7, MP
 9. $F \cdot (G \cdot H)$ 8, 6, Conj
 10. $(F \cdot G) \cdot H$ 9, Assoc

QED

6. 1. $P \vee O$
 2. $Q \supset \sim O$ / $\sim Q \vee P$
 3. $\sim O \supset P$ 1, Com, DN,
 Impl
 4. $Q \supset P$ 2, 3, HS
 5. $\sim Q \supset P$ 4, Impl

QED

7. 1. $T \equiv V$
 2. $\sim T \vee \sim V$ / $\sim T$
 3. $(T \cdot V) \vee (\sim T \cdot \sim V)$ 1, Equiv
 4. $\sim(T \cdot V)$ 2, DM
 5. $\sim T \cdot \sim V$ 3, 4, DS
 6. $\sim T$ 5, Simp

QED

8. 1. $K \supset \sim M$
 2. $(L \cdot N) \supset \sim M$
 3. $(K \vee L) \cdot (K \vee N)$ / $\sim M$
 4. $K \vee (L \cdot N)$ 3, Dist
 5. $\sim M \vee \sim M$ 1, 2, 4, CD
 6. $\sim M$ 5, Taut

QED

- 9.
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|-------------------------------|--------------------------|
| 1. $X \vee Y$ | |
| 2. $Z \supset W$ | |
| 3. $\sim(Y \cdot W)$ | / $\sim(\sim X \cdot Z)$ |
| 4. $\sim Y \vee \sim W$ | 3, DM |
| 5. $Y \vee X$ | 1, Com |
| 6. $\sim\sim Y \vee X$ | 5, DN |
| 7. $\sim Y \supset X$ | 6, Impl |
| 8. $\sim W \supset \sim Z$ | 5, Cont |
| 9. $X \vee \sim Z$ | 7, 8, 4, CD |
| 10. $\sim\sim(X \vee \sim Z)$ | 9, DN |
| 11. $\sim(\sim X \cdot Z)$ | 10, DM, DN |

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- 10.
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|-------------------------------|---------------------|
| 1. $\sim P \equiv Q$ | |
| 2. $\sim Q \equiv \sim S$ | / $\sim S \equiv P$ |
| 3. $\sim S \equiv \sim Q$ | 2, BCom |
| 4. $Q \equiv \sim P$ | 1, BCom |
| 5. $\sim Q \equiv \sim\sim P$ | 4, BCont |
| 6. $\sim Q \equiv P$ | 5, DN |
| 7. $\sim S \equiv P$ | 3, 6, BHS |

QED

- 11.
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|--|---------------------|
| 1. $X \vee Z$ | |
| 2. $Z \supset \sim Y$ | |
| 3. $X \supset Y$ | / $\sim X \equiv Z$ |
| 4. $\sim Y \supset \sim X$ | 3, Cont |
| 5. $Z \supset \sim X$ | 2, 4, HS |
| 6. $\sim X \supset Z$ | 1, DN, Impl |
| 7. $(\sim X \supset Z) \cdot (Z \supset \sim X)$ | 6, 5, Conj |
| 8. $\sim X \equiv Z$ | 7, Equiv |

QED

- 12.
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|--------------------------------|-------------|
| 1. $A \supset B$ | |
| 2. $B \supset \sim A$ | |
| 3. $(A \vee D) \vee E$ | |
| 4. $(D \vee E) \supset F$ | / F |
| 5. $A \supset \sim A$ | 1, 2, HS |
| 6. $\sim A \vee \sim A$ | 5, Impl |
| 7. $\sim A$ | 6, Taut |
| 8. $A \vee (D \vee E)$ | 3, Assoc |
| 9. $\sim A \supset (D \vee E)$ | 8, Dn, Impl |
| 10. $D \vee E$ | 9, 7, MP |
| 11. F | 4, 10, MP |

QED

13. 1. $G \supset (H \supset I)$
 2. $I \supset (J \cdot K)$ / $G \supset (H \supset J)$
 3. $(G \cdot H) \supset I$ 1, Exp
 4. $\sim I \vee (J \cdot K)$ 2, Impl
 5. $(\sim I \vee J) \cdot (\sim I \vee K)$ 4, Dist
 6. $\sim I \vee J$ 5, Simp
 7. $I \supset J$ 6, Impl
 8. $(G \cdot H) \supset J$ 3, 7, HS
 9. $G \supset (H \supset J)$ 8, Impl

QED

14. 1. $(\sim L \vee M) \cdot (L \vee N)$
 2. $\sim O \supset \sim N$ / $M \vee O$
 3. $\sim L \vee M$ 1, Simp
 4. $L \supset M$ 3, Impl
 5. $\sim M \supset \sim L$ 4, Cont
 6. $L \vee N$ 1, Com, Simp
 7. $\sim L \supset N$ 6, DN, Impl
 8. $\sim M \supset N$ 5, 7, HS
 9. $N \supset O$ 2, Cont
 10. $\sim M \supset O$ 8, 9, HS
 11. $M \vee O$ 10, Impl, DN

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15. 1. $(P \supset Q) \cdot (R \supset S)$ / $(P \cdot R) \supset (Q \cdot S)$
 2. $P \supset Q$ 1, Simp
 3. $\sim P \vee Q$ 2, Impl
 4. $(\sim P \vee Q) \vee \sim R$ 3, Add
 5. $\sim P \vee (Q \vee \sim R)$ 4, Assoc
 6. $R \supset S$ 1, Com, Simp
 7. $\sim R \vee S$ 6, Impl
 8. $\sim P \vee (\sim R \vee S)$ 7, Add, Com
 9. $[\sim P \vee (Q \vee \sim R)] \cdot [\sim P \vee (\sim R \vee S)]$ 5, 8, Conj
 10. $\sim P \vee [(Q \vee \sim R) \cdot (\sim R \vee S)]$ 9, Dist
 11. $\sim P \vee [(\sim R \vee Q) \cdot (\sim R \vee S)]$ 10, Com
 12. $\sim P \vee [\sim R \vee (Q \cdot S)]$ 11, Dist
 13. $P \supset [R \supset (Q \cdot S)]$ 12, Impl, Impl
 14. $(P \cdot R) \supset (Q \cdot S)$ 13, Exp

QED

- 16.
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|--|---------------|
| 1. $T \supset W$ | |
| 2. $X \supset Y$ | |
| 3. $T \vee X$ | |
| 4. $T \supset \sim Y$ | |
| 5. $X \supset \sim W$ | |
| 6. $(W \bullet \sim Y) \supset Z$ | |
| 7. $Y \supset (W \vee A)$ | $/ Z \vee A$ |
| 8. $W \vee Y$ | 1, 2, 3, CD |
| 9. $\sim Y \vee \sim W$ | 4, 5, 3, CD |
| 10. $\sim \sim W \vee Y$ | 8, DN |
| 11. $\sim W \supset Y$ | 10, Impl |
| 12. $Y \supset \sim W$ | 9, Impl |
| 13. $(\sim W \supset Y) \bullet (Y \supset \sim W)$ | 11, 12, Conj |
| 14. $\sim W \equiv Y$ | 13, Equiv |
| 15. $(\sim W \bullet Y) \vee (\sim \sim W \bullet \sim Y)$ | 14, Equiv |
| 16. $(\sim W \bullet Y) \vee (W \bullet \sim Y)$ | 15, DN |
| 17. $(W \bullet \sim Y) \vee (\sim W \bullet Y)$ | 16, Com |
| 18. $\sim Y \vee (W \vee A)$ | 7, Impl |
| 19. $(\sim Y \vee W) \vee A$ | 18, Assoc |
| 20. $\sim \sim (\sim Y \vee W) \vee A$ | 19, DN |
| 21. $\sim (\sim Y \vee W) \supset A$ | 20, Impl |
| 22. $(\sim \sim Y \bullet \sim W) \supset A$ | 21, DM |
| 23. $(Y \bullet \sim W) \supset A$ | 22, DN |
| 24. $(\sim W \bullet Y) \supset A$ | 23, Com |
| 25. $Z \vee A$ | 6, 24, 17, CD |

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- 17.
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| 1. $(B \bullet \sim D) \equiv (E \vee F)$ | |
| 2. $\sim(E \equiv \sim G)$ | |
| 3. $(G \vee \sim F) \bullet (G \vee B)$ | |
| 4. $\sim G$ | $/ D$ |
| 5. $G \vee (\sim F \bullet B)$ | 3, Dist |
| 6. $\sim F \bullet B$ | 5, 4, DS |
| 7. $\sim E \equiv \sim G$ | 2, BDM |
| 8. $\sim G \equiv \sim E$ | 7, BCom |
| 9. $\sim E$ | 8, 4, BMP |
| 10. $\sim F$ | 6, Simp |
| 11. $\sim E \bullet \sim F$ | 9, 10, Conj |
| 12. $\sim(E \vee F)$ | 11, DM |
| 13. $(E \vee F) \equiv (B \bullet \sim D)$ | 1, BCom |
| 14. $\sim(B \bullet \sim D)$ | 13, BMT |
| 15. $\sim B \vee \sim \sim D$ | 14, DM |
| 16. $B \bullet \sim F$ | 6, Com |
| 17. B | 16, Simp |
| 18. $\sim \sim B$ | 17, DN |
| 19. $\sim \sim D$ | 15, 18, DS |
| 20. D | 19, DN |

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| 18. | 1. $P \equiv (Q \vee \sim R)$ | |
| | 2. $\sim S \equiv \sim P$ | |
| | 3. $(Q \supset S) \supset T$ | / T |
| | 4. $S \equiv P$ | 2, BCont |
| | 5. $S \equiv (Q \vee \sim R)$ | 4, 1, BHS |
| | 6. $[S \supset (Q \vee \sim R)] \cdot [(Q \vee \sim R) \supset S]$ | 5, Equiv |
| | 7. $[(Q \vee \sim R) \supset S] \cdot [S \supset (Q \vee \sim R)]$ | 6, Com |
| | 8. $(Q \vee \sim R) \supset S$ | 7, Simp |
| | 9. $\sim(Q \vee \sim R) \vee S$ | 8, Impl |
| | 10. $(\sim Q \cdot \sim \sim R) \vee S$ | 9, DM |
| | 11. $S \vee (\sim Q \cdot \sim \sim R)$ | 10, Com |
| | 12. $(S \vee \sim Q) \cdot (S \vee \sim \sim R)$ | 11, Dist |
| | 13. $S \vee \sim Q$ | 12, Simp |
| | 14. $\sim Q \vee S$ | 13, Com |
| | 15. $Q \supset S$ | 14, Impl |
| | 16. T | 3, 15, MP |

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