

# Philosophy 240: Symbolic Logic

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Class #42 - Second-Order Quantification



# The Final

- 4 arguments (4 sentences each) to translate
  - 52%
- 4 different arguments to prove
  - 48%

# Philosophy and Higher-Order Logic

# Second- and Higher-Order Logics

- All logics of higher order than first are called higher-order logics.
- They facilitate many inferences and allow us to represent some claims that we can represent only awkwardly in first-order languages.
  - Shared properties
  - Leibniz's law and excluded middle
  - Analogies
  - Mathematical induction
  - Replace the identity predicate
- Stewart Shapiro argues that second-order logic should be taken as the canonical language for mathematics.
  - Categoricity and non-standard models
- Frege's *Begriffsschrift* is second-order.
- Omega-order logic is type theory

# Against Second-Order Logic

- Some philosophers argue that higher-order logics are not really logic.
- Quine:
  - ▶ First-order logic with identity is canonical.
  - ▶ Second-order logic is, “Set theory in sheep’s clothing” (*Philosophy of Logic*, p 66).
- Set theory is mathematics, not logic.
  - ▶ To Frege’s disappointment



# Interpretations and Existence

- When we interpret first-order logic, we specify a domain for the variables to range over.
- To be is to be the value of a variable.
- For our most general reasoning, we take an unrestricted domain.
  - ▶ the universe
  - ▶ everything there is
- There are blue hats.
  - ▶  $(\exists x)(Bx \cdot Hx)$
- 'Some properties are shared by two people'.
  - ▶  $(\exists X)(\exists x)(\exists y)(Px \cdot Py \cdot x \neq y \cdot Xx \cdot Xy)$
  - ▶ There must exist two people, to satisfy the 'Px' and 'Py'.
  - ▶ There must exist a property, to satisfy the 'Xx' and the 'Xy'.
  - ▶ In other words, the domain for interpreting the second-order quantifier will have properties in it.
  - ▶ So, there are properties.

# The Reification of Properties

- By quantifying over properties, we take properties as objects.
- What are properties?
  - Platonic forms?
  - Eternal ideas?
- Commitments to properties, in addition to objects which have those properties, is metaphysically contentious.
- There are blue things.
- Is blueness also a thing?



# Deflating Second-Order Logic

- We can take properties to be *sets* of objects which have those properties.
- On this extensional interpretation of predicate variables, 'blueness' refers to the collection of all blue things.
- Thus, second-order logic commits us *at least* to the existence of sets.
- We might want to include sets in our ontology.
  - We might think there are mathematical objects.
- We need not include them under the guise of second-order logic.
- We can take them to be values of first-order variables.
- We can count them as among the objects in the universe, in the domain of quantification, rather than sneaking them in through the interpretations of second-order variables.
- We have to look more closely at general principles of theory choice.



# In Favor of Second-Order Logic

- Expressional strength
- Deriving the properties of identity from the second-order axioms, rather than introducing a special predicate with special inferential properties
- Quine favors using schematic predicate letters in lieu of predicate variables.
  - The law of the excluded middle:  $P \vee \sim P$
- I find this approach disingenuous.
- Schematic letters are really meta-linguistic variables.
- Quine is admitting is that we can not formulate second-order claims in our canonical language.
- We must, instead, ascend to a meta-language, using meta-linguistic variables.

# Review Session

Wednesday, 12/17, noon, here

Final Exam

Thursday, December 18

9am - noon