Philosophy 240
Symbolic Logic

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Class #4
Philosophy Friday #1: Conditionals

Logic Tutoring at the QSR

- Spencer, by arrangment through QSR
- Emily Moore
  Tuesday 4-6pm
  Sunday 7-9pm

Natural-Language Conditionals

A. Indicative conditionals: If the Mets lost, then the Cubs won.
B. Conditional questions: If I like logic, what class should I take next?
C. Conditional commands: If you want to pass this class, do the homework.
D. Conditional prescriptions: If you want a good life, you ought to act virtuously.
E. Cookie Conditionals: There are cookies in the jar if you want them.
F. Subjunctive conditionals: If Rod were offered the bribe, he would take it.

- A is a straightforward logical conditional.
  The material interpretation seems pretty good.
- B, C, and D are not propositions; as they stand, they lack truth values.
- We can parse them truth-functionally.
  B'. If you like logic, then you take linear algebra next.
  C'. If you want to pass the class, you do the homework.
  D'. If you want a good life, you act virtuously.
  – These might take a material interpretation well too.
- E isn't really a conditional statement; it's a fraud.
  Like 'Bob and Ray are brothers'; the surface grammar is misleading.
- F is a worry and hints at some deep problems.

The Material Conditional

α	$\supset$	β
1	1	1
1	0	0
0	1	1
0	1	0

1. Why the material interpretation is weird
2. Why we can't do much about the weirdness
   Should we give up truth-functionality?
3. Connections to philosophy of science

The Paradoxes of Material Implication

- Subjunctive conditionals are not the only problems with the material interpretation.
- One problem is that the so-called paradoxes of material implication are logical truths of **PL**.
- To understand that problem, we have to know a little bit about logical truths.

Logical Truths

- *Logical truths* are privileged sentences of a logical system.
  - The theorems
- In an axiomatic system we choose a small set of privileged sentences that we call *axioms*.
  - Euclidean geometry
  - Newtonian mechanics
- The *theorems* of a formal system are provable from its axioms.
- PL has no axioms, but lots of theorems anyway.
  - Logical truths
- We identify any formal system with its theorems.

  "Demarcate the totality of logical truths, in whatever terms, and you have in those terms specified the logic" (Quine, *Philosophy of Logic*, p 80.)
- In classical propositional logic, the theorems are called *tautologies*.
- Tautologies are true under any interpretation of their component variables.

  $P \supset P$

  $[(P \supset (Q \supset R)] \supset [(P \supset Q) \supset (P \supset R)]$

The Paradoxes

- The paradoxes of material implication are that statements of the following forms are tautologies:

  $\alpha \supset (\beta \supset \alpha)$

  $\sim\alpha \supset (\alpha \supset \beta)$

  $(\alpha \supset \beta) \vee (\beta \supset \alpha)$

- Such statements are unobvious or counter-intuitive.
- Awkward consequences

  'If Martians have infra-red vision, then Obama is president' is true.

  'If Bush is still president, then Venusians have a colony on the dark side of Mercury' is true.

  Either 'Neptunians love to wassail' entails 'Saturnians love to foxtrot' or 'Saturnians love to foxtrot' entails 'Neptunians love to wassail'.

  One of the following is true:

  – 'It is raining' entails 'Chickens are robot spies from Pluto'.

  – 'Chickens are robot spies from Pluto' entails 'It is raining'.

Benefits of the Material Interpretation

- Simplicity, elegance
- Truth-Functional Compositionality: the truth value of any complex sentence is completely dependent on the truth value of its component parts.
- Imagine you were using logic to program a computer, or a robot.
  - We do not want the program to stall on an empty truth value.
  - We want it to have rules for how to proceed in any case.
- We should see if there is a better truth-functional alternative.

Nicod's Criterion

A constraint on the first two lines of the material interpretation

- Many scientific laws are conditional in form.
- Nicod's criterion captures how such scientific laws are confirmed.
  - Evidence confirms a law if it satisfies both the antecedent and consequent.
  - Evidence disconfirms a law if it satisfies the antecedent, and fails to satisfy the consequent.
- All swans are white.
  - If something is a swan, then it is white.
  - When we find a white swan, which satisfies the antecedent and the consequent, it confirms the claim.
  - If we were to find a black swan, which satisfies the antecedent but falsifies the consequent, then it would disconfirm the claim.
- Coulomb's Law: $F = k\ |q_1 q_2| / r^2$.
  - If two particles have a certain amount of charge and a certain distance between them, then they have a certain, calculable force between them.
  - If we were to find two particles which did not have the force between them that the formula on the right side of Coulomb's Law says should hold, and we could not find over-riding laws to explain this discrepancy, we would seek a revision of Coulomb's Law.

Nicod's Criterion and the Second Two Rows

- According to Nicod's criterion, instances which do not satisfy the antecedent are irrelevant to confirmation or disconfirmation.
- A white dog and a black dog and a blue pen have no effect on our confidence in the claim that all swans are white.
- Call a conditional in which the antecedent is false a *counterfactual conditional*.
- Nicod's criterion says nothing about counterfactual conditionals.
- We are considering alternatives to the material interpretation of the conditional.
- The point of mentioning Nicod's criterion was to say that we should leave the first two lines of the truth table alone.

The Immutability of the Last Two Rows of the Truth Table for the Material Conditional

Option A

α	⊃	β
1	1	1
1	0	0
0	1	1
0	0	0

Option B

α	⊃	β
1	1	1
1	0	0
0	0	1
0	1	0

Option C

α	⊃	β
1	1	1
1	0	0
0	0	1
0	0	0

- Option A gives the conditional the same truth-values as the consequent.
- Option B gives the conditional the same truth-values as a biconditional.
- Option C gives the conditional the same truth-values as the conjunction.
- Thus, the truth table for the material conditional is the only one possible with those first two lines that doesn’t merely replicate a truth table we already have.

Counterfactual Dependent Conditionals

Option A

α	⊃	β
1	1	1
1	0	0
0	1	1
0	0	0

Option B

α	⊃	β
1	1	1
1	0	0
0	0	1
0	1	0

Option C

α	⊃	β
1	1	1
1	0	0
0	0	1
0	0	0

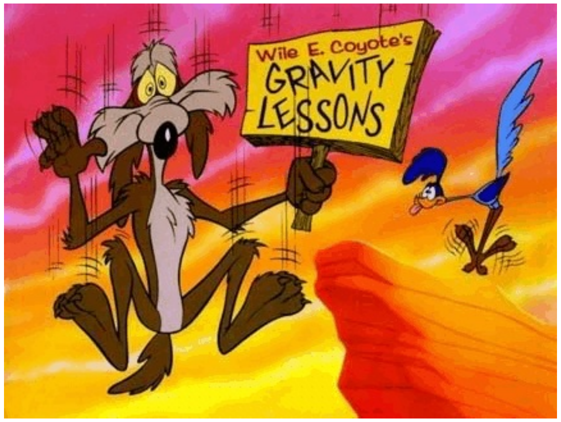
- Moreover, none of these three options helps us.
- 'If I were to jump out of the window right now, I would fall to the ground.'

  Option A says that this sentence is falsified when I don't jump out the window and I don't fall to the ground.

  Options B and C say that it is falsified when I don't jump out of the window and I do fall to the ground.

  Neither case seems to falsify the sentence, as it is intended.

Falsification



- The only time that sentence is falsified, as on Nicod's criterion, is in the second line of the truth table.
- We must stick with the original truth table, if we want the conditional to be truth-functional.
- If the problem were just the oddities of the paradoxes of the material conditional, we might bite the bullet.
- But, the problem is deeper.

Subjunctive and Counterfactual Conditionals

- ‘If Rod were offered the bribe, he would take it.’
  - If Rod takes the bribe, then it is true; if he refuses the bribe, then it is false.
  - If Rod is never offered the bribe, then it remains true.
  - So far, so good: the material interpretation is satisfactory for at least one counterfactual conditional.
- But contrast:
  - S: If I were to jump out of the window right now, I would fall to the ground.
  - S': If I were to jump out of the window right now, I would flutter to the moon.
- I am not now jumping out of the window.
- S is true and S' is false.
- Some conditionals with false antecedents are false!
- Goodman’s Example
  - If that piece of butter had been heated to 150°F, it would not have melted.

Non-Truth-Functional Operators

The Two-State Solution

- The logical aspects of the natural-language conditional may be expressed by the truth-functional ‘⊃’ .
- Other aspects of the natural-language conditional might not be truth-functional.
- We could introduce a new operator, strict implication, ⇒.
- Statements of the form ‘α ⊃ β’ could continue to be truth-functional
- Statements of the form ‘α ⇒ β’ would be non-truth-functional.
- If I were to jump out of the window right now, I would flutter to the moon.
  - We have already seen that the ⊃ does not work for this claim.
  - We could regiment it as ‘J ⇒ F’.
  - ‘J ⇒ F’ would lack a standard truth-value in the third and fourth rows.

Modal Interpretations

- C.I. Lewis defined ‘α ⇒ β’ as ‘□(α ⊃ β)’.
- The ‘□’ is a modal operator.
  - Used for formal theories of knowledge, moral properties, tenses, or knowledge.’
  - Philosophy Friday #5: October 31
- For Lewis’s suggestion, strict implication, we use an alethic interpretation.
  - ‘□’ means ‘necessarily’.
- For strict implication, ‘α ⇒ β’ is true iff it is necessarily the case that the consequent is true whenever the antecedent is.

Modals and Scientific Laws

- A scientific law is naturally taken as describing a necessary, causal relation.
- When we say that event A causes event B, we imply that A necessitates B, that B could not fail to occur, given A.
- To say that lighting the stove causes the water to boil is to say that, given the stability of background conditions, the water has no choice but to boil.
- S: If I were to jump out of the window right now, I would fall to the ground.
  - True, since it's a law of physics
- S': If I were to jump out of the window right now, I would flutter to the moon.
  - False, since it's contrary to the laws of physics.
- Thus, we might distinguish the two senses of the conditional by saying that material implication represents logical connections and strict implication attempts to regiment causal connections.

Causation and Strict Implication

- Causal laws are often conditional, indicating dispositional properties.
- ‘If this salt had been placed in water, it would have dissolved.’
  - dispositional property of salt
- Other dispositional properties, like irritability, flammability, and flexibility, refer to properties interesting to scientists.
- Psychological properties, are often explained as dispositions to behave.
  - Believing that it is cold outside
- ‘This marble counter is soluble in water.’
  - If we never place the counter in water, then it comes out true on the material interpretation.
- To be flammable is just, by definition, to have certain counterfactual properties.
  - Pajamas are flammable just in case they would burn if subjected to certain conditions.
- The laws of science depend essentially on precisely the counterfactual conditionals that the logic of the material conditional gets wrong.

The Two-State Solution

α	⊃	β
1	1	1
1	0	0
0	1	1
0	1	0

Material Implication

If you paint my house, I'll give you $3K.
If Rod were offered the bribe, he'd take it.
If I like chocolate, then Mary likes kale.

α	→	β
1	1	1
1	0	0
0		1
0		0

Strict Implication

If this piece of steel were heated to 150°F, then it would melt.
If this piece of butter were heated to 150°F, then it would melt.

Strict Implication

Not a Logical Relation

- The truth value of the bottom two rows in strict implication depends on the content of the claims.
  - Not strictly logical
  - Depends on causal laws
- "The principle that permits inference of 'That match lights' from 'That match is scratched. That match is dry enough. Enough oxygen is present. Etc.' is not a law of logic but what we call a natural or physical or causal law" (Goodman, 8-9).
- So, how do we know when we have a law?

Laws and Accidental Generalizations

- The problem of giving an analysis of the logic of conditionals is intimately related to the problem of distinguishing laws from accidental generalizations.
- Compare:
  1. There are no balls of uranium one mile in diameter.
  2. There are no balls of gold one mile in diameter.
- The explanation of 1 refers to scientific laws about critical mass.
- The explanation of 2 is merely accidental.
- In order to know that difference, you must know the laws which govern the universe.
- The problem of knowing the laws of nature is thus inextricably linked to the problem of understanding the logic of the natural-language conditional.
- Our ability to know which events or properties are necessary and which are contingent is severely limited.

Summing Up

- The material interpretation of the conditional is the best available truth function.
  - We may have just to accept the counter-intuitive paradoxes of material implication.
  - Again, independent conditionals are just weird anyway.
- Some conditionals, especially dependent counterfactual ones (e.g. scientific laws) are not best understood materially (i.e. truth-functionally).
  - The proper analysis of counterfactual conditionals is not a logical matter.
- We have gone far from just understanding the logic of our language.
- We are now engaged in a pursuit of the most fundamental features of scientific discourse.
- *For our technical work, parse all conditionals truth-functionally.*