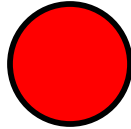# Philosophy 240:
Symbolic Logic

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Class #36 - Color Incompatibility
The Limits of Logic

Two Inferences

A. This spot is red
Therefore, this spot is not blue

B. This spot is red.
Therefore, this spot has a color.

- The premises and conclusions are all different propositions, so propositional logic will not facilitate the inferences.

- Neither will predicate logic, as the argument stand:
    Argument A: Ra / ~Ba
    Argument B: Ra / Ca

A Suggestion

Argument A: 1. Ra / ~Ba

Argument B: 1. Ra / Ca

- What if we add premises?
    To Argument A: $(\forall x)(Rx \supset \sim Bx)$
    To Argument B: $(\forall x)(Rx \supset Cx)$

- Then the conclusions will follow.

- Argument A
    1. Ra
    2. $(\forall x)(Rx \supset \sim Bx)$ / ~Ba
    3. Ra $\supset$ ~Ba
    4. ~Ba

- Argument B
    1. Ra
    2. $(\forall x)(Rx \supset Cx)$ / Ca
    3. Ra $\supset$ Ca
    4. Ca

Adding A Premise to Facilitate the Inference

1. Ra
2. (∀x)(Rx ⊃ ~Bx) / ~Ba
3. Ra ⊃ ~Ba
4. ~Ba
QED

- On our intended interpretation, the conclusion does follows from the premises, as a matter of logic.

- But, a valid inference will be valid on any interpretation.

- Consider this interpretation:
  - Rx = {Presidents of the United States}
  - Bx = {People Born in Kenya}
  - a = Barack Obama

- It does not follow *as a matter of logic* from the fact that Obama is president that he was not born in Kenya.
  - The added premise is an empirical claim.
 - But, we want the original inferences to follow as matters of logic.

Meaning Postulates

- We can call the added premise a meaning postulate.

- I can stipulate anything I want.
  - ($\forall$x)(Rx $\supset$ ~Bx)

- All Beethoven symphonies are boring.
  - ($\forall$x)[(Sx • Wbx) $\supset$ Bx]
  - It follows from this premise as a matter of logic that the *Eroica* (Beethoven's third symphony) is boring.
  - But, it is not a logical truth that the *Eroica* is boring.

- We want the fact that this spot is not blue to follow as a matter of logic from the fact that it is red.
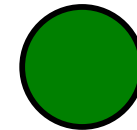
Katz on Color Incompatibility

"The Problem in Twentieth-Century Philosophy"
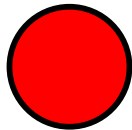
1. The spot is red and blue.
2. The spot is red.
3. The spot is not blue.
4. The spot has a color.
5. The spot is green

Propositions 1-5 seem to be simple propositions.

But there are logical relations among them.

- ▸ 1 is a contradiction.
- ▸ 2 entails 3 and 4.
- ▸ 2 and 5 are incompatible.

The Generalized Problem

- "It is a general problem about the extralogical vocabulary of the language and about all the semantic properties and relations of the language" (Katz, 548).

- Consider: 'Bachelors are single'.
  - ‣ $(\forall x)(Bx \supset Sx)$
  - ‣ not a logical truth

- There is a logical relation among the terms 'bachelor' and 'single'.

- The logic we have been studying does not show those relations.
  - ‣ On some interpretations, this sentence will be true.
  - ‣ On some interpretations, this sentence will be false.

- Indeed, the propositions, being simple, do not seem to have structural or logical relations at all.

- That, at least, is the view which many philosophers in the twentieth century have held.

Wittgenstein's *Tractatus*

- Quine and others take first-order logic to be canonical, the language in which we express our ontological commitments.

- The original philosophical application of Frege's logic to broader philosophical purposes came from Ludwig Wittgenstein, in his 1919 *Tractatus Logico-Philosophicus*.

- According to the *Tractatus,* the world is a collection of independent atomic facts combined according to logical principles.

- If we could get clear about the correct logic, Wittgenstein argued, then we could have a complete, accurate picture of the world in our best, most austere language.

- The *Tractatus* was highly influential in Europe between the two wars, as the foundation of logical empiricism.
  - The Vienna and Berlin Circles
  - A.J. Ayer's *Language, Truth, and Logic*

The Picture Theory

- The *Tractatus* presents an atomistic picture theory of meaning on which language mirrors the world.

- Language and the world are both governed by logical rules.

- The world is a collection of independent states of affairs.
  - §1.2. The world divides into facts.
  - §2.06. From the existence or non-existence of one state of affairs, it is impossible to infer the existence or non-existence of another.

- Language consists of atomic statements of those facts, connected into more complex statements by logical principles.

- Language mirrors the world by providing a logical structure which is isomorphic to the structure of the world.
  - §2.16. If a fact is to be a picture, it must have something in common with what it depicts.
  - §2.17. What a picture must have in common with reality, in order to be able to depict it - correctly or incorrectly - in the way it does, is its pictorial form.

- Since language and logic have the same form as the world, we can know about the fundamental structure of reality by examining the fundamental structures of language and logic.
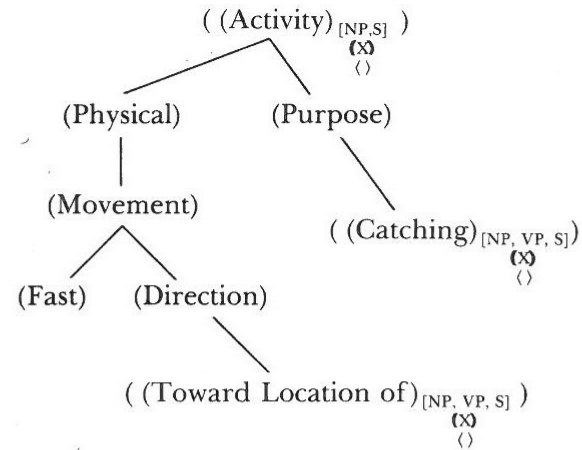
Frege's Microscope

I believe I can make the relationship of my *Begriffsschrift* to ordinary language clearest if I compare it to that of the microscope to the eye. The latter, due to the range of its applicability, due to the flexibility with which it is able to adapt to the most diverse circumstances, has a great superiority over the microscope. Considered as an optical instrument, it admittedly reveals many imperfections, which usually remain unnoticed only because of its intimate connection with mental life. But as soon as scientific purposes place great demands on sharpness of resolution, the eye turns out to be inadequate. The microscope, on the other hand, is perfectly suited for such purposes... (Frege, Preface to *Begriffsschrift*)

The Problem

- What are the atoms?
  - ▸ In the world: facts
  - ▸ In language???
    - – They must be independent and simple
    - – Representations of sense data?

- *Tractatus* §6.3751: It is clear that the logical product of two elementary propositions can neither be a tautology nor a contradiction. The statement that a point in the visual field has two different colors at the same time is a contradiction.

- Katz's solution: we need a formal theory that analyzes language more deeply than predicate logic.
  - ▸ Semantic theory
  - ▸ Decompositional sense structure
  - ▸ "A new way out must reject Carnap's assumption that the external, logical structure of extralogical words is the source of analyticity, contradiction, and analytic entailment in connection with sentences like (1)-(6). It must assume instead that such properties and relations derive from the internal, sense structure of extralogical words" (Katz 553).

(19)

$((\text{Activity})_{[NP,S]})$
(X)
( )

(Physical)   (Purpose)

(Movement)

$((\text{Catching})_{[NP, VP, S]})$
(X)
( )

(Fast)   (Direction)

$((\text{Toward Location of})_{[NP, VP, S]})$
(X)
( )

SEMANTIC REPRESENTATION of
'TO CHASE'

fr. KATZ, COGITATIONS, p 85

Atomism and Holism

- Carnap and Katz attempt to save atomism by exploring the logic of semantic entailments.
  - ‣ We can extend our logic so that there is a formal representation of the required entailments.
    - – Carnap: meaning postulates
    - – Katz: decompositional sense structure
- Quine gives up the belief that there are elementary propositions.
  - ‣ Also senses (meanings)
  - ‣ He denies that there are any logical connections among 1-6.
  - ‣ The connections are loose, perhaps causal connections.
  - ‣ All entailments are holistic, involve large swaths of theory.

Quine and Sellars and Holism

- It is obvious that truth in general depends on both language and extralinguistic fact. The statement "Brutus killed Caesar" would be false if the world had been different in certain ways, but it would also be false if the word "killed" happened rather to have the sense of "begat." Hence, the temptation to suppose in general that the truth of a statement is somehow analyzable into a linguistic component and a factual component. Given this supposition, it next seems reasonable that in some statements the factual component should be null; and these are the analytic statements. But, for all its a priori reasonableness, a boundary between analytic and synthetic statements simply has not been drawn (Quine, "Two Dogmas of Empiricism," 70).

- Our statements about the external world face the tribunal of sense experience not individually but only as a corporate body ("Two Dogmas of Empiricism" 41).

- One couldn't have observational knowledge of *any* fact unless one knew many *other* things as well (Sellars, "Does Empirical Knowledge Have a Foundation?" 123).

- One couldn't form the concept of *being green*, and, by parity of reasoning, of the other colors, unless he already had them (Sellars, "Does Empirical Knowledge Have a Foundation?" 120).

Have a Great Brrreak!