Philosophy 240: Symbolic Logic — Hamilton College
Fall 2014 — Russell Marcus

Solutions to Practice Problems for Test #5

These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

I. Derivations

1.

1.	$Ab \vee Bc$	
2.	$(\forall x)\sim Bx$	/ $(\exists x)Ax$
3.	$\sim Bc$	2, UI
4.	Ab	1,3 Com, DS
5.	$(\exists x)Ax$	4, EG

QED

2.

1.	$(\forall x)(Rx \supset Ox)$	
2.	$(\exists x)\sim Ox$	
3.	$(\forall x)(\sim Rx \supset Px)$	/ $(\exists x)Px$
4.	$\sim Oa$	2, EI
5.	$Ra \supset Oa$	1, UI
6.	$\sim Ra$	5, 4, MT
7.	$\sim Ra \supset Pa$	3, UI
8.	Pa	7, 6, MP
9.	$(\exists x)Px$	8, EG

QED

3.

1.	$(\forall x)(Fx \supset Gx)$	
2.	$(\forall y)(Gy \supset Hy)$	/ $(\forall z)(\sim Hz \supset \sim Fz)$
3.	$Fx \supset Gx$	1, UI
4.	$Gx \supset Hx$	2, UI
5.	$Fx \supset Hx$	3, 4, HS
6.	$\sim Hx \supset \sim Fx$	5, Cont
7.	$(\forall z)(\sim Hz \supset \sim Fz)$	6, UG

QED

4.

1.	$(\exists x)(Ax \cdot Bx) \supset (\forall x)\ Dx$	
2.	$\sim Da$	/ $(\forall x)(Ax \supset \sim Bx)$
3.	$(\exists x)\sim Dx$	2, EG
4.	$\sim(\forall x)Dx$	3, QE
5.	$\sim(\exists x)(Ax \cdot Bx)$	1, 4, MT
6.	$(\forall x)\sim(Ax \cdot Bx)$	5, QE
7.	$(\forall x)(\sim Ax \vee \sim Bx)$	6, DM
8.	$(\forall x)(Ax \supset \sim Bx)$	7, Impl

QED

5.

1.	(∀y)[Ay ⊃ (~By ⊃ Dy)]	
2.	~Ba	/ Aa ⊃ Da
	3. Aa	ACP
	4. Aa ⊃ (~Ba ⊃ Da)	1, UI
	5. ~Ba ⊃ Da	4, 3, MP
	6. Da	5, 2, MP
7.	Aa ⊃ Da	3-6, CP

QED

6.

1.	(∀x)(Qx ⊃ ~Px)	/ (∃x)Px ⊃ ~(∀x)Qx
	2. (∃x)Px	ACP
	3. Pa	2, EI
	4. Qa ⊃ ~Pa	1, UI
	5. ~Qa	4, 3, DN, MT
	6. (∃x)~Qx	5, EG
	7. ~(∀x)Qx	6, QE
8.	(∃x)Px ⊃ ~(∀x)Qx	2-7, CP

QED

7.

1.	(∀x)[Ax ⊃ (Bx · Dx)]	
2.	(∀x)[(Ax · Dx) ⊃ Ex]	
3.	(∀x)(Ex ⊃ ~Dx)	/ ~Aa
	4. Aa	AIP
	5. Aa ⊃ (Ba · Da)	1, UI
	6. Ba · Da	5, 4, MP
	7. Da	6, Com, Simp
	8. Aa · Da	4, 7, Conj
	9. (Aa · Da) ⊃ Ea	2, UI
	10. Ea	9, 8, MP
	11. Ea ⊃ ~Da	3, UI
	12. ~Da	11, 10, MP
	13. Da · ~Da	7, 12, Conj
14.	~Aa	4-13, IP

QED

8.

1.	(∀x)(Ax ⊃ Bx)	
2.	(∀x)[Bx ⊃ (Ax ⊃ ~Fx)]	
3.	(∀x)[(~Cx · Dx) ⊃ Fx]	/ (∀x)[Ax ⊃ (Cx ∨ ~Dx)]
| 4.	Ax	ACP
| 5.	Ax ⊃ Bx	1, UI
| 6.	Bx	5, 4, MP
| 7.	Bx ⊃ (Ax ⊃ ~Fx)	2, UI
| 8.	Ax ⊃ ~Fx	7, 6, MP
| 9.	~Fx	8, 4, MP
| 10.	(~Cx · Dx) ⊃ Fx	3, UI
| 11.	~(~Cx · Dx)	10, 9, MT
| 12.	Cx ∨ ~Dx	11, DM, DN
13.	Ax ⊃ (Cx ∨ ~Dx)	4-12, CP
14.	(∀x)[Ax ⊃ (Cx ∨ ~Dx)]	13, UG

QED

9.

1.	(∃x)Gx ⊃ (∀x)(Fx ⊃ Dx)	
2.	(∃x)(Gx · ~Dx)	/ ~(∀x)Fx
3.	Ga · ~Da	2, EI
4.	Ga	3, Simp
5.	(∃x)Gx	4, EG
6.	(∀x)(Fx ⊃ Dx)	1, 5, MP
7.	Fa ⊃ Da	6, UI
8.	~Da	3, Com, Simp
9.	~Fa	7, 8, MT
10.	(∃x)~Fx	9, EG
11.	~(∀x)Fx	10, QE

QED

10.

1.	(∃x)Qx ⊃ (∀x)(Rx ⊃ Sx)	
2.	(∀x)~Qx ⊃ (∃x)Sx	
3.	(∀x) Rx	/ (∃x)Sx
| 4.	~(∃x)Sx	AIP
| 5.	~(∀x)~Qx	2, 4, MT
| 6.	(∃x)Qx	5, QE
| 7.	(∀x)(Rx ⊃ Sx)	1, 6, MP
| 8.	Rx ⊃ Sx	7, UI
| 9.	Rx	3, UI
| 10.	Sx	8, 9, MP
| 11.	(∃x)Sx	10, EG
| 12.	(∃x)Sx · ~(∃x)Sx	11, 4, Conj
13.	(∃x)Sx	4-12, IP, DN

QED

II. Invalidity

1. There is a counterexample in a 1-member universe, where:
Aa: True; Ba: False; Ca: False.

2. There is a counterexample in a 2-member universe, where:
Ea: True; Fa: False; Ga: False;
Eb: True or False; Fb: True; Gb: True

3. There is a counterexample in a 2-member universe, where:
Pa: True; Qa: False; Ra: False
Pb: False; Qb: True; Rb: False

Alternate solutions are possible.