

Solutions to Practice Problems for Test #6

I. Translations

1.  $(\forall x)(Ax \supset Px)$
2.  $(\forall x)[(Tx \cdot Ax) \supset Px]$
3.  $(\forall x)(Ax \supset Mnx)$
4.  $(\forall x)(Ppx \supset Mnx)$
5.  $(\forall x)(Sxn \supset Mnx)$
6.  $(\forall x)(\sim Mxx \supset Mnx)$
7.  $(\forall x)(\forall y)(Sxy \supset \sim Syx)$
8.  $(\forall x)\{(Ax \cdot Px) \supset (\exists y)[(Ty \cdot Py) \cdot Rxy]\} \supset (\exists x)[Tx \cdot (\forall y)(Ay \supset Sxy)]$
9.  $Bam \cdot (\forall x)(Bxm \supset x=a)$   
 $(\exists x)(Nx \cdot Pex)$   
 $\sim(\exists x)(Nx \cdot Pax) \quad / \sim Bem$
10.  $(\forall x)(\forall y)(Rxy \supset \sim x=y) \quad / (\forall x)\sim Rxx$
11.  $(\forall x)(\forall y)(\forall z)(x=y \vee y=z \vee x=z)$   
 $(\exists x)(\sim x=c \cdot Jx) \quad / (\exists x)(\exists y)[\sim x=y \cdot (\forall z)(z=x \vee z=y)]$
12.  $(\exists x)[Bxc \cdot (\forall y)(Byc \supset y=x) \cdot Jx] \quad / (\exists x)(Bxc)$
13.  $\sim Jm \cdot \sim Jb \cdot (\forall x)[(\sim x=m \cdot \sim x=b) \supset Jx]$   
 $\sim Ja \quad / a=m \vee a=b$
14.  $Af(b) \cdot \sim Ag(c)$
15.  $\sim(\exists x)Bxf(c,e)$
16.  $Tc \supset \{(Jg(c) \cdot Rg(c)) \cdot [Af(c) \cdot (\exists x)(Nx \cdot Pf(c)x)]\}$
17.  $(\exists X)(Xn \cdot \sim Xp)$
18.  $(\forall x)(\forall y)[(Rx \cdot Ry) \supset (\exists X)(Xx \cdot Xy)]$
19.  $(\exists X)\{(\forall x)(\forall y)(\forall z)[(Xxy \cdot Xyz) \supset Xxz] \cdot (\forall x)(\forall y)(Xxy \equiv \sim Xyx)\}$
20.  $(\forall x)x=x \quad / (\exists X)(\forall x)Xxx$

II. Derivations

Note: These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

- |    |  |   |
|----|--|---|
| 1. | 1. $(\forall x)(\exists y)(\sim Ax \vee By)$ | $/ (\forall x)Ax \supset (\exists y)By$ |
|    | 2. $(\forall x)Ax$                           | ACP                                     |
|    | 3. $(\exists y)(\sim Ax \vee By)$            | 1, UI                                   |
|    | 4. $\sim Ax \vee Ba$                         | 3, EI                                   |
|    | 5. $Ax$                                      | 2, UI                                   |
|    | 6. $Ba$                                      | 4, 5, DN, DS                            |
|    | 7. $(\exists y)By$                           | 6, EG                                   |
|    | 8. $(\forall x)Ax \supset (\exists y)By$     | 2-7 CP                                  |

QED

- 2.
- |  |                                    |
|--|------------------------------------|
| 1. $(\forall x)(\exists y)Axy \supset (\forall x)(\exists y)Bxy$ |                                    |
| 2. $(\exists x)(\forall y)\sim Bxy$                              | $/ (\exists x)(\forall y)\sim Axy$ |
| 3. $\sim(\forall x)(\exists y)Bxy$                               | 2, QE, QE                          |
| 4. $\sim(\forall x)(\exists y)Axy$                               | 1, 3, MT                           |
| 5. $(\exists x)(\forall y)\sim Axy$                              | 4, QE, QE                          |

QED

- 3.
- |   |   |
|---|---|
| 1. $(\forall x)[(Ax \vee Bx) \supset (Dx \cdot Kx)]$                |   |
| 2. $(\forall x)\{(Dx \vee Lx) \supset [(Dx \cdot Nx) \supset Px]\}$ | $/ (\forall x)[Ax \supset (Nx \supset Px)]$ |
| 3. $Ax$   | ACP   |
| 4. $Ax \vee Bx$   | 3, Add                                      |
| 5. $(Ax \vee Bx) \supset (Dx \cdot Kx)$                             | 1, UI                                       |
| 6. $Dx \cdot Kx$  | 5, 4, MP                                    |
| 7. $Dx$   | 6, Simp                                     |
| 8. $Dx \vee Lx$   | 7, Add                                      |
| 9. $(Dx \vee Lx) \supset [(Dx \cdot Nx) \supset Px]$                | 2, UI                                       |
| 10. $(Dx \cdot Nx) \supset Px$                                      | 9, 8, MP                                    |
| 11. $Nx$  | ACP   |
| 12. $Dx \cdot Nx$   | 7, 11, Conj                                 |
| 13. $Px$  | 10, 12, MP                                  |
| 14. $Nx \supset Px$   | 11-13, CP                                   |
| 15. $Ax \supset (Nx \supset Px)$                                    | 3-14, CP                                    |
| 16. $(\forall x)[Ax \supset (Nx \supset Px)]$                       | 15, UG                                      |

QED

- 4.
- |   |                         |
|---|-------------------------|
| 1. $\sim(\exists x)(Axa \cdot \sim Bxb)$  |                         |
| 2. $\sim(\exists x)(Dxd \cdot Dbx)$       |                         |
| 3. $(\forall x)(Bex \supset Dxd)$         | $/ \sim(Aea \cdot Dgd)$ |
| 4. $Aea \cdot Dgd$                        | AIP                     |
| 5. $(\forall x)\sim(Axa \cdot \sim Bxb)$  | 1, QE                   |
| 6. $(\forall x)(\sim Axa \vee Bxb)$       | 5, DM, DN               |
| 7. $\sim Aea \vee Beb$                    | 6, UI                   |
| 8. $Aea$                                  | 4, Simp                 |
| 9. $Beb$                                  | 7, 8, DN, DS            |
| 10. $(\forall x)\sim(Dxd \cdot Dbx)$      | 2, QE                   |
| 11. $(\forall x)(\sim Dxd \vee \sim Dbx)$ | 10, DM                  |
| 12. $\sim Dgd \vee \sim Dbg$              | 11, UI                  |
| 13. $Dgd$                                 | 4, Com, Simp            |
| 14. $\sim Dbg$                            | 12, 13, DN, DS          |
| 15. $Beb \supset Dbg$                     | 3, UI                   |
| 16. $Dbg$                                 | 15, 9, MP               |
| 17. $Dbg \cdot \sim Dbg$                  | 16, 14, Conj            |
| 18. $\sim(Aea \cdot Dgd)$                 | 4-17, IP                |

QED

- |    |  |  |
|----|--|--|
| 5. | 1. $(\forall x)(Ax \supset Bx)$  | $/ (\forall x)[(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)]$ |
|    | 2. $(\exists y)(Ay \cdot Cxy)$   | ACP  |
|    | 3. $Aa \cdot Cxa$  | 2, EI  |
|    | 4. $Aa$  | 3, Simp  |
|    | 5. $Aa \supset Ba$   | 1, UI  |
|    | 6. $Ba$  | 5, 4, MP   |
|    | 7. $Cxa$   | 3, Com, Simp   |
|    | 8. $Ba \cdot Cxa$  | 6, 7, Conj   |
|    | 9. $(\exists z)(Bz \cdot Cxz)$   | 8, EG  |
|    | 10. $(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)$              | 2-9, CP  |
|    | 11. $(\forall x)[(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)]$ | 10, UG   |

QED

- |    |   |              |
|----|---|--------------|
| 6. | 1. $(\exists x)(Nx \cdot Pjx \cdot Ix)$                         |              |
|    | 2. $Nc \cdot Pjc \cdot (\forall x)[(Nx \cdot Pjx) \supset x=c]$ | $/ Ic$       |
|    | 3. $Na \cdot Pja \cdot Ia$                                      | 1, EI        |
|    | 4. $(\forall x)[(Nx \cdot Pjx) \supset x=c]$                    | 2, Com, Simp |
|    | 5. $(Na \cdot Pja) \supset a=c$                                 | 4, UI        |
|    | 6. $Na \cdot Pja$   | 3, Simp      |
|    | 7. $a=c$  | 5, 6, MP     |
|    | 8. $Ia$   | 3, Com, Simp |
|    | 9. $Ic$   | 8, 7, IDi    |

QED

- |    |  |  |
|----|--|--|
| 7. | 1. $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot y \neq x) \supset Dxy]\}$ | $/ (\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot \sim Ty) \supset Dxy]\}$ |
|    | 2. $Ma \cdot Ta \cdot (\forall y)[(My \cdot \sim y=a) \supset Day]$                | 1, EI  |
|    | 3. $My \cdot \sim Ty$  | ACP  |
|    | 4. $(\forall y)[(My \cdot \sim y=a) \supset Day]$                                  | 2, Com, Simp   |
|    | 5. $(My \cdot \sim y=a) \supset Day$   | 4, UI  |
|    | 6. $y=a$   | AIP  |
|    | 7. $Ta$  | 2, Simp  |
|    | 8. $\sim Ty$   | 3, Com, Simp   |
|    | 9. $Ty$  | 7, 6, IDi  |
|    | 10. $Ty \cdot \sim Ty$   | 9, 8, Conj   |
|    | 11. $\sim y=a$   | 6-10, IP   |
|    | 12. $My$   | 3, Simp  |
|    | 13. $My \cdot \sim y=a$  | 12, 11, Conj   |
|    | 14. $Day$  | 5, 13, MP  |
|    | 15. $(My \cdot \sim Ty) \supset Day$   | 3-14, CP   |
|    | 16. $(\forall y)[(My \cdot \sim Ty) \supset Day]$                                  | 15, UG   |
|    | 17. $Ma \cdot Ta$  | 2, Simp  |
|    | 18. $Ma \cdot Ta \cdot (\forall y)[(My \cdot \sim Ty) \supset Day]$                | 17, 16, Conj   |
|    | 19. $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot \sim Ty) \supset Dxy]\}$ | 18, EG   |

QED

8. 1.  $(\forall x)(\forall y)(\forall z)[(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (x=y \vee y=z \vee x=z)]$   
 2.  $(\exists x)(\exists y)(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Rx \cdot Ry \cdot x \neq y)$   
 3.  $(\forall x)(Rx \supset \sim Cx)$  /  $(Sa \cdot Ca) \supset \sim La$   
 4.  $Sa \cdot Ca$  ACP  
 5.  $La$  AIP  
 6.  $(\exists y)(Sb \cdot Lb \cdot Sy \cdot Ly \cdot Rb \cdot Ry \cdot b \neq y)$  3, EI  
 7.  $Sb \cdot Lb \cdot Sc \cdot Lc \cdot Rb \cdot Rc \cdot b \neq c$  6, EI  
 8.  $Sb \cdot Lb \cdot Sc \cdot Lc$  7, Simp  
 9.  $Sa$  4, Simp  
 10.  $Sa \cdot La$  9, 5, Conj  
 11.  $Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc$  10, 8, Conj  
 12.  $(\forall y)(\forall z)[(Sa \cdot La \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (a=y \vee y=z \vee a=z)]$  1, UI  
 13.  $(\forall z)[(Sa \cdot La \cdot Sb \cdot Lb \cdot Sz \cdot Lz) \supset (a=b \vee b=z \vee a=z)]$  12, UI  
 14.  $(Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc) \supset (a=b \vee b=c \vee a=c)$  13, UI  
 15.  $a=b \vee b=c \vee a=c$  14, 11, MP  
 16.  $\sim b=c$  7, Simp  
 17.  $a=b \vee a=c$  17, 16, Com, DS  
 18.  $Ra \supset \sim Ca$  3, UI  
 19.  $Ca$  4, Com, Simp  
 20.  $\sim Ra$  18, 19, DN, MT  
 21.  $Rb$  7, Simp  
     22.  $a=b$  AIP  
     23.  $\sim Rb$  20, 22, IDi  
     24.  $Rb \cdot \sim Rb$  21, 24, Conj  
 25.  $\sim a=b$  22-24, IP  
 26.  $a=c$  17, 25, DS  
 27.  $Rc$  7, Simp  
 28.  $Rc \supset \sim Cc$  3, UI  
 29.  $\sim Cc$  28, 27, MP  
 30.  $Cc$  19, 26, IDi  
 31.  $Cc \cdot \sim Cc$  30, 29, Conj  
 33.  $\sim La$  5-31, IP  
 34.  $(Sa \cdot Ca) \supset \sim La$  4-33, CP

QED

9. 1.  $(\forall x)(\forall y)f(x,y)=f(y,x)$   
 2.  $(\forall x)f(x,o)=o$  /  $(\forall x)f(o,x)=o$   
 3.  $f(x,o)=o$  2, UI  
 4.  $(\forall y)f(o,y)=f(y,o)$  1, UI  
 5.  $f(o,x)=f(x,o)$  4, UI  
 6.  $f(o,x)=o$  5, 3, IDi  
 7.  $(\forall x)f(o,x)=o$  6, UG

QED

- 10.
- |   |                       |
|---|-----------------------|
| 1. $(\forall x)(\forall y)(Bxy \equiv Lyx)$               |                       |
| 2. $(\forall x)Bf(x)x$                                    | $/ (\forall x)Lxf(x)$ |
| 3. $(\forall y)(Bf(x)y \equiv Lyf(x))$                    | 1, UI                 |
| 4. $Bf(x)x \equiv Lxf(x)$                                 | 3, UI                 |
| 5. $Bf(x)x$   | 2, UI                 |
| 6. $[Bf(x)x \supset Lxf(x)] \cdot [Lxf(x) \equiv Bf(x)x]$ | 4, Equiv              |
| 7. $Bf(x)x \supset Lxf(x)$                                | 6, Simp               |
| 8. $Lxf(x)$   | 7, 5, MP              |
| 9. $(\forall x)Lxf(x)$                                    | 8, UG                 |

QED

- 11.
- |  |   |
|--|---|
| 1. $(\forall x)(\forall y)(\exists z)Sf(x)yz$                                    |   |
| 2. $(\forall x)(\forall y)(\forall z)[Sxyz \supset \sim(Cxyz \vee Mzyx)]$        | $/ (\exists x)(\exists y)(\exists z)\sim Mzg(y)f(g(x))$ |
| 3. $(\forall y)(\exists z)Sf(g(x))yz$  | 1, UI   |
| 4. $(\exists z)Sf(g(x))g(y)z$  | 3, UI   |
| 5. $Sf(g(x))g(y)a$   | 4, EI   |
| 6. $(\forall y)(\forall z)[Sf(g(x))yz \supset \sim(Cf(g(x))yz \vee Mzyf(g(x)))]$ | 2, UI   |
| 7. $(\forall z)[Sf(g(x))g(y)z \supset \sim(Cf(g(x))g(y)z \vee Mzg(y)f(g(x)))]$   | 6, UI   |
| 8. $Sf(g(x))g(y)a \supset \sim(Cf(g(x))g(y)a \vee Mag(y)f(g(x)))$                | 7, UI   |
| 9. $\sim(Cf(g(x))g(y)a \vee Mag(y)f(g(x)))$                                      | 8, 5, MP  |
| 10. $\sim Cf(g(x))g(y)a \cdot \sim Mag(y)f(g(x))$                                | 9, DM   |
| 11. $\sim Mag(y)f(g(x))$   | 10, Com, Simp   |
| 12. $(\exists z)\sim Mzg(y)f(g(x))$  | 11, EG  |
| 13. $(\exists y)(\exists z)\sim Mzg(y)f(g(x))$                                   | 12, EG  |
| 14. $(\exists x)(\exists y)(\exists z)\sim Mzg(y)f(g(x))$                        | 13, EG  |

QED