Philosophy 240: Symbolic Logic — Hamilton College
Fall 2014 — Russell Marcus

Practice Problems for Test #6

I. Translations.

Use the following legend to translate the sentences below. For questions 1-13, do not use any functions.

a: Al
b: Bud
c: Cindy
e: Ed
m: Megha
n: Nietzsche
p: Plato

f(x): the father of x
g(x): the mother of x
f(x,y): the only son of x and y

Ax: x is an altruist
Jx: x is joyful
Nx: x is a novel
Px: x is a philosopher
Rx: x is Russian
Tx: x is thoughtful

Bxy: x is a brother of y
Mxy: x mocks y
Pxy: x produces y
Rxy: x is richer than y
Sxy: x is smarter than y

1. All altruists are philosophers.
2. All thoughtful altruists are philosophers.
3. Nietzsche mocks all altruists.
4. Nietzsche mocks everything that Plato produces.
5. Nietzsche mocks everything smarter than him.
6. Nietzsche mocks a thing if it does not mock itself.
7. If one thing is smarter than a second, then the second is not smarter than the first.
8. If all altruist philosophers are richer than some thoughtful philosopher, then something thoughtful is smarter than all altruists.
9. Megha's only brother is Al. Ed produces novels. Al doesn't. So, Ed isn't Megha's brother.
10. If one thing is richer than a second, then the two aren't identical. So, nothing is richer than itself.
11. There are at most two things. Something other than Cindy is joyful. So, there are exactly two things.
12. The brother of Cindy is joyful. So, Cindy has a brother.
13. Everything is joyful, except Megha and Bud. Al is not joyful. So, Al is either Megha or Bud.
14. Bud's father is an altruist, but Cindy's mother is not.
15. The only son of Cindy and Ed has no brother.
16. If Cindy is thoughtful, then her mother is a joyful Russian and her father is an altruist who produces novels.
17. There are properties that Nietzsche has that Plato lacks.
18. All Russians have something in common.
19. Some transitive relations are asymmetric.
20. Everything is self-identical. Therefore, there is some relation that everything has to itself.

II. Derivations. Derive the conclusions of each of the following arguments.

1. 1. (∀x)(∃y)(~Ax ∨ By) / (∀x)Ax ⊃ (∃y)By

2. 1. (∀x)(∃y)Axy ⊃ (∀x)(∃y)Bxy
   2. (∃x)(∀y)~Bxy / (∃x)(∀y)~Axy

3. 1. (∀x)[(Ax ∨ Bx) ⊃ (Dx · Kx)]
   2. (∀x){(Dx ∨ Lx) ⊃ [(Dx· Nx) ⊃ Px]} / (∀x)[Ax ⊃ (Nx ⊃ Px)]

4. 1. ~(∃x)(Axa · ~Bxb)
   2. ~(∃x)(Dxd · Dbx)
   3. (∀x)(Bex ⊃ Dxg) / ~(Aea · Dgd)

5. 1. (∀x)(Ax ⊃ Bx) / (∀x)[(∃y)(Ay · Cxy) ⊃ (∃z)(Bz · Cxz)]

6. 1. (∃x)(Nx · Pjx · Ix)
   2. Nc · Pjc · (∀x)[(Nx · Pjx) ⊃ x=c] / Ic

7. 1. (∃x){Mx · Tx · (∀y)[(My · y≠x) ⊃ Dxy] / (∃x){Mx · Tx · (∀y)[(My · ~Ty) ⊃Dxy]}

8. 1. (∀x)(∀y)(∀z)[(Sx · Lx · Sy · Ly · Sz · Lz) ⊃ (x=y ∨ y=z ∨ x=z)]
   2. (∃x)(∃y)(Sx · Lx · Sy · Ly · Rx · Ry · x≠y)
   3. (∀x)(Rx ⊃ ~Cx) / (Sa · Ca) ⊃ ~La

9. 1. (∀x)(∀y)f(x,y)=f(y,x)
   2. (∀x)f(x,o)=o / (∀x)f(o,x)=o

10. 1. (∀x)(∀y)(Bxy ≡ Lyx)
    2. (∀x)Bf(x)x / (∀x)Lxf(x)

11. 1. (∀x)(∀y)(∃z)Sf(x)yz
    2. (∀x)(∀y)(∀z)[Sxyz ⊃ ~(Cxyz ∨ Mzyx)] / (∃x)(∃y)(∃z)~Mzg(y)f(g(x))

There will be no derivations in second-order logic on the test.