

Practice Proofs for Test #3

1. 1. $G \supset H$
 2. $G \supset I$ / $G \supset (H \cdot I)$

2. 1. $D \supset E$
 2. $E \supset (F \cdot G)$
 3. $\sim F \vee \sim G$ / $\sim D$

3. 1. $(I \cdot E) \supset \sim F$
 2. $F \vee (G \cdot H)$
 3. $I \equiv E$ / $I \supset G$

4. 1. $(P \vee Q) \vee (\sim R \vee S)$
 2. $\sim Q \cdot (\sim S \cdot \sim P)$ / $\sim R$

5. 1. $(A \cdot B) \supset C$
 2. $(F \cdot D) \supset E$
 3. $A \cdot D$
 4. $B \vee F$ / $C \vee E$

6. Prove that ' $R \supset \{(R \supset (S \cdot T)) \supset T\}$ ' is a logical truth.

7. Prove that ' $A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$ ' is a logical truth

8. 1. $J \equiv K$
 2. $\sim K \equiv \sim L$
 3. $L \supset \sim M$
 4. $\sim L \supset M$ / $J \equiv \sim M$

9. 1. $\sim F \supset (G \cdot H)$
 2. $F \supset (I \vee J)$
 3. $\sim[G \cdot (K \cdot \sim J)]$
 4. $K \equiv (L \cdot \sim J)$ / $K \supset I$

10. 1. $A \supset B$
 2. $B \supset D$
 3. $D \supset A$
 4. $A \supset \sim D$ / $\sim A \cdot \sim D$

11. 1. M / $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$

12. 1. $(A \vee B) \supset \sim(F \cdot D)$
 2. $\sim(A \cdot \sim D)$
 3. $\sim F \supset \sim(C \cdot D)$
 4. $C \vee A$ / $A \equiv \sim C$
13. Prove that ' $(P \cdot Q) \supset [(P \vee R) \cdot (Q \vee R)]$ ' is a logical truth.
14. Prove that ' $(A \supset B) \vee (\sim A \supset C)$ ' is a logical truth.
15. 1. $(T \supset W) \cdot (X \supset Y)$
 2. $T \vee X$
 3. $(T \supset \sim Y) \cdot (X \supset \sim W)$
 4. $(W \cdot \sim Y) \supset Z$
 5. $Y \supset (W \vee A)$ / $Z \vee A$
16. 1. $(\sim S \vee T) \cdot (\sim S \vee U)$
 2. $(T \vee S) \vee U$ / $T \vee (U \vee X)$

Sample Solutions to Practice Problems for Test #3

1. 1. $G \supset H$
 2. $G \supset I$ / $G \supset (H \cdot I)$
 3. G ACP
 4. H 1, 3, MP
 5. I 2, 3, MP
 6. $H \cdot I$ 4, 5, Conj
 7. $G \supset (H \cdot I)$ 3-6, CP
 QED

2. 1. $D \supset E$
 2. $E \supset (F \cdot G)$
 3. $\sim F \vee \sim G$ / $\sim D$
 4. D AIP
 5. E 1, 4, MP
 6. $F \cdot G$ 2, 5, MP
 7. $\sim(F \cdot G)$ 3, DM
 8. $(F \cdot G) \cdot \sim(F \cdot G)$ 6, 7, Conj
 9. $\sim D$ 4-8, IP
 QED

3. 1. $(I \cdot E) \supset \sim F$
 2. $F \vee (G \cdot H)$
 3. $I \equiv E$ / $I \supset G$
 4. I ACP
 5. E 3, 4, BMP
 6. $I \cdot E$ 4, 5, Conj
 7. $\sim F$ 1, 6, MP
 8. $G \cdot H$ 2, 7, DS
 9. G 8, Simp
 10. $I \supset G$ 5-9, CP

4. 1. $(P \vee Q) \vee (\sim R \vee S)$
 2. $\sim Q \cdot (\sim S \cdot \sim P)$ / $\sim R$
 3. $\sim Q$ 2, Simp
 4. $(Q \vee P) \vee (\sim R \vee S)$ 1, Com
 5. $Q \vee [P \vee (\sim R \vee S)]$ 4, Assoc
 6. $P \vee (\sim R \vee S)$ 5, 3, DS
 7. $\sim S \cdot \sim P$ 2, Com, Simp
 8. $\sim P$ 7, Com, Simp
 9. $\sim R \vee S$ 6, 8, DS
 10. $\sim S$ 7, Simp
 11. $\sim R$ 9, 10, Com, DS
 QED

- 5.
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|----------------------------|---------------|
| 1. $(A \cdot B) \supset C$ | |
| 2. $(F \cdot D) \supset E$ | |
| 3. $A \cdot D$ | |
| 4. $B \vee F$ | / $C \vee E$ |
| 5. $\sim(C \vee E)$ | AIP |
| 6. $\sim C \cdot \sim E$ | 5, DM |
| 7. $\sim C$ | 6, Simp |
| 8. $\sim(A \cdot B)$ | 1, 7, MT |
| 9. $\sim A \vee \sim B$ | 8, DM |
| 10. A | 3, Simp |
| 11. $\sim B$ | 9, 10, DN, DS |
| 12. F | 4, 11, DS |
| 13. D | 3, Com, Simp |
| 14. $F \cdot D$ | 12, 13, Conj |
| 15. E | 2, 14, MP |
| 16. $\sim E$ | 6, Com, Simp |
| 17. $E \cdot \sim E$ | 15, 16, Conj |
| 18. $C \vee E$ | 5-17, IP, DN |

QED

- 6.
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| 1. R | | ACP |
| 2. $R \supset (S \cdot T)$ | | ACP |
| 3. $S \cdot T$ | | 2, 1, MP |
| 4. $T \cdot S$ | | 3, Com |
| 5. T | | 4, Simp |
| 6. $[R \supset (S \cdot T)] \supset T$ | | 2-5, CP |
| 7. $R \supset \{[R \supset (S \cdot T)] \supset T\}$ | | 1-6, CP |

QED

- 7.
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|--|--|--------------|
| 1. $\sim\{A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]\}$ | | AIP |
| 2. $\sim A \cdot \sim [(\sim A \vee B) \cdot (\sim A \vee C)]$ | | 1, DM |
| 3. $\sim A$ | | 2, Simp |
| 4. $\sim [(\sim A \vee B) \cdot (\sim A \vee C)]$ | | 2, Com, Simp |
| 5. $\sim[\sim A \vee (B \cdot C)]$ | | 4, Dist |
| 6. $A \cdot \sim(B \cdot C)$ | | 5, DM, DN |
| 7. A | | 6, Simp |
| 8. $A \cdot \sim A$ | | 7, 3, Conj |
| 9. $A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$ | | 1-8, IP, DN |

QED

- 8.
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|--|---------------------|
| 1. $J \equiv K$ | |
| 2. $\sim K \equiv \sim L$ | |
| 3. $L \supset \sim M$ | |
| 4. $\sim L \supset M$ | / $J \equiv \sim M$ |
| 5. $K \equiv L$ | 2, BCont |
| 6. $J \equiv L$ | 1, 3, BHS |
| 7. $\sim M \supset L$ | 4, Cont, DN |
| 8. $(L \supset \sim M) \cdot (\sim M \supset L)$ | 3, 7, Conj |
| 9. $L \equiv \sim M$ | 8, Equiv |
| 10. $J \equiv \sim M$ | 6, 9, BHS |

QED

9. 1. $\sim F \supset (G \cdot H)$
 2. $F \supset (I \vee J)$
 3. $\sim [G \cdot (K \cdot \sim J)]$
 4. $K \equiv (L \cdot \sim J)$ / $K \supset I$
 5. K ACP
 6. $L \cdot \sim J$ 4, 5, BMP
 7. $\sim J \cdot L$ 6, Com
 8. $\sim J$ 7, Simp
 9. $K \cdot \sim J$ 5, 8, Conj
 10. $\sim G \vee \sim (K \cdot \sim J)$ 3, DM
 11. $\sim (K \cdot \sim J) \vee \sim G$ 10, Com
 12. $\sim \sim (K \cdot \sim J)$ 9, DN
 13. $\sim G$ 11, 12, DS
 14. $\sim I$ AIP
 15. $\sim I \cdot \sim J$ 14, 8, Conj
 16. $\sim (I \vee J)$ 15, DM
 17. $\sim F$ 2, 16, MT
 18. $G \cdot H$ 1, 17, MP
 19. G 18, Simp
 20. $G \cdot \sim G$ 19, 13, Conj
 21. I 14-20, IP
 22. $K \supset I$ 5-21, CP

QED

10. 1. $A \supset B$
 2. $B \supset D$
 3. $D \supset A$
 4. $A \supset \sim D$ / $\sim A \cdot \sim D$
 5. $A \supset D$ 1, 2, HS
 6. $\sim D \supset \sim A$ 5, Cont
 7. $A \supset \sim A$ 4, 6, HS
 8. $\sim A \vee \sim A$ 7, Impl
 9. $\sim A$ 8, Taut
 10. $\sim D$ 3, 9, MT
 11. $\sim A \cdot \sim D$ 9, 10, Conj

QED

11. 1. M / $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$
 2. N ACP
 3. O ACP
 4. P ACP
 5. $M \cdot N$ 1, 2, Conj
 6. $O \cdot P$ 3, 4, Conj
 7. $(M \cdot N) \cdot (O \cdot P)$ 5, 6, Conj
 8. $P \supset [(M \cdot N) \cdot (O \cdot P)]$ 4-7, CP
 9. $O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}$ 3-8, CP
 10. $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$ 2-9, CP

QED

12.	1. $(A \vee B) \supset \sim(F \cdot D)$	
	2. $\sim(A \cdot \sim D)$	
	3. $\sim F \supset \sim(C \cdot D)$	
	4. $C \vee A$	$/ A \equiv \sim C$
	5. A	ACP
	6. $A \vee B$	5, Add
	7. $\sim(F \cdot D)$	6, 1, 6, MP
	8. $\sim F \vee \sim D$	7, DM
	9. $\sim A \vee \sim \sim D$	2, DM
	10. $\sim \sim D$	9, 5, DN, DM
	11. $\sim F$	8, 10, Com, DS
	12. $\sim(C \cdot D)$	3, 11, MP
	13. $\sim C \vee \sim D$	12, DM
	14. $\sim C$	13, 10, Com, DS
	15. $A \supset \sim C$	5-14, CP
	16. $\sim C \supset A$	4, DN, Impl
	17. $(A \supset \sim C) \cdot (\sim C \supset A)$	15, 16, Conj
	18. $A \equiv \sim C$	17, Equiv

QED

13.	1. $P \cdot Q$	ACP
	2. P	1, Simp
	3. $P \vee R$	2, Add
	4. Q	1, Com, Simp
	5. $Q \vee R$	4, Add
	6. $(P \vee R) \cdot (Q \vee R)$	3, 5, Conj
	7. $(P \cdot Q) \supset [(P \vee R) \cdot (Q \vee R)]$	1-6, IP

QED

14.	1. $\sim[(A \supset B) \vee (\sim A \supset C)]$	AIP
	2. $\sim(A \supset B) \cdot \sim(\sim A \supset C)$	1, DM
	3. $\sim(A \supset B)$	2, Simp
	4. $\sim(\sim A \vee B)$	3, Impl
	5. $A \cdot \sim B$	4, DM, DN
	6. $\sim(\sim A \supset C)$	2, Com, Simp
	7. $\sim(A \vee C)$	6, Impl, DN
	8. $\sim A \cdot \sim C$	8, DM
	9. A	5, Simp
	10. $\sim A$	8, Simp
	11. $A \cdot \sim A$	9, 10, Conj
	12. $(A \supset B) \vee (\sim A \supset C)$	1-11, IP, DN

QED

15. 1. $(T \supset W) \cdot (X \supset Y)$
 2. $T \vee X$
 3. $(T \supset \sim Y) \cdot (X \supset \sim W)$
 4. $(W \cdot \sim Y) \supset Z$
 5. $Y \supset (W \vee A)$ / $Z \vee A$
 6. $W \vee Y$ 1, 2, CD
 7. $\sim Y \vee \sim W$ 3, 2, CD
 8. $\sim(Z \vee A)$ AIP
 9. $\sim Z \cdot \sim A$ 8, DM
 10. $\sim Z$ 9, Simp
 11. $\sim(W \cdot \sim Y)$ 4, 10, MT
 12. $\sim W \vee Y$ 11, DM, DN
 13. $\sim W \vee \sim Y$ 7, Com
 14. $(\sim W \vee Y) \cdot (\sim W \vee \sim Y)$ 12, 13, Conj
 15. $\sim W \vee (Y \cdot \sim Y)$ 14, Dist
 16. $Y \cdot \sim Y$ AIP
 17. $\sim(Y \cdot \sim Y)$ 16, IP
 18. $\sim W$ 15, 17, Com, DS
 19. Y 6, 18, DS
 20. $W \vee A$ 5, 19, MP
 21. A 20, 18, DS
 22. $\sim A$ 9, Com, Simp
 23. $A \cdot \sim A$ 21, 22, Conj
 24. $Z \vee A$ 8-23, IP, DN

QED

16. 1. $(\sim S \vee T) \cdot (\sim S \vee U)$
 2. $(T \vee S) \vee U$ / $T \vee (U \vee X)$
 3. $\sim(T \vee U)$ AIP
 4. $\sim T \cdot \sim U$ 3, DM
 5. $\sim T$ 4, Simp
 6. $T \vee (S \vee U)$ 2, Assoc
 7. $S \vee U$ 6, 5, DS
 8. $\sim U$ 4, Com, Simp
 9. S 7, 8, Com, DS
 10. $\sim S \vee T$ 1, Simp
 11. T 10, 9, DN, DS
 12. $T \cdot \sim T$ 11, 5, Conj
 13. $T \vee U$ 3-12, IP, DN
 14. $(T \vee U) \vee X$ 13, Add
 15. $T \vee (U \vee X)$ 14, Assoc

QED