Philosophy 240
Symbolic Logic

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Fall 2013

Class #32 - Translation Using Relational Predicates

Limits of Monadic Predicates

Consider:

1. Andrew is taller than Bob.
2. Bob is taller than Charles.
3. For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z.

So, Andrew is taller than Charles.

1. Ta
2. Yb
3. ???

/ Ya

Relational (Polyadic) Predicates

- Dyadic:
  - Txy: x is taller than y
  - Kxy: x knows y
  - Bxy: x believes y
  - Dxy: x does y
- Triadic:
  - Gxyz: x gives y to z
  - Kxyz: x kisses y in z
  - Bxyz: x is between y and z
- We can construct four-place and higher-place predicates, too.

Choosing Your Predicates

- Andrés loves Beatriz
  - La
  - Lab
- Camila gave David the earring.
  - Gc
  - Gcde
- There is something blue over there now.
  - $(\exists x)Bxabct$
- By using a relational predicate, we reveal more logical structure.
- The more logical structure we reveal, the more we can facilitate inferences.

Full First-Order Logic

- We are now using **F**, Full First-Order Predicate Logic, rather than **M**.
- For the purposes of this course, the differences between **F** and **M** are minor.
- Beyond this course, the differences between **M** and **F** are significant; we have breached a barrier.
- **M** admits of a decision procedure: there is a way of deciding, for any given formula, whether it is a theorem or not.
- **F** is not decidable.
- There are formulas for which there are no effective methods for deciding whether they are theorems or not.

Syntax for M and **F**

Vocabulary for **M** and **F**

Capital letters A...Z used as predicates
Lower case letters (singular terms)
  a, b, c,...u are constants.
  v, w, x, y, z are variables.
Five connectives: ~, •, ∨, ⊃ ≡
Quantifier Symbols: ∃, ∀
Punctuation: ( ), [ ], { }

Formation Rules for Wffs of **M**

1. A predicate (capital letter) followed by a singular term (lower-case letter) is a wff.
2. For any variable β, if α is a wff that does not contain either '(∃β)' or '(∀β)', then '(∃β)α' and '(∀β)α' are wffs.
3. If α is a wff, so is ~α.
4. If α and β are wffs, then so are:
   (α · β)
   (α ∨ β)
   (α ⊃ β)
   (α ≡ β)
5. These are the only ways to make wffs.

Formation Rules for Wffs of **F**

1. **A predicate followed by n singular terms is a wff.**
2. For any variable β, if α is a wff that does not contain either '(∃β)' or '(∀β)', then '(∃β)α' and '(∀β)α' are wffs.
3. If α is a wff, so is ~α.
4. If α and β are wffs, then so are:
   (α · β)
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Semantics for F

- Recall that there are four steps for providing a semantics for **M**.
  - Step 1. Specify a set to serve as a domain of interpretation.
  - Step 2. Assign a member of the domain to each constant.
  - Step 3. Assign some set of objects in the domain to each predicate.
  - Step 4. Use the customary truth tables for the interpretation of the connectives.
- The introduction of relational predicates for **F** requires adjustment only to Step 3.
- We assign sets of ordered n-tuples to each relational predicate.
  - **New** Step 3. Assign a set of ordered n-tuples of objects in the domain to each n-place predicate.
    - Taking a 1-tuple (single) of objects to be just an object

N-Tuples

- An n-tuple is an n-place relation.
  - an ordered sequence of objects
  - Singles are objects themselves
  - doubles, triples, quadruples...
  - a set with structure
- Sets are not ordered.
  - {1, 2} = {2, 1}
- N-tuples are ordered
  - <1, 2, 5> ≠ <2, 1, 5> ≠ <5, 2, 1>
- For the semantics for **F**, an n-place predicate is assigned sets of ordered n-tuples
- Domain = {1, 2, 3, 4, 5}
  - Nx: {1, 2, 3, 4, 5}
  - Ex: {2, 4}
  - Ox: {1, 3, 5}
  - Gxy: {<2,1>, <3,1>, <4,1>, <5,1>, <3,2>, <4,2>, <5,2>, <4,3>, <5,3>, <5,4>}
  - Lxy: {<1,2>, <1,3>, <1,4>, <1,5>, <2,3>, <2,4>, <2,5>, <3,4>, <3,5>, <4,5>}

Satisfaction and Truth

- Objects in the domain (still) can satisfy one-place predicates.
- Ordered n-tuples may satisfy relational predicates.
- A wff will be satisfiable if there are objects in the domain of quantification which stand in the relations indicated in the wff.
- A wff will be true for an interpretation if all objects in the domain of quantification stand in the relations indicated in the wff.
- And, still, a wff will be logically true if it is true for every interpretation.

A Sample Theory and Interpretation

1. $Pa \bullet Pb$
2. $Wa \bullet {\sim}Wb$
3. Oab
4. Obc
5. $(\exists x)(Px \bullet Oxb)$
6. $(\exists x)(Px \bullet Obx)$
7. $(\forall x)[Wx \supset (\exists y)(Py \bullet Oyx)]$

- Domain: {Bob Simon, Rick Werner, Katheryn Doran, Todd Franklin, Marianne Janack, Russell Marcus, Theresa Lopez}
- Constants
  - a: Katheryn Doran
  - b: Bob Simon
  - c: Russell Marcus
- Predicates
  - Px: {Bob Simon, Rick Werner, Katheryn Doran, Todd Franklin, Marianne Janack, Russell Marcus, Theresa Lopez}
  - Wx: {Katheryn Doran, Marianne Janack, Theresa Lopez}
  - Oxy: {<Bob Simon, Rick Werner>, <Bob Simon, Katheryn Doran>, <Bob Simon, Todd Franklin>, <Bob Simon, Marianne Janack>, <Bob Simon, Russell Marcus>, <Rick Werner, Katheryn Doran>, <Rick Werner, Todd Franklin>, <Rick Werner, Marianne Janack>, <Rick Werner, Russell Marcus>, <Katheryn Doran, Todd Franklin>, <Katheryn Doran, Marianne Janack>, <Katheryn Doran, Russell Marcus>, <Todd Franklin, Marianne Janack>, <Todd Franklin, Russell Marcus>, <Marianne Janack, Russell Marcus>, <Bob Simon, Theresa Lopez>, <Rick Werner, Theresa Lopez>, <Katheryn Doran, Theresa Lopez>, <Todd Franklin, Theresa Lopez>, <Marianne Janack, Theresa Lopez>, <Russell Marcus, Theresa Lopez>}

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- 1 and 2 are true.

A Sample Theory and Interpretation

1. Pa • Pb
2. Wa • ~Wb
3. Oab
4. Obc
5. (∃x)(Px • Oxb)
6. (∃x)(Px • Obx)
7. (∀x)[Wx ⊃ (∃y)(Py • Oyx)]

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- 1 and 2 are true.
- 3 is false.

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- 1 and 2 are true.
- 3 is false.
- 4 is true.

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- 1 and 2 are true.
- 3 is false while 4 is true.
- 5 is false but 6 and 7 are true.

Some Translations

1. John loves Mary (Lxy: x loves y)
   - Ljm
2. Tokyo isn't smaller than New York. (Sxy: x is smaller than y)
   - ~Stn
3. Marco was introduced to Paco by Erika. (Ixyz: x introduced y to z)
   - Ipme
4. America took California from Mexico. (Txyz: x was taken by y from z)
   - Tcam

Our Original Argument

Consider:

1. Bob is taller than Charles.
2. Andrew is taller than Bob.
3. For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z.

So, Andrew is taller than Charles.

1. Tbc
2. Tab
3. ???

/ Tac

Quantifiers and Relational Predicates

Bxy: x is bigger than y

- Joe is bigger than some thing.
  $(\exists x)Bjx$
- Something is bigger than Joe.
  $(\exists x)Bxj$
- Joe is bigger than everything.
  $(\forall x)Bjx$
- Everything is bigger than Joe.
  $(\forall x)Bxj$

Overlapping Quantifiers

Lxy: x loves y

- Everything loves something.
  $(\forall x)(\exists y)Lxy$
- Something loves everything.
  $(\exists x)(\forall y)Lxy$
- $(\forall x)(\exists y)Lyx$
  Everything is loved by something.
- $(\exists x)(\forall y)Lyx$
  Something is loved by everything.

Our Original Argument

Finally Translated

Consider:

1. Bob is taller than Charles.
2. Andrew is taller than Bob.
3. For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z.

So, Andrew is taller than Charles.

1. Tbc
2. Tab
3. $(\forall x)(\forall y)(\forall z)[(Txy \bullet Tyz) \supset Txz]$ / Tac

More Examples

Something teaches Plato. (Txy: x teaches y)

- $(\exists x)Txp$

Someone teaches Plato. (Px: x is a person)

- $(\exists x)(Px \bullet Txp)$

Plato teaches everyone.

- $(\forall x)(Px \supset Tpx)$

Everyone teaches something.

- $(\forall x)[Px \supset (\exists y)Txy]$

Some people teach themselves.

- $(\exists x)(Px \bullet Txx)$

There are teachers.

- $(\exists x)(\exists y)Txy$

There are students.

- $(\exists x)(\exists y)Tyx$

Skilled teachers are interesting.

- $(\forall x)[(\exists y)Txy \supset (Sx \supset Ix)]$

Skilled teachers are better than unskilled teachers.

- $(\forall x)\{[(\exists y)Txy \bullet Sx] \supset \{(\forall z)[(\exists w)Tzw \bullet {\sim}Sz] \supset Bxz\}\}$

Wide and Narrow Scope

- Wide: $(\exists x)(\exists y)[(Px \bullet Py) \bullet Lxy]$
- Narrow: $(\exists x)[Px \bullet (\exists y)(Py \bullet Lxy)]$
- Give your quantifiers as narrow a scope as possible.
- Not equivalent:
  - $(\forall x)[Px \supset (\exists y)(Py \bullet Qxy)]$
    – ‘all people love someone’
  - $(\exists y)(\forall x)[Px \supset (Py \bullet Qxy)]$
    – ‘there is someone everyone loves’