

Solutions to Practice Problems for Test #6

I. Translations

1. $(\forall x)(Ax \supset Px)$
2. $(\forall x)[(Tx \cdot Ax) \supset Px]$
3. $(\forall x)(Ax \supset Mnx)$
4. $(\forall x)(Ppx \supset Mnx)$
5. $(\forall x)(Sxn \supset Mnx)$
6. $(\forall x)(\sim Mxx \supset Mnx)$
7. $(\forall x)(\forall y)(Sxy \supset \sim Syx)$
8. $(\forall x)\{(Ax \cdot Px) \supset (\exists y)[(Ty \cdot Py) \cdot Rxy]\} \supset (\exists x)[Tx \cdot (\forall y)(Ay \supset Sxy)]$
9. $Bam \cdot (\forall x)(Bxm \supset x=a)$
 $(\exists x)(Nx \cdot Pex)$
 $\sim(\exists x)(Nx \cdot Pax) \quad / \sim Bem$
10. $(\forall x)(\forall y)(Rxy \supset \sim x=y) \quad / (\forall x)\sim Rxx$
11. $(\forall x)(\forall y)(\forall z)(x=y \vee y=z \vee x=z)$
 $(\exists x)(\sim x=c \cdot Jx) \quad / (\exists x)(\exists y)[\sim x=y \cdot (\forall z)(z=x \vee z=y)]$
12. $(\exists x)[Bxc \cdot (\forall y)(Byc \supset y=x) \cdot Jx] \quad / (\exists x)(Bxc)$
13. $(\forall x)[(\sim x=m \cdot \sim x=b) \supset Jx]$
 $\sim Ja \quad / a=m \vee a=b$
14. $Af(b) \cdot \sim Ag(c)$
15. $\sim(\exists x)Bxf(c,e)$
16. $Tc \supset \{(Jg(c) \cdot Rg(c)) \cdot [Af(c) \cdot (\exists x)(Nx \cdot Pf(c)x)]\}$
17. $(\exists X)(Xn \cdot \sim Xp)$
18. $(\forall x)(\forall y)[(Rx \cdot Ry) \supset (\exists X)(Xx \cdot Xy)]$
19. $(\exists X)\{(\forall x)(\forall y)(\forall z)[(Xxy \cdot Xyz) \supset Xxz] \cdot (\forall x)(\forall y)(Xxy \supset \sim Xyx)\}$
20. $(\forall x)x=x \quad / (\exists X)Xxx$

II. Derivations

Note: These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

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|----|--|---|
| 1. | 1. $(\forall x)(\exists y)(\sim Ax \vee By)$ | $/ (\forall x)Ax \supset (\exists y)By$ |
| | 2. $(\forall x)Ax$ | ACP |
| | 3. $(\exists y)(\sim Ax \vee By)$ | 1, UI |
| | 4. $\sim Ax \vee Ba$ | 3, EI |
| | 5. Ax | 2, UI |
| | 6. Ba | 4, 5, DN, DS |
| | 7. $(\exists y)By$ | 6, EG |
| | 8. $(\forall x)Ax \supset (\exists y)By$ | 2-7 CP |

QED

2. 1. $(\forall x)(\exists y)Axy \supset (\forall x)(\exists y)Bxy$
 2. $(\exists x)(\forall y)\sim Bxy$ / $(\exists x)(\forall y)\sim Axy$
 3. $\sim(\forall x)(\exists y)Bxy$ 2, QE, QE
 4. $\sim(\forall x)(\exists y)Axy$ 1, 3, MT
 5. $(\exists x)(\forall y)\sim Axy$ 4, QE, QE

QED

3. 1. $(\forall x)[(Ax \vee Bx) \supset (Dx \cdot Kx)]$
 2. $(\forall x)\{(Dx \vee Lx) \supset [(Dx \cdot Nx) \supset Px]\}$ / $(\forall x)[Ax \supset (Nx \supset Px)]$
 3. Ax ACP
 4. $Ax \vee Bx$ 3, Add
 5. $(Ax \vee Bx) \supset (Dx \cdot Kx)$ 1, UI
 6. $Dx \cdot Kx$ 5, 4, MP
 7. Dx 6, Simp
 8. $Dx \vee Lx$ 7, Add
 9. $(Dx \vee Lx) \supset [(Dx \cdot Nx) \supset Px]$ 2, UI
 10. $(Dx \cdot Nx) \supset Px$ 9, 8, MP
 11. Nx ACP
 12. $Dx \cdot Nx$ 7, 11, Conj
 13. Px 10, 12, MP
 14. $Nx \supset Px$ 11-13, CP
 15. $Ax \supset (Nx \supset Px)$ 3-14, CP
 16. $(\forall x)[Ax \supset (Nx \supset Px)]$ 15, UG

QED

4. 1. $\sim(\exists x)(Axa \cdot \sim Bxb)$
 2. $\sim(\exists x)(Dxd \cdot Dbx)$
 3. $(\forall x)(Bex \supset Dxd)$ / $\sim(Aea \cdot Dgd)$
 4. $Aea \cdot Dgd$ AIP
 5. $(\forall x)\sim(Axa \cdot \sim Bxb)$ 1, QE
 6. $(\forall x)(\sim Axa \vee Bxb)$ 5, DM, DN
 7. $\sim Aea \vee Bba$ 6, UI
 8. Aea 4, Simp
 9. Bba 7, 8, DN, DS
 10. $(\forall x)\sim(Dxd \cdot Dbx)$ 2, QE
 11. $(\forall x)(\sim Dxd \vee \sim Dbx)$ 10, DM
 12. $\sim Dgd \vee \sim Dbg$ 11, UI
 13. Dgd 4, Com, Simp
 14. $\sim Dbg$ 12, 13, DN, DS
 15. $Bba \supset Dbg$ 3, UI
 16. Dbg 15, 9, MP
 17. $Dbg \cdot \sim Dbg$ 16, 14, Conj
 18. $\sim(Aea \cdot Dgd)$ 4-17, IP

QED

5. 1. $(\forall x)(Ax \supset Bx)$ / $(\forall x)[(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)]$
 2. $(\exists y)(Ay \cdot Cxy)$ ACP
 3. $Aa \cdot Cxa$ 2, EI
 4. Aa 3, Simp
 5. $Aa \supset Ba$ 1, UI
 6. Ba 5, 4, MP
 7. Cxa 3, Com, Simp
 8. $Ba \cdot Cxa$ 6, 7, Conj
 9. $(\exists z)(Bz \cdot Cxz)$ 8, EG
 10. $(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)$ 2-9, CP
 11. $(\forall x)[(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)]$ 10, UG

QED

6. 1. $(\exists x)(Nx \cdot Pjx \cdot Ix)$
 2. $Nc \cdot Pjc \cdot (\forall x)[(Nx \cdot Pjx) \supset x=c]$ / Ic
 3. $Na \cdot Pja \cdot Ia$ 1, EI
 4. $(\forall x)[(Nx \cdot Pjx) \supset x=c]$ 2, Com, Simp
 5. $(Na \cdot Pja) \supset a=c$ 4, UI
 6. $Na \cdot Pja$ 3, Simp
 7. $a=c$ 5, 6, MP
 8. Ia 3, Com, Simp
 9. Ic 8, 7, IDi

QED

7. 1. $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot y \neq x) \supset Dxy]\}$ / $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot \sim Ty) \supset Dxy]\}$
 2. $Ma \cdot Ta \cdot (\forall y)[(My \cdot \sim y=a) \supset Day]$ 1, EI
 3. $My \cdot \sim Ty$ ACP
 4. $(\forall y)[(My \cdot \sim y=a) \supset Day]$ 2, Com, Simp
 5. $(My \cdot \sim y=a) \supset Day$ 4, UI
 6. $y=a$ AIP
 7. Ta 2, Simp
 8. $\sim Ty$ 3, Com, Simp
 9. Ty 7, 6, IDi
 10. $Ty \cdot \sim Ty$ 9, 8, Conj
 11. $\sim y=a$ 6-10, IP
 12. My 3, Simp
 13. $My \cdot \sim y=a$ 12, 11, Conj
 14. Day 5, 13, MP
 15. $(My \cdot \sim Ty) \supset Day$ 3-14, CP
 16. $(\forall y)[(My \cdot \sim Ty) \supset Day]$ 15, UG
 17. $Ma \cdot Ta$ 2, Simp
 18. $Ma \cdot Ta \cdot (\forall y)[(My \cdot \sim Ty) \supset Day]$ 17, 16, Conj
 19. $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot \sim Ty) \supset Dxy]\}$ 18, EG

QED

8. 1. $(\forall x)(\forall y)(\forall z)[(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (x=y \vee y=z \vee x=z)]$
 2. $(\exists x)(\exists y)(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Rx \cdot Ry \cdot x \neq y)$
 3. $(\forall x)(Rx \supset \sim Cx)$ / $(Sa \cdot Ca) \supset \sim La$
 4. $Sa \cdot Ca$ ACP
 5. La AIP
 6. $(\exists y)(Sb \cdot Lb \cdot Sy \cdot Ly \cdot Rb \cdot Ry \cdot b \neq y)$ 3, EI
 7. $Sb \cdot Lb \cdot Sc \cdot Lc \cdot Rb \cdot Rc \cdot b \neq c$ 6, EI
 8. $Sb \cdot Lb \cdot Sc \cdot Lc$ 7, Simp
 9. Sa 4, Simp
 10. $Sa \cdot La$ 9, 5, Conj
 11. $Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc$ 10, 8, Conj
 12. $(\forall y)(\forall z)[(Sa \cdot La \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (a=y \vee y=z \vee a=z)]$ 1, UI
 13. $(\forall z)[(Sa \cdot La \cdot Sb \cdot Lb \cdot Sz \cdot Lz) \supset (a=b \vee b=z \vee a=z)]$ 12, UI
 14. $(Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc) \supset (a=b \vee b=c \vee a=c)$ 13, UI
 15. $a=b \vee b=c \vee a=c$ 14, 11, MP
 16. $\sim b=c$ 7, Simp
 17. $a=b \vee a=c$ 17, 16, Com, DS
 18. $Ra \supset \sim Ca$ 3, UI
 19. Ca 4, Com, Simp
 20. $\sim Ra$ 18, 19, DN, MT
 21. Rb 7, Simp
 22. $a=b$ AIP
 23. $\sim Rb$ 20, 22, IDi
 24. $Rb \cdot \sim Rb$ 21, 24, Conj
 25. $\sim a=b$ 22-24, IP
 26. $a=c$ 17, 25, DS
 27. Rc 7, Simp
 28. $Rc \supset \sim Cc$ 3, UI
 29. $\sim Cc$ 28, 27, MP
 30. Cc 19, 26, IDi
 31. $Cc \cdot \sim Cc$ 30, 29, Conj
 33. $\sim La$ 5-31, IP
 34. $(Sa \cdot Ca) \supset \sim La$ 4-33, CP
- QED
9. 1. $(\forall x)(\forall y)f(x,y)=f(y,x)$
 2. $(\forall x)f(x,o)=o$ / $(\forall x)f(o,x)=o$
 3. $f(x,o)=o$ 2, UI
 4. $(\forall y)f(o,y)=f(y,o)$ 1, UI
 5. $f(o,x)=f(x,o)$ 4, UI
 6. $f(o,x)=o$ 5, 3, IDi
 7. $(\forall x)f(o,x)=o$ 6, UG
- QED

- 10.
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|---|-----------------------|
| 1. $(\forall x)(\forall y)(Bxy \equiv Lyx)$ | |
| 2. $(\forall x)Bf(x)x$ | / $(\forall x)Lxf(x)$ |
| 3. $(\forall y)(Bf(x)y \equiv Lyf(x))$ | 1, UI |
| 4. $Bf(x)x \equiv Lxf(x)$ | 3, UI |
| 5. $Bf(x)x$ | 2, UI |
| 6. $[Bf(x)x \supset Lxf(x)] \bullet [Lxf(x) \equiv Bf(x)x]$ | 4, Equiv |
| 7. $Bf(x)x \supset Lxf(x)$ | 6, Simp |
| 8. $Lxf(x)$ | 7, 5, MP |
| 9. $(\forall x)Lxf(x)$ | 8, UG |

QED

- 11.
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|--|---|
| 1. $(\forall x)(\forall y)(\exists z)Sf(x)yz$ | |
| 2. $(\forall x)(\forall y)(\forall z)[Sxyz \supset \sim(Cxyz \vee Mzyx)]$ | / $(\exists x)(\exists y)(\exists z)\sim Mzg(y)f(g(x))$ |
| 3. $(\forall y)(\exists z)Sf(g(x))yz$ | 1, UI |
| 4. $(\exists z)Sf(g(x))g(y)z$ | 3, UI |
| 5. $Sf(g(x))g(y)a$ | 4, EI |
| 6. $(\forall y)(\forall z)[Sf(g(x))yz \supset \sim(Cf(g(x))yz \vee Mzyf(g(x)))]$ | 2, UI |
| 7. $(\forall z)[Sf(g(x))g(y)z \supset \sim(Cf(g(x))g(y)z \vee Mzg(y)f(g(x)))]$ | 6, UI |
| 8. $Sf(g(x))g(y)a \supset \sim(Cf(g(x))g(y)a \vee Mag(y)f(g(x)))$ | 7, UI |
| 9. $\sim(Cf(g(x))g(y)a \vee Mag(y)f(g(x)))$ | 8, 5, MP |
| 10. $\sim Cf(g(x))g(y)a \bullet \sim Mag(y)f(g(x))$ | 9, DM |
| 11. $\sim Mag(y)f(g(x))$ | 10, Com, Simp |
| 12. $(\exists z)\sim Mzg(y)f(g(x))$ | 11, EG |
| 13. $(\exists y)(\exists z)\sim Mzg(y)f(g(x))$ | 12, EG |
| 14. $(\exists x)(\exists y)(\exists z)\sim Mzg(y)f(g(x))$ | 13, EG |

QED