

Solutions to Practice Problems for Test #5

These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

I. Derivations

1. 1. $Ab \vee Bc$
2. $(\forall x)\sim Bx$ / $(\exists x)Ax$
3. $\sim Bc$ 2, UI
4. Ab 1,3 Com, DS
5. $(\exists x)Ax$ 4, EG

QED

2. 1. $(\forall x)(Rx \supset Ox)$
2. $(\exists x)\sim Ox$
3. $(\forall x)(\sim Rx \supset Px)$ / $(\exists x)Px$
4. $\sim Oa$ 2, EI
5. $Ra \supset Oa$ 1, UI
6. $\sim Ra$ 5, 4, MT
7. $\sim Ra \supset Pa$ 3, UI
8. Pa 7, 6, MP
9. $(\exists x)Px$ 8, EG

QED

3. 1. $(\forall x)(Fx \supset Gx)$
2. $(\forall y)(Gy \supset Hy)$ / $(\forall z)(\sim Hz \supset \sim Fz)$
3. $Fx \supset Gx$ 1, UI
4. $Gx \supset Hx$ 2, UI
5. $Fx \supset Hx$ 3, 4, HS
6. $\sim Hx \supset \sim Fx$ 5, Cont
7. $(\forall z)(\sim Hz \supset \sim Fz)$ 6, UG

QED

4. 1. $(\exists x)(Ax \cdot Bx) \supset (\forall x) Dx$
2. $\sim Da$ / $(\forall x)(Ax \supset \sim Bx)$
3. $(\exists x)\sim Dx$ 2, EG
4. $\sim(\forall x)Dx$ 3, QE
5. $\sim(\exists x)(Ax \cdot Bx)$ 1, 4, MT
6. $(\forall x)\sim(Ax \cdot Bx)$ 5, QE
7. $(\forall x)(\sim Ax \vee \sim Bx)$ 6, DM
8. $(\forall x)(Ax \supset \sim Bx)$ 7, Impl

QED

- 8.
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|---|---|
| 1. $(\forall x)(Ax \supset Bx)$ | |
| 2. $(\forall x)[Bx \supset (Ax \supset \sim Fx)]$ | |
| 3. $(\forall x)[(\sim Cx \cdot Dx) \supset Fx]$ | $/ (\forall x)[Ax \supset (Cx \vee \sim Dx)]$ |
| 4. Ax | ACP |
| 5. $Ax \supset Bx$ | 1, UI |
| 6. Bx | 5, 4, MP |
| 7. $Bx \supset (Ax \supset \sim Fx)$ | 2, UI |
| 8. $Ax \supset \sim Fx$ | 7, 6, MP |
| 9. $\sim Fx$ | 8, 4, MP |
| 10. $(\sim Cx \cdot Dx) \supset Fx$ | 3, UI |
| 11. $\sim(\sim Cx \cdot Dx)$ | 10, 9, MT |
| 12. $Cx \vee \sim Dx$ | 11, DM, DN |
| 13. $Ax \supset (Cx \vee \sim Dx)$ | 4-12, CP |
| 14. $(\forall x)[Ax \supset (Cx \vee \sim Dx)]$ | 13, UG |

QED

- 9.
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|---|-----------------------|
| 1. $(\exists x)Gx \supset (\forall x)(Fx \supset Dx)$ | |
| 2. $(\exists x)(Gx \cdot \sim Dx)$ | $/ \sim(\forall x)Fx$ |
| 3. $Ga \cdot \sim Da$ | 2, EI |
| 4. Ga | 3, Simp |
| 5. $(\exists x)Gx$ | 4, EG |
| 6. $(\forall x)(Fx \supset Dx)$ | 1, 5, MP |
| 7. $Fa \supset Da$ | 6, UI |
| 8. $\sim Da$ | 3, Com, Simp |
| 9. $\sim Fa$ | 7, 8, MT |
| 10. $(\exists x)\sim Fx$ | 9, EG |
| 11. $\sim(\forall x)Fx$ | 10, QE |

QED

- 10.
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|---|-------------------|
| 1. $(\exists x)Qx \supset (\forall x)(Rx \supset Sx)$ | |
| 2. $(\forall x)\sim Qx \supset (\exists x)Sx$ | |
| 3. $(\forall x) Rx$ | $/ (\exists x)Sx$ |
| 4. $\sim(\exists x)Sx$ | AIP |
| 5. $\sim(\forall x)\sim Qx$ | 2, 4, MT |
| 6. $(\exists x)Qx$ | 5, QE |
| 7. $(\forall x)(Rx \supset Sx)$ | 1, 6, MP |
| 8. $Rx \supset Sx$ | 7, UI |
| 9. Rx | 3, UI |
| 10. Sx | 8, 9, MP |
| 11. $(\exists x)Sx$ | 10, EG |
| 12. $(\exists x)Sx \cdot \sim(\exists x)Sx$ | 11, 4, Conj |
| 13. $(\exists x)Sx$ | 4-12, IP, DN |

QED

II. Invalidity

1. Invalid in a 1-member universe, where:

Aa: True; Ba: False; Ca: False.

2. Invalid in a 2-member universe, where:

Ea: True; Fa: False; Ga: False;

Eb: True or False; Fb: True; Gb: True

3. Invalid in a 3-member universe, where:

Pa: True; Qa: False; Ra: False

Pb: False; Qb: True; Rb: False

Pc: False; Qc: False; Rc: True or False

Alternate solutions are possible.