

Reference Sheet for *What Follows*

Names of Languages

PL: Propositional Logic
M: Monadic (First-Order) Predicate Logic
F: Full (First-Order) Predicate Logic
FF: Full (First-Order) Predicate Logic with functors
S: Second-Order Predicate Logic

Basic Truth Tables

$\sim$	α
0	1
1	0

α	$\cdot$	β
1	1	1
1	0	0
0	0	1
0	0	0

α	$\vee$	β
1	1	1
1	1	0
0	1	1
0	0	0

α	$\supset$	β
1	1	1
1	0	0
0	1	1
0	1	0

α	$\equiv$	β
1	1	1
1	0	0
0	0	1
0	1	0

Rules of Inference

Modus Ponens (MP)

$\alpha \supset \beta$
 α / β

Conjunction (Conj)

α
 β / $\alpha \cdot \beta$

Modus Tollens (MT)

$\alpha \supset \beta$
 $\sim \beta$ / $\sim \alpha$

Addition (Add)

α / $\alpha \vee \beta$

Disjunctive Syllogism (DS)

$\alpha \vee \beta$
 $\sim \alpha$ / β

Simplification (Simp)

$\alpha \cdot \beta$ / α

Hypothetical Syllogism (HS)

$\alpha \supset \beta$
 $\beta \supset \gamma$ / $\alpha \supset \gamma$

Constructive Dilemma (CD)

$(\alpha \supset \beta) \cdot (\gamma \supset \delta)$
 $\alpha \vee \gamma$ / $\beta \vee \delta$

Rules of Equivalence

DeMorgan's Laws (DM)

$$\sim(\alpha \cdot \beta) \equiv \sim\alpha \vee \sim\beta$$

$$\sim(\alpha \vee \beta) \equiv \sim\alpha \cdot \sim\beta$$

Association (Assoc)

$$\alpha \vee (\beta \vee \gamma) \equiv (\alpha \vee \beta) \vee \gamma$$

$$\alpha \cdot (\beta \cdot \gamma) \equiv (\alpha \cdot \beta) \cdot \gamma$$

Distribution (Dist)

$$\alpha \cdot (\beta \vee \gamma) \equiv (\alpha \cdot \beta) \vee (\alpha \cdot \gamma)$$

$$\alpha \vee (\beta \cdot \gamma) \equiv (\alpha \vee \beta) \cdot (\alpha \vee \gamma)$$

Commutativity (Com)

$$\alpha \vee \beta \equiv \beta \vee \alpha$$

$$\alpha \cdot \beta \equiv \beta \cdot \alpha$$

Double Negation (DN)

$$\alpha \equiv \sim\sim\alpha$$

Contraposition (Cont)

$$\alpha \supset \beta \equiv \sim\beta \supset \sim\alpha$$

Material Implication (Impl)

$$\alpha \supset \beta \equiv \sim\alpha \vee \beta$$

Material Equivalence (Equiv)

$$\alpha \equiv \beta \equiv (\alpha \supset \beta) \cdot (\beta \supset \alpha)$$

$$\alpha \equiv \beta \equiv (\alpha \cdot \beta) \vee (\sim\alpha \cdot \sim\beta)$$

Exportation (Exp)

$$\alpha \supset (\beta \supset \gamma) \equiv (\alpha \cdot \beta) \supset \gamma$$

Tautology (Taut)

$$\alpha \equiv \alpha \cdot \alpha$$

$$\alpha \equiv \alpha \vee \alpha$$

Six Derived Rules for the Biconditional

Rules of Inference

Biconditional Modus Ponens (BMP)

$$\alpha \equiv \beta$$

$$\alpha \quad / \beta$$

Biconditional Modus Tollens (BMT)

$$\alpha \equiv \beta$$

$$\sim\alpha \quad / \sim\beta$$

Biconditional Hypothetical Syllogism (BHS)

$$\alpha \equiv \beta$$

$$\beta \equiv \gamma \quad / \alpha \equiv \gamma$$

Rules of Equivalence

Biconditional DeMorgan's Law (BDM)

$$\sim(\alpha \equiv \beta) \equiv \sim\alpha \equiv \beta$$

Biconditional Commutativity (BCom)

$$\alpha \equiv \beta \equiv \beta \equiv \alpha$$

Biconditional Contraposition (BCont)

$$\alpha \equiv \beta \equiv \sim\alpha \equiv \sim\beta$$

Rules for Quantifier Instantiation and Generalization

Universal Instantiation (UI)

$$\frac{(\forall\alpha)\mathcal{T}\alpha}{\mathcal{T}\beta} \quad \text{for any variable } \alpha, \text{ any formula } \mathcal{T} \text{ containing } \alpha, \text{ and any singular term } \beta$$

Universal Generalization (UG)

$$\frac{\mathcal{T}\beta}{(\forall\alpha)\mathcal{T}\alpha} \quad \begin{array}{l} \text{for any variable } \beta, \text{ any formula } \mathcal{T} \text{ containing } \beta, \text{ and} \\ \text{for any variable } \alpha \end{array}$$

Never UG within the scope of an assumption for conditional or indirect proof on a variable that is free in the first line of the assumption.

Never UG on a variable when there is a constant present, and the variable was free when the constant was introduced.

Existential Generalization (EG)

$$\frac{\mathcal{T}\beta}{(\exists\alpha)\mathcal{T}\alpha} \quad \begin{array}{l} \text{for any singular term } \beta, \text{ any formula } \mathcal{T} \text{ containing } \beta, \text{ and} \\ \text{for any variable } \alpha \end{array}$$

Existential Instantiation (EI)

$$\frac{(\exists\alpha)\mathcal{T}\alpha}{\mathcal{T}\beta} \quad \begin{array}{l} \text{for any variable } \alpha, \text{ any formula } \mathcal{T} \text{ containing } \alpha, \text{ and} \\ \text{any new constant } \beta \end{array}$$

Quantifier Equivalence (QE)

$$\begin{array}{lll} (\forall\alpha)\mathcal{T}\alpha & \Leftrightarrow & \sim(\exists\alpha)\sim\mathcal{T}\alpha \\ (\exists\alpha)\mathcal{T}\alpha & \Leftrightarrow & \sim(\forall\alpha)\sim\mathcal{T}\alpha \\ (\forall\alpha)\sim\mathcal{T}\alpha & \Leftrightarrow & \sim(\exists\alpha)\mathcal{T}\alpha \\ (\exists\alpha)\sim\mathcal{T}\alpha & \Leftrightarrow & \sim(\forall\alpha)\mathcal{T}\alpha \end{array}$$

Rules of Passage

For all variables α and all formulas Γ and Δ :

$$\begin{array}{ll} \text{RP1: } (\exists\alpha)(\Gamma \vee \Delta) & \rightleftharpoons (\exists\alpha)\Gamma \vee (\exists\alpha)\Delta \\ \text{RP2: } (\forall\alpha)(\Gamma \bullet \Delta) & \rightleftharpoons (\forall\alpha)\Gamma \bullet (\forall\alpha)\Delta \end{array}$$

For all variables α , all formulas Γ containing α , and all formulas Δ not containing α :

$$\begin{array}{ll} \text{RP3: } (\exists\alpha)(\Delta \bullet \Gamma\alpha) & \rightleftharpoons \Delta \bullet (\exists\alpha)\Gamma\alpha \\ \text{RP4: } (\forall\alpha)(\Delta \bullet \Gamma\alpha) & \rightleftharpoons \Delta \bullet (\forall\alpha)\Gamma\alpha \\ \text{RP5: } (\exists\alpha)(\Delta \vee \Gamma\alpha) & \rightleftharpoons \Delta \vee (\exists\alpha)\Gamma\alpha \\ \text{RP6: } (\forall\alpha)(\Delta \vee \Gamma\alpha) & \rightleftharpoons \Delta \vee (\forall\alpha)\Gamma\alpha \\ \text{RP7: } (\exists\alpha)(\Delta \supset \Gamma\alpha) & \rightleftharpoons \Delta \supset (\exists\alpha)\Gamma\alpha \\ \text{RP8: } (\forall\alpha)(\Delta \supset \Gamma\alpha) & \rightleftharpoons \Delta \supset (\forall\alpha)\Gamma\alpha \\ \text{RP9: } (\exists\alpha)(\Gamma\alpha \supset \Delta) & \rightleftharpoons (\forall\alpha)\Gamma\alpha \supset \Delta \\ \text{RP10: } (\forall\alpha)(\Gamma\alpha \supset \Delta) & \rightleftharpoons (\exists\alpha)\Gamma\alpha \supset \Delta \end{array}$$

Here are versions of each of the rules that are less-meta-linguistic and maybe easier to read:

$$\begin{array}{ll} \text{RP1: } (\exists x)(Px \vee Qx) & \rightleftharpoons (\exists x)Px \vee (\exists x)Qx \\ \text{RP2: } (\forall x)(Px \bullet Qx) & \rightleftharpoons (\forall x)Px \bullet (\forall x)Qx \\ \text{RP3: } (\exists x)(\mathcal{F} \bullet Px) & \rightleftharpoons \mathcal{F} \bullet (\exists x)Px \\ \text{RP4: } (\forall x)(\mathcal{F} \bullet Px) & \rightleftharpoons \mathcal{F} \bullet (\forall x)Px \\ \text{RP5: } (\exists x)(\mathcal{F} \vee Px) & \rightleftharpoons \mathcal{F} \vee (\exists x)Px \\ \text{RP6: } (\forall x)(\mathcal{F} \vee Px) & \rightleftharpoons \mathcal{F} \vee (\forall x)Px \\ \text{RP7: } (\exists x)(\mathcal{F} \supset Px) & \rightleftharpoons \mathcal{F} \supset (\exists x)Px \\ \text{RP8: } (\forall x)(\mathcal{F} \supset Px) & \rightleftharpoons \mathcal{F} \supset (\forall x)Px \\ \text{RP9: } (\exists x)(Px \supset \mathcal{F}) & \rightleftharpoons (\forall x)Px \supset \mathcal{F} \\ \text{RP10: } (\forall x)(Px \supset \mathcal{F}) & \rightleftharpoons (\exists x)Px \supset \mathcal{F} \end{array}$$

Rules Governing the Identity Predicate (ID)

IDr. Reflexivity: $\alpha=\alpha$

IDs. Symmetry: $\alpha=\beta \rightleftharpoons \beta=\alpha$

IDI. Indiscernibility of Identicals

$$\begin{array}{l} \mathcal{F}\alpha \\ \alpha=\beta \quad / \quad \mathcal{F}\beta \end{array}$$