

Practice Proofs for Test #3

1.     1.  $G \supset H$   
       2.  $G \supset I$                      /  $G \supset (H \cdot I)$
2.     1.  $D \supset E$   
       2.  $E \supset (F \cdot G)$   
       3.  $\sim F \vee \sim G$                  /  $\sim D$
3.     1.  $(I \cdot E) \supset \sim F$   
       2.  $F \vee (G \cdot H)$   
       3.  $I \equiv E$                      /  $I \supset G$
4.     1.  $(P \vee Q) \vee (\sim R \vee S)$   
       2.  $\sim Q \cdot (\sim S \cdot \sim P)$          /  $\sim R$
5.     1.  $(A \cdot B) \supset C$   
       2.  $(F \cdot D) \supset E$   
       3.  $A \cdot D$   
       4.  $B \vee F$                      /  $C \vee E$
6.     Prove that ' $R \supset [(R \supset S) \supset S]$ ' is a logical truth.
7.     Prove that ' $A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$ ' is a logical truth
8.     1.  $J \equiv K$   
       2.  $\sim K \equiv \sim L$                  /  $J \equiv L$
9.     1.  $\sim F \supset (G \cdot H)$   
       2.  $F \supset (I \vee J)$   
       3.  $\sim[G \cdot (K \cdot \sim J)]$   
       4.  $K \equiv (L \cdot \sim J)$              /  $K \supset I$
10.    1.  $A \supset B$   
       2.  $B \supset D$   
       3.  $D \supset A$   
       4.  $A \supset \sim D$                    /  $\sim A \cdot \sim D$
11.    1.  $M$                              /  $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$
12.    1.  $(A \vee B) \supset \sim(F \cdot D)$   
       2.  $\sim(A \cdot \sim D)$   
       3.  $\sim F \supset \sim(C \cdot D)$   
       4.  $C \vee A$                      /  $A \equiv \sim C$
13.    Prove that ' $(P \cdot Q) \supset [(P \vee R) \cdot (Q \vee R)]$ ' is a logical truth.

14. Prove that  $(A \supset B) \vee (\sim A \supset C)$  is a logical truth.

15.    1.  $(T \supset W) \bullet (X \supset Y)$   
      2.  $T \vee X$   
      3.  $(T \supset \sim Y) \bullet (X \supset \sim W)$   
      4.  $(W \bullet \sim Y) \supset Z$   
      5.  $Y \supset (W \vee A)$                     /  $Z \vee A$

16.    1.  $(\sim S \vee T) \bullet (\sim S \vee U)$   
      2.  $(T \vee S) \vee U$                     /  $T \vee (U \vee X)$

Sample Solutions to Practice Problems for Test #3

- 1.
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|----------------------------|---------------------------|
| 1. $G \supset H$           |                           |
| 2. $G \supset I$           | $/ G \supset (H \cdot I)$ |
| 3. $G$                     | ACP                       |
| 4. $H$                     | 1, 3, MP                  |
| 5. $I$                     | 2, 3, MP                  |
| 6. $H \cdot I$             | 4, 5, Conj                |
| 7. $G \supset (H \cdot I)$ | 3-6, CP                   |

QED

- 2.
- |                                        |            |
|----------------------------------------|------------|
| 1. $D \supset E$                       |            |
| 2. $E \supset (F \cdot G)$             |            |
| 3. $\sim F \vee \sim G$                | $/ \sim D$ |
| 4. $D$                                 | AIP        |
| 5. $E$                                 | 1, 4, MP   |
| 6. $F \cdot G$                         | 2, 5, MP   |
| 7. $\sim(F \cdot G)$                   | 3, DM      |
| 8. $(F \cdot G) \cdot \sim(F \cdot G)$ | 6, 7, Conj |
| 9. $\sim D$                            | 4-8, IP    |

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- 3.
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|---------------------------------|-----------------|
| 1. $(I \cdot E) \supset \sim F$ |                 |
| 2. $F \vee (G \cdot H)$         |                 |
| 3. $I \equiv E$                 | $/ I \supset G$ |
| 4. $I$                          | ACP             |
| 5. $E$                          | 3, 4, BMP       |
| 6. $I \cdot E$                  | 4, 5, Conj      |
| 7. $\sim F$                     | 1, 6, MP        |
| 8. $G \cdot H$                  | 2, 7, DS        |
| 9. $G$                          | 8, Simp         |
| 10. $I \supset G$               | 5-9, CP         |

- 4.
- |                                         |                |
|-----------------------------------------|----------------|
| 1. $(P \vee Q) \vee (\sim R \vee S)$    |                |
| 2. $\sim Q \cdot (\sim S \cdot \sim P)$ | $/ \sim R$     |
| 3. $\sim Q$                             | 2, Simp        |
| 4. $(Q \vee P) \vee (\sim R \vee S)$    | 1, Com         |
| 5. $Q \vee [P \vee (\sim R \vee S)]$    | 4, Assoc       |
| 6. $P \vee (\sim R \vee S)$             | 5, 3, DS       |
| 7. $\sim S \cdot \sim P$                | 2, Com, Simp   |
| 8. $\sim P$                             | 7, Com, Simp   |
| 9. $\sim R \vee S$                      | 6, 8, DS       |
| 10. $\sim S$                            | 7, Simp        |
| 11. $\sim R$                            | 9, 10, Com, DS |

QED

- 5.
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|----------------------------|---------------|
| 1. $(A \cdot B) \supset C$ |               |
| 2. $(F \cdot D) \supset E$ |               |
| 3. $A \cdot D$             |               |
| 4. $B \vee F$              | / $C \vee E$  |
| 5. $\sim(C \vee E)$        | AIP           |
| 6. $\sim C \cdot \sim E$   | 5, DM         |
| 7. $\sim C$                | 6, Simp       |
| 8. $\sim(A \cdot B)$       | 1, 7, MT      |
| 9. $\sim A \vee \sim B$    | 8, DM         |
| 10. $A$                    | 3, Simp       |
| 11. $\sim B$               | 9, 10, DN, DS |
| 12. $F$                    | 4, 11, DS     |
| 13. $D$                    | 3, Com, Simp  |
| 14. $F \cdot D$            | 12, 13, Conj  |
| 15. $E$                    | 2, 14, MP     |
| 16. $\sim E$               | 6, Com, Simp  |
| 17. $E \cdot \sim E$       | 15, 16, Conj  |
| 18. $C \vee E$             | 5-17, IP, DN  |

QED

- 6.
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|------------------------------------------|----------|
| 1. $R$                                   | ACP      |
| 2. $R \supset S$                         | ACP      |
| 3. $S$                                   | 2, 1, MP |
| 4. $(R \supset S) \supset S$             | 2-3, CP  |
| 5. $R \supset [(R \supset S) \supset S]$ | 1-4, CP  |

QED

- 7.
- |                                                               |              |
|---------------------------------------------------------------|--------------|
| 1. $\sim\{A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]\}$   | AIP          |
| 2. $\sim A \cdot \sim[(\sim A \vee B) \cdot (\sim A \vee C)]$ | 1, DM        |
| 3. $\sim A$                                                   | 2, Simp      |
| 4. $\sim[(\sim A \vee B) \cdot (\sim A \vee C)]$              | 2, Com, Simp |
| 5. $\sim[\sim A \vee (B \cdot C)]$                            | 4, Dist      |
| 6. $A \cdot \sim(B \cdot C)$                                  | 5, DM, DN    |
| 7. $A$                                                        | 6, Simp      |
| 8. $A \cdot \sim A$                                           | 7, 3, Conj   |
| 9. $A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$           | 1-8, IP, DN  |

QED

- 8.
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|---------------------------|----------------|
| 1. $J \equiv K$           |                |
| 2. $\sim K \equiv \sim L$ | / $J \equiv L$ |
| 3. $K \equiv L$           | 2, BCont       |
| 4. $J \equiv L$           | 1, 3, BHS      |

QED

- 9.
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|----------------------------------------|-----------------|
| 1. $\sim F \supset (G \cdot H)$        |                 |
| 2. $F \supset (I \vee J)$              |                 |
| 3. $\sim[G \cdot (K \cdot \sim J)]$    |                 |
| 4. $K \equiv (L \cdot \sim J)$         | / $K \supset I$ |
| 5. $K$                                 | ACP             |
| 6. $L \cdot \sim J$                    | 4, 5, BMP       |
| 7. $\sim J \cdot L$                    | 6, Com          |
| 8. $\sim J$                            | 7, Simp         |
| 9. $K \cdot \sim J$                    | 5, 8, Conj      |
| 10. $\sim G \vee \sim(K \cdot \sim J)$ | 3, DM           |
| 11. $\sim(K \cdot \sim J) \vee \sim G$ | 10, Com         |
| 12. $\sim\sim(K \cdot \sim J)$         | 9, DN           |
| 13. $\sim G$                           | 11, 12, DS      |
| 14. $\sim I$                           | AIP             |
| 15. $\sim I \cdot \sim J$              | 14, 8, Conj     |
| 16. $\sim(I \vee J)$                   | 15, DM          |
| 17. $\sim F$                           | 2, 16, MT       |
| 18. $G \cdot H$                        | 1, 17, MP       |
| 19. $G$                                | 18, Simp        |
| 20. $G \cdot \sim G$                   | 19, 13, Conj    |
| 21. $I$                                | 14-20, IP       |
| 22. $K \supset I$                      | 5-21, CP        |

QED

- 10.
- |                            |                         |
|----------------------------|-------------------------|
| 1. $A \supset B$           |                         |
| 2. $B \supset D$           |                         |
| 3. $D \supset A$           |                         |
| 4. $A \supset \sim D$      | / $\sim A \cdot \sim D$ |
| 5. $A \supset D$           | 1, 2, HS                |
| 6. $\sim D \supset \sim A$ | 5, Cont                 |
| 7. $A \supset \sim A$      | 4, 6, HS                |
| 8. $\sim A \vee \sim A$    | 7, Impl                 |
| 9. $\sim A$                | 8, Taut                 |
| 10. $\sim D$               | 3, 9, MT                |
| 11. $\sim A \cdot \sim D$  | 9, 10, Conj             |

QED

- 11.
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|-----------------------------------------------------------------------------|--|--|--|---------------------------------------------------------------------------|
| 1. $M$                                                                      |  |  |  | / $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$ |
| 2. $N$                                                                      |  |  |  | ACP                                                                       |
| 3. $O$                                                                      |  |  |  | ACP                                                                       |
| 4. $P$                                                                      |  |  |  | ACP                                                                       |
| 5. $M \cdot N$                                                              |  |  |  | 1, 2, Conj                                                                |
| 6. $O \cdot P$                                                              |  |  |  | 3, 4, Conj                                                                |
| 7. $(M \cdot N) \cdot (O \cdot P)$                                          |  |  |  | 5, 6, Conj                                                                |
| 8. $P \supset [(M \cdot N) \cdot (O \cdot P)]$                              |  |  |  | 4-7, CP                                                                   |
| 9. $O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}$                |  |  |  | 3-8, CP                                                                   |
| 10. $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$ |  |  |  | 2-9, CP                                                                   |

QED

|     |                                                     |                     |
|-----|-----------------------------------------------------|---------------------|
| 12. | 1. $(A \vee B) \supset \sim(F \bullet D)$           |                     |
|     | 2. $\sim(A \bullet \sim D)$                         |                     |
|     | 3. $\sim F \supset \sim(C \bullet D)$               |                     |
|     | 4. $C \vee A$                                       | / $A \equiv \sim C$ |
|     | 5. A                                                | ACP                 |
|     | 6. $A \vee B$                                       | 5, Add              |
|     | 7. $\sim(F \bullet D)$                              | 6, 1, 6, MP         |
|     | 8. $\sim F \vee \sim D$                             | 7, DM               |
|     | 9. $\sim A \vee \sim \sim D$                        | 2, DM               |
|     | 10. $\sim \sim D$                                   | 9, 5, DN, DM        |
|     | 11. $\sim F$                                        | 8, 10, Com, DS      |
|     | 12. $\sim(C \bullet D)$                             | 3, 11, MP           |
|     | 13. $\sim C \vee \sim D$                            | 12, DM              |
|     | 14. $\sim C$                                        | 13, 10, Com, DS     |
|     | 15. $A \supset \sim C$                              | 5-14, CP            |
|     | 16. $\sim C \supset A$                              | 4, DN, Impl         |
|     | 17. $(A \supset \sim C) \bullet (\sim C \supset A)$ | 15, 16, Conj        |
|     | 18. $A \equiv \sim C$                               | 17, Equiv           |

QED

|     |                                                            |              |
|-----|------------------------------------------------------------|--------------|
| 13. | 1. $P \bullet Q$                                           | ACP          |
|     | 2. P                                                       | 1, Simp      |
|     | 3. $P \vee R$                                              | 2, Add       |
|     | 4. Q                                                       | 1, Com, Simp |
|     | 5. $Q \vee R$                                              | 4, Add       |
|     | 6. $(P \vee R) \bullet (Q \vee R)$                         | 3, 5, Conj   |
|     | 7. $(P \bullet Q) \supset [(P \vee R) \bullet (Q \vee R)]$ | 1-6, IP      |

QED

|     |                                                       |              |
|-----|-------------------------------------------------------|--------------|
| 14. | 1. $\sim[(A \supset B) \vee (\sim A \supset C)]$      | AIP          |
|     | 2. $\sim(A \supset B) \bullet \sim(\sim A \supset C)$ | 1, DM        |
|     | 3. $\sim(A \supset B)$                                | 2, Simp      |
|     | 4. $\sim(\sim A \vee B)$                              | 3, Impl      |
|     | 5. $A \bullet \sim B$                                 | 4, DM, DN    |
|     | 6. $\sim(\sim A \supset C)$                           | 2, Com, Simp |
|     | 7. $\sim(A \vee C)$                                   | 6, Impl, DN  |
|     | 8. $\sim A \bullet \sim C$                            | 7, DM        |
|     | 9. A                                                  | 5, Simp      |
|     | 10. $\sim A$                                          | 8, Simp      |
|     | 11. $A \bullet \sim A$                                | 9, 10, Conj  |
|     | 12. $(A \supset B) \vee (\sim A \supset C)$           | 1-11, IP, DN |

QED

- 15.
- |                                                  |                 |
|--------------------------------------------------|-----------------|
| 1. $(T \supset W) \cdot (X \supset Y)$           |                 |
| 2. $T \vee X$                                    |                 |
| 3. $(T \supset \sim Y) \cdot (X \supset \sim W)$ |                 |
| 4. $(W \cdot \sim Y) \supset Z$                  |                 |
| 5. $Y \supset (W \vee A)$                        | $/ Z \vee A$    |
| 6. $W \vee Y$                                    | 1, 2, CD        |
| 7. $\sim Y \vee \sim W$                          | 3, 2, CD        |
| 8. $\sim(Z \vee A)$                              | AIP             |
| 9. $\sim Z \cdot \sim A$                         | 8, DM           |
| 10. $\sim Z$                                     | 9, Simp         |
| 11. $\sim(W \cdot \sim Y)$                       | 4, 10, MT       |
| 12. $\sim W \vee Y$                              | 11, DM, DN      |
| 13. $\sim W \vee \sim Y$                         | 7, Com          |
| 14. $(\sim W \vee Y) \cdot (\sim W \vee \sim Y)$ | 12, 13, Conj    |
| 15. $\sim W \vee (Y \cdot \sim Y)$               | 14, Dist        |
| 16. $Y \cdot \sim Y$                             | AIP             |
| 17. $\sim(Y \cdot \sim Y)$                       | 16, IP          |
| 18. $\sim W$                                     | 15, 17, Com, DS |
| 19. $Y$                                          | 6, 18, DS       |
| 20. $W \vee A$                                   | 5, 19, MP       |
| 21. $A$                                          | 20, 18, DS      |
| 22. $\sim A$                                     | 9, Com, Simp    |
| 23. $A \cdot \sim A$                             | 21, 22, Conj    |
| 24. $Z \vee A$                                   | 8-23, IP, DN    |

QED

- 16.
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|--------------------------------------------|-----------------------|
| 1. $(\sim S \vee T) \cdot (\sim S \vee U)$ |                       |
| 2. $(T \vee S) \vee U$                     | $/ T \vee (U \vee X)$ |
| 3. $\sim(T \vee U)$                        | AIP                   |
| 4. $\sim T \cdot \sim U$                   | 3, DM                 |
| 5. $\sim T$                                | 4, Simp               |
| 6. $T \vee (S \vee U)$                     | 2, Assoc              |
| 7. $S \vee U$                              | 6, 5, DS              |
| 8. $\sim U$                                | 4, Com, Simp          |
| 9. $S$                                     | 7, 8, Com, DS         |
| 10. $\sim S \vee T$                        | 1, Simp               |
| 11. $T$                                    | 10, 9, DN, DS         |
| 12. $T \cdot \sim T$                       | 11, 5, Conj           |
| 13. $T \vee U$                             | 3-12, IP, DN          |
| 14. $(T \vee U) \vee X$                    | 13, Add               |
| 15. $T \vee (U \vee X)$                    | 14, Assoc             |

QED