

Practice with Proofs (§2.1 - §2.5)

1. 1. $A \supset B$
 2. $\sim B$
 3. $\sim A \supset \sim C$ / $\sim C$
2. 1. $(A \vee B) \supset \sim C$
 2. $D \supset \sim E$
 3. $C \vee D$
 4. E / $\sim B$
3. 1. $\sim(A \cdot B)$
 2. $\sim(\sim B \vee C)$ / $\sim A \vee \sim D$
4. 1. $A \supset B$
 2. $(E \vee D) \supset \sim B$ / $A \supset (\sim E \cdot \sim D)$
5. 1. $\sim(G \cdot H) \supset I$
 2. $\sim(I \vee J)$
 3. $\sim J \supset F$ / $(F \cdot G) \cdot H$
6. 1. $P \vee O$
 2. $Q \supset \sim O$ / $\sim Q \vee P$
7. 1. $T \equiv V$
 2. $\sim T \vee \sim V$ / $\sim T$
8. 1. $(K \supset \sim M) \cdot [(L \cdot N) \supset \sim M]$
 2. $(K \vee L) \cdot (K \vee N)$ / $\sim M$
9. 1. $(X \vee Y) \cdot (Z \supset W)$
 2. $\sim(Y \cdot W)$ / $\sim(\sim X \cdot Z)$
10. 1. $X \vee Z$
 2. $Z \supset \sim Y$
 3. $X \supset Y$ / $\sim X \equiv Z$
11. 1. $A \supset B$
 2. $B \supset \sim A$
 3. $(A \vee D) \vee E$
 4. $(D \vee E) \supset F$ / F
12. 1. $G \supset (H \supset I)$
 2. $I \supset (J \cdot K)$ / $G \supset (H \supset J)$
13. 1. $(\sim L \vee M) \cdot (L \vee N)$
 2. $\sim O \supset \sim N$ / $M \vee O$
14. 1. $(P \supset Q) \cdot (R \supset S)$
 / $(P \cdot R) \supset (Q \cdot S)$
15. 1. $(T \supset W) \cdot (X \supset Y)$
 2. $T \vee X$
 3. $(T \supset \sim Y) \cdot (X \supset \sim W)$
 4. $(W \cdot \sim Y) \supset Z$
 5. $Y \supset (W \vee A)$ / $Z \vee A$

Sample Solutions

1. 1. $A \supset B$
 2. $\sim B$
 3. $\sim A \supset \sim C$ / $\sim C$
 4. $\sim A$ 1, 2, MT
 5. $\sim C$ 3, 4, MP
 QED
2. 1. $(A \vee B) \supset \sim C$
 2. $D \supset \sim E$
 3. $C \vee D$
 4. E / $\sim B$
 5. $\sim \sim E$ 4, DN
 6. $\sim D$ 2, 5, MT
 7. $D \vee C$ 3, Com
 8. C 7, 6, DS
 9. $\sim \sim C$ 8, DN
 10. $\sim(A \vee B)$ 1, 9, MT
 11. $\sim A \bullet \sim B$ 10, DM
 12. $\sim B \bullet \sim A$ 11, Com
 13. $\sim B$ 12, Simp
 QED
3. 1. $\sim(A \bullet B)$
 2. $\sim(\sim B \vee C)$ / $\sim A \vee \sim D$
 3. $\sim \sim B \bullet \sim C$ 2, DM
 4. $\sim \sim B$ 3, Simp
 5. $\sim A \vee \sim B$ 1, DM
 6. $\sim B \vee \sim A$ 5, Com
 7. $\sim A$ 6, 4, DS
 8. $\sim A \vee \sim D$ 7, Add
 QED
4. 1. $A \supset B$
 2. $(E \vee D) \supset \sim B$ / $A \supset (\sim E \bullet \sim D)$
 3. $B \supset \sim(E \vee D)$ 2, Trans, DN
 4. $A \supset \sim(E \vee D)$ 1, 3, HS
 5. $A \supset (\sim E \bullet \sim D)$ 4, DM
 QED

5. 1. $\sim(G \cdot H) \supset I$
 2. $\sim(I \vee J)$
 3. $\sim J \supset F$ / $(F \cdot G) \cdot H$
 4. $\sim I \bullet \sim J$ 2, DM
 5. $\sim I$ 4, Simp
 6. $G \cdot H$ 1, 5, MT, DN
 7. $\sim J$ 4, Com, Simp
 8. F 3, 7, MP
 9. $F \cdot (G \cdot H)$ 8, 6, Conj
 10. $(F \cdot G) \cdot H$ 9, Assoc
 QED
6. 1. $P \vee O$
 2. $Q \supset \sim O$ / $\sim Q \vee P$
 3. $\sim O \supset P$ 1, Com, DN,
 Impl
 4. $Q \supset P$ 2, 3, HS
 5. $\sim Q \supset P$ 4, Impl
 QED
7. 1. $T \equiv V$
 2. $\sim T \vee \sim V$ / $\sim T$
 3. $(T \cdot V) \vee (\sim T \cdot \sim V)$ 1, Equiv
 4. $\sim(T \cdot V)$ 2, DM
 5. $\sim T \cdot \sim V$ 3, 4, DS
 6. $\sim T$ 5, Simp
 QED
8. 1. $(K \supset \sim M) \cdot [(L \cdot N) \supset \sim M]$
 2. $(K \vee L) \cdot (K \vee N)$ / $\sim M$
 3. $K \vee (L \cdot N)$ 2, Dist
 4. $\sim M \vee \sim M$ 1, 3, CD
 5. $\sim M$ 4, Taut
 QED

9. 1. $(X \vee Y) \cdot (Z \supset W)$
 2. $\sim(Y \cdot W)$ / $\sim(\sim X \cdot Z)$
 3. $\sim Y \vee \sim W$ 2, DM
 4. $(Y \vee X) \cdot (Z \supset W)$ 1, Com
 5. $(\sim Y \supset X) \cdot (Z \supset W)$ 4, DN, Impl
 6. $(\sim Y \supset X) \cdot (\sim W \supset \sim Z)$ 5, Trans
 7. $X \vee \sim Z$ 6, 3, CD
 8. $\sim\sim(X \vee \sim Z)$ 7, DN
 9. $\sim(\sim X \cdot Z)$ 8, DM, DN

QED

10. 1. $X \vee Z$
 2. $Z \supset \sim Y$
 3. $X \supset Y$ / $\sim X \equiv Z$
 4. $\sim Y \supset \sim X$ 3, Trans
 5. $Z \supset \sim X$ 2, 4, HS
 6. $\sim X \supset Z$ 1, DN, Impl
 7. $(\sim X \supset Z) \cdot (Z \supset \sim X)$ 6, 5, Conj
 8. $\sim X \equiv Z$ 7, Equiv

QED

11. 1. $A \supset B$
 2. $B \supset \sim A$
 3. $(A \vee D) \vee E$
 4. $(D \vee E) \supset F$ / F
 5. $A \supset \sim A$ 1, 2, HS
 6. $\sim A \vee \sim A$ 5, Impl
 7. $\sim A$ 6, Taut
 8. $A \vee (D \vee E)$ 3, Assoc
 9. $\sim A \supset (D \vee E)$ 8, Dn, Impl
 10. $D \vee E$ 9, 7, MP
 11. F 4, 10, MP

QED

12. 1. $G \supset (H \supset I)$
 2. $I \supset (J \cdot K)$ / $G \supset (H \supset J)$
 3. $(G \cdot H) \supset I$ 1, Exp
 4. $\sim I \vee (J \cdot K)$ 2, Impl
 5. $(\sim I \vee J) \cdot (\sim I \vee K)$ 4, Dist
 6. $\sim I \vee J$ 5, Simp
 7. $I \supset J$ 6, Impl
 8. $(G \cdot H) \supset J$ 3, 7, HS
 9. $G \supset (H \supset J)$ 8, Impl

QED

- 13.
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|---------------------------------------|--------------|
| 1. $(\sim L \vee M) \cdot (L \vee N)$ | |
| 2. $\sim O \supset \sim N$ | / $M \vee O$ |
| 3. $\sim L \vee M$ | 1, Simp |
| 4. $L \supset M$ | 3, Impl |
| 5. $\sim M \supset \sim L$ | 4, Trans |
| 6. $L \vee N$ | 1, Com, Simp |
| 7. $\sim L \supset N$ | 6, DN, Impl |
| 8. $\sim M \supset N$ | 5, 7, HS |
| 9. $N \supset O$ | 2, Trans |
| 10. $\sim M \supset O$ | 8, 9, HS |
| 11. $M \vee O$ | 10, Impl, DN |

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- 14.
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|--|-------------------------------------|
| 1. $(P \supset Q) \cdot (R \supset S)$ | / $(P \cdot R) \supset (Q \cdot S)$ |
| 2. $P \supset Q$ | 1, Simp |
| 3. $\sim P \vee Q$ | 2, Impl |
| 4. $(\sim P \vee Q) \vee \sim R$ | 3, Add |
| 5. $\sim P \vee (Q \vee \sim R)$ | 4, Assoc |
| 6. $R \supset S$ | 1, Com, Simp |
| 7. $\sim R \vee S$ | 6, Impl |
| 8. $\sim P \vee (\sim R \vee S)$ | 7, Add, Com |
| 9. $[\sim P \vee (Q \vee \sim R)] \cdot [\sim P \vee (\sim R \vee S)]$ | 5, 8, Conj |
| 10. $\sim P \vee [(Q \vee \sim R) \cdot (\sim R \vee S)]$ | 9, Dist |
| 11. $\sim P \vee [(\sim R \vee Q) \cdot (\sim R \vee S)]$ | 10, Com |
| 12. $\sim P \vee [\sim R \vee (Q \cdot S)]$ | 11, Dist |
| 13. $P \supset [R \supset (Q \cdot S)]$ | 12, Impl, Impl |
| 14. $(P \cdot R) \supset (Q \cdot S)$ | 13, Exp |

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- 15.
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|---|-----------------|
| 1. $(T \supset W) \cdot (X \supset Y)$ | |
| 2. $T \vee X$ | |
| 3. $(T \supset \sim Y) \cdot (X \supset \sim W)$ | |
| 4. $(W \cdot \sim Y) \supset Z$ | |
| 5. $Y \supset (W \vee A)$ | / $Z \vee A$ |
| 6. $W \vee Y$ | 1, 2, CD |
| 7. $\sim Y \vee \sim W$ | 3, 2, CD |
| 8. $\sim W \supset Y$ | 6, DN, Impl |
| 9. $Y \supset \sim W$ | 7, Impl |
| 10. $(\sim W \supset Y) \cdot (Y \supset \sim W)$ | 8, 9, Conj |
| 11. $\sim W \equiv Y$ | 10, Equiv |
| 12. $(W \cdot \sim Y) \vee (\sim W \cdot Y)$ | 11, Equiv, DN |
| 13. $\sim Y \vee (W \vee A)$ | 5, Impl |
| 14. $(\sim Y \vee W) \vee A$ | 13, Assoc |
| 15. $\sim(\sim Y \vee W) \supset A$ | 14, Impl, DN |
| 16. $(\sim W \cdot Y) \supset A$ | 15, DM, DN, Com |
| 17. $[(W \cdot \sim Y) \supset Z] \cdot [(\sim W \cdot Y) \supset A]$ | 4, 16, Conj |
| 18. $Z \vee A$ | 17, 12, CD |

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