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**Philosophy 240**  
***Symbolic Logic***

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Class 2: Translation and Wffs  
(§1.3-§1.4)

# Compositionality

Gabriel García Márquez  
from “The Last Voyage of the Ghost Ship”

Now they're going to see who I am, he said to himself in his strong new man's voice, many years after had seen the huge ocean liner without lights **and** without any sound which passed by the village one night like a great uninhabited palace, longer than the whole village and much taller than the steeple of the church, **and** it sailed by in the darkness toward the colonial city on the the other side of the bay that had been fortified against buccaneers, with its old slave port and the rotating light, whose gloomy beams transfigured the village into a lunar encampment of glowing houses **and** streets of volcanic deserts every fifteen seconds...

# Five Connectives

Identified by syntax (shape)

- Squiggle  $\sim$
- Dot  $\bullet$
- Wedge  $\vee$
- Hook  $\supset$ 
  - ▶ or horseshoe
- Triple-bar  $\equiv$

# Five Connectives

By logical operations

- Negation  $\sim$
- Conjunction  $\cdot$
- Disjunction  $\vee$
- Material Implication  $\supset$
- Material Biconditional  $\equiv$

# Negation

a unary operator

- Some negation indicators
  - ▶ Not
  - ▶ It is not the case that
  - ▶ It is not true that
  - ▶ It is false that
- John will take the train
  - ▶ John won't take the train.
  - ▶ It's not the case that John will take the train.
  - ▶ John takes the train...not!
- Sample Negations
  - ▶  $\sim R$
  - ▶  $\sim(P \cdot Q)$
  - ▶  $\sim\{(A \vee B) \supset C\} \cdot \sim D$

# Conjunction

- Some conjunction indicators
  - ▶ and
  - ▶ but
  - ▶ also
  - ▶ however
  - ▶ yet
  - ▶ still
  - ▶ moreover
  - ▶ although
  - ▶ nevertheless
  - ▶ both
- Some English sentences which we can represent as conjunctions.
  - ▶ Angelina walks the dog and Brad cleans the floors.
  - ▶ Although Angelina walks the dog, Brad cleans the floors.
  - ▶ Bob and Ray are comedians.
    - Not: Bob and Ray are brothers.
  - ▶ Carolyn is nice, but Emily is really nice.
- Sample Conjunctions
  - ▶  $P \cdot \sim Q$
  - ▶  $(A \supset B) \cdot (B \supset A)$
  - ▶  $(P \vee \sim Q) \cdot \sim[P \equiv (Q \cdot R)]$

# Disjunction

- Some disjunction indicators
  - ▶ or
  - ▶ either
  - ▶ unless
- Some English sentences which we can represent as disjunctions.
  - ▶ Either Paco makes the Website, or Matt does.
  - ▶ Jared or Rene will go to the party.
  - ▶ Justin doesn't feed the kids unless Carolyn asks him to.
- Sample Disjunctions
  - ▶  $\sim P \vee Q$
  - ▶  $(A \supset B) \vee (B \supset A)$
  - ▶  $(P \vee \sim Q) \vee \sim [P \equiv (Q \bullet R)]$

# Material Implication (The Conditional)

- Some material implication indicators
  - ▶ if
  - ▶ only if
  - ▶ only when
  - ▶ is a necessary condition for
  - ▶ is a sufficient condition for
  - ▶ implies
  - ▶ entails
  - ▶ provided that
  - ▶ given that
  - ▶ on the condition that
  - ▶ in case



# Translating Conditionals

A: You join me; B: I go to the movies

- |   |                            |
|---|----------------------------|
| 1. If you join me, then I go to the movies.                   | 1. If A then B             |
| 2. You join me if I go to the movies.                         | 2. If B then A             |
| 3. You join me only if (only when) I go to the movies.        | 3. A only if (only when) B |
| 4. Your joining me is a necessary condition for my going.     | 4. A is necessary for B    |
| 5. Your joining me is a sufficient condition for my going.    | 5. A is sufficient for B   |
| 6. A necessary condition of your joining me is my going.      | 6. B is necessary for A    |
| 7. A sufficient condition for your joining me is my going.    | 7. B is sufficient for A   |
| 8. Your joining me entails (implies) that I go to the movies. | 8. A entails (implies) B   |
| 9. You join me given (provided, on the condition) that I go.  | 9. A given B               |

# Necessary and Sufficient Conditions

- Sufficient conditions are antecedents
- Necessary conditions are consequents
- SUN
- “I just love basketball. Playing ball is sufficient for my happiness.”
  - $P \supset H$
- “I only go with you if you buy my ticket. Your buying the ticket is necessary for me to go.”
  - $G \supset B$

# Sample Conditionals

- $\sim P \supset Q$
- $(A \supset B) \supset (B \supset A)$
- $(P \vee \sim Q) \supset \sim [P \equiv (Q \cdot R)]$

# The Material Biconditional

- Some biconditional indicators
  - ▶ if and only if
  - ▶ is a necessary and sufficient condition for
  - ▶ just in case.
- 'A  $\equiv$  B' is short for '(A  $\supset$  B)  $\cdot$  (B  $\supset$  A)'
- An English sentence we can represent as a biconditional
  - ▶ You'll be successful just in case you work hard and are lucky.
- Sample biconditionals
  - ▶  $\sim P \equiv Q$
  - ▶  $(A \supset B) \equiv (B \supset A)$
  - ▶  $(P \vee \sim Q) \equiv \sim[P \equiv (Q \cdot R)]$

# Ambiguous Cases

You may have salad or potatoes and carrots.

- $(S \vee P) \cdot C$
- $S \vee (P \cdot C)$
- You may have salad or potatoes, and carrots.
  - $(S \vee P) \cdot C$
- You may have salad, or potatoes and carrots.
  - $S \vee (P \cdot C)$

# Syntax of PL

- Capital English letters, used as propositional variables
  - A ... Z
- Five connectives
  - $\sim$ ,  $\bullet$ ,  $\vee$ ,  $\supset$ ,  $\equiv$
- Punctuation
  - (, ), [, ], {, }

# Wffs

- baker and aebkr
- Some wffs
  - ▶  $P \cdot Q$
  - ▶  $(\sim P \vee Q) \supset \sim R$
- Not wffs
  - ▶  $\cdot P Q$
  - ▶  $Pq \vee R\sim$

# Formation rules for wffs

1. A single capital English letter is a wff.
2. If  $\alpha$  is a wff, so is  $\sim\alpha$ .
3. If  $\alpha$  and  $\beta$  are wffs, then so are:
  - $(\alpha \cdot \beta)$
  - $(\alpha \vee \beta)$
  - $(\alpha \supset \beta)$
  - $(\alpha \equiv \beta)$
  - ▶ By convention, you may drop the outermost brackets.
4. These are the only ways to make wffs.



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# HW for Wednesday

- Translation from English to PL and back
- wffs