

Solutions to Practice Problems for Test #6

I. Translations

1.  $(\forall x)(Fx \supset Px)$
2.  $(\forall x)[(Gx \cdot Fx) \supset Px]$
3.  $(\forall x)(Fx \supset Mnx)$
4.  $(\forall x)(Wpx \supset Mnx)$
5.  $(\forall x)(Sxn \supset Mnx)$
6.  $(\forall x)(\sim Mxx \supset Mnx)$
7.  $(\forall x)(\forall y)(Sxy \supset \sim Syx)$
8.  $(\forall x)\{(Fx \cdot Px) \supset (\exists y)[(Gy \cdot Py) \cdot Rxy]\} \supset (\exists x)[Gx \cdot (\forall y)(Fy \supset Sxy)]$
9.  $Bam \cdot (\forall x)(Bxm \supset x=a)$   
 $(\exists x)(Nx \cdot Wex)$   
 $\sim(\exists x)(Nx \cdot Wax) \quad / \sim Bem$
10.  $(\forall x)(\forall y)(Rxy \supset \sim x=y) \quad / (\forall x)\sim Rxx$
11.  $(\forall x)(\forall y)(\forall z)(x=y \vee y=z \vee x=z)$   
 $(\exists x)(\sim x=c \cdot Hx) \quad / (\exists x)(\exists y)[\sim x=y \cdot (\forall z)(z=x \vee z=y)]$
12.  $(\exists x)[Bxc \cdot (\forall y)(Byc \supset y=x) \cdot Hx] \quad / (\exists x)(Bxc)$
13.  $(\forall x)[(\sim x=m \cdot \sim x=b) \supset Hx]$   
 $\sim Ha \quad / a=m \vee a=b$
14.  $Ff(b) \cdot \sim Fg(c)$
15.  $\sim(\exists x)Bxf(c,e)$
16.  $Gc \supset \{(Hg(c) \cdot Rg(c)) \cdot [Ff(c) \cdot (\exists x)(Nx \cdot Wf(c)x)]\}$
17.  $(\exists X)(Xn \cdot \sim Xp)$
18.  $(\forall x)(\forall y)[(Rx \cdot Ry) \supset (\exists X)(Xx \cdot Xy)]$
19.  $(\exists X)\{(\forall x)(\forall y)(\forall z)[(Xxy \cdot Xyz) \supset Xxz] \cdot (\forall x)(\forall y)(Xxy \supset \sim Xyx)\}$
20.  $(\forall x)x=x \quad / (\exists X)Xxx$

II. Derivations

Note: These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

1.  $(\forall x)(\exists y)(\sim Fx \vee Gy) \quad / (\forall x)Fx \supset (\exists y)Gy$ 

2. $(\forall x)Fx$	ACP
3. $(\exists y)(\sim Fx \vee Gy)$	1, UI
4. $\sim Fx \vee Ga$	3, EI
5. $Fx$	2, UI
6. $Ga$	4, 5, DN, DS
7. $(\exists y)Gy$	6, EG
8.  $(\forall x)Fx \supset (\exists y)Gy \quad / 2-7 \text{ CP}$

QED

2. 1.  $(\forall x)(\exists y)Fxy \supset (\forall x)(\exists y)Gxy$   
 2.  $(\exists x)(\forall y)\sim Gxy$  /  $(\exists x)(\forall y)\sim Fxy$   
 3.  $\sim(\forall x)(\exists y)Gxy$  2, QE, QE  
 4.  $\sim(\forall x)(\exists y)Fxy$  1, 3, MT  
 5.  $(\exists x)(\forall y)\sim Fxy$  4, QE, QE

QED

3. 1.  $(\forall x)[(Fx \vee Gx) \supset (Hx \cdot Kx)]$   
 2.  $(\forall x)\{(Hx \vee Lx) \supset [(Hx \cdot Nx) \supset Px]\}$  /  $(\forall x)[Fx \supset (Nx \supset Px)]$   
     | 3. Fx ACP  
     | 4.  $Fx \vee Gx$  3, Add  
     | 5.  $(Fx \vee Gx) \supset (Hx \cdot Kx)$  1, UI  
     | 6.  $Hx \cdot Kx$  5, 4, MP  
     | 7. Hx 6, Simp  
     | 8.  $Hx \vee Lx$  7, Add  
     | 9.  $(Hx \vee Lx) \supset [(Hx \cdot Nx) \supset Px]$  2, UI  
     | 10.  $(Hx \cdot Nx) \supset Px$  9, 8, MP  
         | 11. Nx ACP  
         | 12.  $Hx \cdot Nx$  7, 11, Conj  
         | 13. Px 10, 12, MP  
     | 14.  $Nx \supset Px$  11-13, CP  
 15.  $Fx \supset (Nx \supset Px)$  3-14, CP  
 16.  $(\forall x)[Fx \supset (Nx \supset Px)]$  15, UG

QED

4. 1.  $\sim(\exists x)(Axa \cdot \sim Bxb)$   
 2.  $\sim(\exists x)(Dxd \cdot Dbx)$   
 3.  $(\forall x)(Bex \supset Dxd)$  /  $\sim(Aea \cdot Dgd)$   
     | 4.  $Aea \cdot Dgd$  AIP  
     | 5.  $(\forall x)\sim(Axa \cdot \sim Bxb)$  1, QE  
     | 6.  $(\forall x)(\sim Axa \vee Bxb)$  5, DM, DN  
     | 7.  $\sim Aea \vee Beb$  6, UI  
     | 8. Aea 4, Simp  
     | 9. Beb 7, 8, DN, DS  
     | 10.  $(\forall x)\sim(Dxd \cdot Dbx)$  2, QE  
     | 11.  $(\forall x)(\sim Dxd \vee \sim Dbx)$  10, DM  
     | 12.  $\sim Dgd \vee \sim Dbg$  11, UI  
     | 13. Dgd 4, Com, Simp  
     | 14.  $\sim Dbg$  12, 13, DN, DS  
     | 15.  $Beb \supset Dbg$  3, UI  
     | 16. Dbg 15, 9, MP  
     | 17.  $Dbg \cdot \sim Dbg$  16, 14, Conj  
 18.  $\sim(Aea \cdot Dgd)$  4-17, IP

QED

- 5.
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|----|--|--|--|
| 1. | $(\forall x)(Ax \supset Bx)$   |  | $/ (\forall x)[(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)]$ |
|    | 2. $(\exists y)(Ay \cdot Cxy)$   |  | ACP  |
|    | 3. $Aa \cdot Cxa$  |  | 2, EI  |
|    | 4. $Aa$  |  | 3, Simp  |
|    | 5. $Aa \supset Ba$   |  | 1, UI  |
|    | 6. $Ba$  |  | 5, 4, MP   |
|    | 7. $Cxa$   |  | 3, Com, Simp   |
|    | 8. $Ba \cdot Cxa$  |  | 6, 7, Conj   |
|    | 9. $(\exists z)(Bz \cdot Cxz)$   |  | 8, EG  |
|    | 10. $(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)$              |  | 2-9, CP  |
|    | 11. $(\forall x)[(\exists y)(Ay \cdot Cxy) \supset (\exists z)(Bz \cdot Cxz)]$ |  | 10, UG   |

QED

- 6.
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|----|---|--|--------------|
| 1. | $(\exists x)(Nx \cdot Wjx \cdot Ix)$                            |  | $/ Ic$       |
|    | 2. $Nc \cdot Wjc \cdot (\forall x)[(Nx \cdot Wjx) \supset x=c]$ |  | 1, EI        |
|    | 3. $Na \cdot Wja \cdot Ia$                                      |  | 2, Com, Simp |
|    | 4. $(\forall x)[(Nx \cdot Wjx) \supset x=c]$                    |  | 4, UI        |
|    | 5. $(Na \cdot Wja) \supset a=c$                                 |  | 3, Simp      |
|    | 6. $Na \cdot Wja$   |  | 5, 6, MP     |
|    | 7. $a=c$  |  | 3, Com, Simp |
|    | 8. $Ia$   |  | 8, 7, IDi    |
|    | 9. $Ic$   |  |              |

QED

- 7.
- |    |  |  |  |
|----|--|--|--|
| 1. | $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot y \neq x) \supset Hxy]\}$    |  | $/ (\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot \sim Ty) \supset Hxy]\}$ |
|    | 2. $Ma \cdot Ta \cdot (\forall y)[(My \cdot \sim y=a) \supset Hay]$                |  | 1, EI  |
|    | 3. $My \cdot \sim Ty$  |  | ACP  |
|    | 4. $(\forall y)[(My \cdot \sim y=a) \supset Hay]$                                  |  | 2, Com, Simp   |
|    | 5. $(My \cdot \sim y=a) \supset Hay$   |  | 4, UI  |
|    | 6. $y=a$   |  | AIP  |
|    | 7. $Ta$  |  | 2, Simp  |
|    | 8. $\sim Ty$   |  | 3, Com, Simp   |
|    | 9. $Ty$  |  | 7, 6, IDi  |
|    | 10. $Ty \cdot \sim Ty$   |  | 9, 8, Conj   |
|    | 11. $\sim y=a$   |  | 6-10, IP   |
|    | 12. $My$   |  | 3, Simp  |
|    | 13. $My \cdot \sim y=a$  |  | 12, 11, Conj   |
|    | 14. $Hay$  |  | 5, 13, MP  |
|    | 15. $(My \cdot \sim Ty) \supset Hay$   |  | 3-14, CP   |
|    | 16. $(\forall y)[(My \cdot \sim Ty) \supset Hay]$                                  |  | 15, UG   |
|    | 17. $Ma \cdot Ta$  |  | 2, Simp  |
|    | 18. $Ma \cdot Ta \cdot (\forall y)[(My \cdot \sim Ty) \supset Hay]$                |  | 17, 16, Conj   |
|    | 19. $(\exists x)\{Mx \cdot Tx \cdot (\forall y)[(My \cdot \sim Ty) \supset Hxy]\}$ |  | 18, EG   |

QED

8. 1.  $(\forall x)(\forall y)(\forall z)[(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (x=y \vee y=z \vee x=z)]$   
 2.  $(\exists x)(\exists y)(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Rx \cdot Ry \cdot x \neq y)$   
 3.  $(\forall x)(Rx \supset \sim Cx)$  /  $(Sa \cdot Ca) \supset \sim La$
- |   |                        |                 |
|---|------------------------|-----------------|
| 4. $Sa \cdot Ca$  |                        | ACP             |
| 5. $La$   |                        | AIP             |
| 6. $(\exists y)(Sb \cdot Lb \cdot Sy \cdot Ly \cdot Rb \cdot Ry \cdot b \neq y)$                                |                        | 3, EI           |
| 7. $Sb \cdot Lb \cdot Sc \cdot Lc \cdot Rb \cdot Rc \cdot b \neq c$   |                        | 6, EI           |
| 8. $Sb \cdot Lb \cdot Sc \cdot Lc$  |                        | 7, Simp         |
| 9. $Sa$   |                        | 4, Simp         |
| 10. $Sa \cdot La$   |                        | 9, 5, Conj      |
| 11. $Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc$   |                        | 10, 8, Conj     |
| 12. $(\forall y)(\forall z)[(Sa \cdot La \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (a=y \vee y=z \vee a=z)]$ |                        | 1, UI           |
| 13. $(\forall z)[(Sa \cdot La \cdot Sb \cdot Lb \cdot Sz \cdot Lz) \supset (a=b \vee b=z \vee a=z)]$            |                        | 12, UI          |
| 14. $(Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc) \supset (a=b \vee b=c \vee a=c)$                         |                        | 13, UI          |
| 15. $a=b \vee b=c \vee a=c$   |                        | 14, 11, MP      |
| 16. $\sim b=c$  |                        | 7, Simp         |
| 17. $a=b \vee a=c$  |                        | 17, 16, Com, DS |
| 18. $Ra \supset \sim Ca$  |                        | 3, UI           |
| 19. $Ca$  |                        | 4, Com, Simp    |
| 20. $\sim Ra$   |                        | 18, 19, DN, MT  |
| 21. $Rb$  |                        | 7, Simp         |
|   | 22. $a=b$              | AIP             |
|   | 23. $\sim Rb$          | 20, 22, IDi     |
|   | 24. $Rb \cdot \sim Rb$ | 21, 24, Conj    |
| 25. $\sim a=b$  |                        | 22-24, IP       |
| 26. $a=c$   |                        | 17, 25, DS      |
| 27. $Rc$  |                        | 7, Simp         |
| 28. $Rc \supset \sim Cc$  |                        | 3, UI           |
| 29. $\sim Cc$   |                        | 28, 27, MP      |
| 30. $Cc$  |                        | 19, 26, IDi     |
| 31. $Cc \cdot \sim Cc$  |                        | 30, 29, Conj    |
| 33. $\sim La$   |                        | 5-31, IP        |
| 34. $(Sa \cdot Ca) \supset \sim La$   |                        | 4-33, CP        |

QED

9. 1.  $(\forall x)(\forall y)f(x,y)=f(y,x)$   
 2.  $(\forall x)f(x,o)=o$  /  $(\forall x)f(o,x)=o$   
 3.  $f(x,o)=o$  2, UI  
 4.  $(\forall y)f(o,y)=f(y,o)$  1, UI  
 5.  $f(o,x)=f(x,o)$  4, UI  
 6.  $f(o,x)=o$  5, 3, IDi  
 7.  $(\forall x)f(o,x)=o$  6, UG

QED

- 10.
- |   |                       |
|---|-----------------------|
| 1. $(\forall x)(\forall y)(Gxy \equiv Lyx)$               |                       |
| 2. $(\forall x)Gf(x)x$                                    | $/ (\forall x)Lxf(x)$ |
| 3. $(\forall y)(Gf(x)y \equiv Lyf(x))$                    | 1, UI                 |
| 4. $Gf(x)x \equiv Lxf(x)$                                 | 3, UI                 |
| 5. $Gf(x)x$   | 2, UI                 |
| 6. $[Gf(x)x \supset Lxf(x)] \cdot [Lxf(x) \equiv Gf(x)x]$ | 4, Equiv              |
| 7. $Gf(x)x \supset Lxf(x)$                                | 6, Simp               |
| 8. $Lxf(x)$   | 7, 5, MP              |
| 9. $(\forall x)Lxf(x)$                                    | 8, UG                 |

QED

- 11.
- |  |   |
|--|---|
| 1. $(\forall x)(\forall y)(\exists z)Sf(x)yz$                                    |   |
| 2. $(\forall x)(\forall y)(\forall z)[Sxyz \supset \sim(Cxyz \vee Mzyx)]$        | $/ (\exists x)(\exists y)(\exists z)\sim Mzg(y)f(g(x))$ |
| 3. $(\forall y)(\exists z)Sf(g(x))yz$  | 1, UI   |
| 4. $(\exists z)Sf(g(x))g(y)z$  | 3, UI   |
| 5. $Sf(g(x))g(y)a$   | 4, EI   |
| 6. $(\forall y)(\forall z)[Sf(g(x))yz \supset \sim(Cf(g(x))yz \vee Mzyf(g(x)))]$ | 2, UI   |
| 7. $(\forall z)[Sf(g(x))g(y)z \supset \sim(Cf(g(x))g(y)z \vee Mzg(y)f(g(x)))]$   | 6, UI   |
| 8. $Sf(g(x))g(y)a \supset \sim(Cf(g(x))g(y)a \vee Mag(y)f(g(x)))$                | 7, UI   |
| 9. $\sim(Cf(g(x))g(y)a \vee Mag(y)f(g(x)))$                                      | 8, 5, MP  |
| 10. $\sim Cf(g(x))g(y)a \cdot \sim Mag(y)f(g(x))$                                | 9, DM   |
| 11. $\sim Mag(y)f(g(x))$   | 10, Com, Simp   |
| 12. $(\exists z)\sim Mzg(y)f(g(x))$  | 11, EG  |
| 13. $(\exists y)(\exists z)\sim Mzg(y)f(g(x))$                                   | 12, EG  |
| 14. $(\exists x)(\exists y)(\exists z)\sim Mzg(y)f(g(x))$                        | 13, EG  |

QED