Solutions to Practice Problems for Test #5

These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

I. Derivations

1.
1.	Ab ∨ Bc	
2.	(∀x)~Bx	/ (∃x)Ax
3.	~Bc	2, UI
4.	Ab	1,3 Com, DS
5.	(∃x)Ax	4, EG

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2.
1.	(∀x)(Rx ⊃ Ox)	
2.	(∃x)~Ox	
3.	(∀x)(~Rx ⊃ Px)	/ (∃x)Px
4.	~Oa	2, EI
5.	Ra ⊃ Oa	1, UI
6.	~Ra	5, 4, MT
7.	~Ra ⊃ Pa	3, UI
8.	Pa	7, 6, MP
9.	(∃x)Px	8, EG

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3.
1.	(∀x)(Fx ⊃ Gx)	
2.	(∀y)(Gy ⊃ Hy)	/ (∀z)(~Hz ⊃ ~Fz)
3.	Fx ⊃ Gx	1, UI
4.	Gx ⊃ Hx	2, UI
5.	Fx ⊃ Hx	3, 4, HS
6.	~Hx ⊃ ~Fx	5, Cont
7.	(∀z)(~Hz ⊃ ~Fz)	6, UG

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4.
1.	(∃x)(Ax · Bx) ⊃ (∀x) Dx	
2.	~Da	/ (∀x)(Ax ⊃ ~Bx)
3.	(∃x)~Dx	2, EG
4.	~(∀x)Dx	3, QE
5.	~(∃x)(Ax · Bx)	1, 4, MT
6.	(∀x)~(Ax · Bx)	5, QE
7.	(∀x)(~Ax ∨ ~Bx)	6, DM
8.	(∀x)(Ax ⊃ ~Bx)	7, Impl

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5.

1.	(∀y)[Ay ⊃ (~By ⊃ Dy)]	
2.	~Ba	/ Aa ⊃ Da
| 3.	Aa	ACP
| 4.	Aa ⊃ (~Ba ⊃ Da)	1, UI
| 5.	~Ba ⊃ Da	4, 3, MP
| 6.	Da	5, 2, MP
7.	Aa ⊃ Da	3-6, CP

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6.

1.	(∀x)(Qx ⊃ ~Px)	/ (∃x)Px ⊃ ~(∀x)Qx
| 2.	(∃x)Px	ACP
| 3.	Pa	2, EI
| 4.	Qa ⊃ ~Pa	1, UI
| 5.	~Qa	4, 3, DN, MT
| 6.	(∃x)~Qx	5, EG
| 7.	~(∀x)Qx	6, QE
8.	(∃x)Px ⊃ ~(∀x)Qx	2-7, CP

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7.

1.	(∀x)[Ax ⊃ (Bx · Dx)]	
2.	(∀x)[(Ax · Dx) ⊃ Ex]	
3.	(∀x)(Ex ⊃ ~Dx)	/ ~Aa
| 4.	Aa	AIP
| 5.	Aa ⊃ (Ba · Da)	1, UI
| 6.	Ba · Da	5, 4, MP
| 7.	Da	6, Com, Simp
| 8.	Aa · Da	4, 7, Conj
| 9.	(Aa · Da) ⊃ Ea	2, UI
| 10.	Ea	9, 8, MP
| 11.	Ea ⊃ ~Da	3, UI
| 12.	~Da	11, 10, MP
| 13.	Da · ~Da	7, 12, Conj
14.	~Aa	4-13, CP

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8.

1.	(∀x)(Ax ⊃ Bx)	
2.	(∀x)[Bx ⊃ (Ax ⊃ ~Fx)]	
3.	(∀x)[(~Cx · Dx) ⊃ Fx]	/ (∀x)[Ax ⊃ (Cx ∨ ~Dx)]
| 4.	Ax	ACP
| 5.	Ax ⊃ Bx	1, UI
| 6.	Bx	5, 4, MP
| 7.	Bx ⊃ (Ax ⊃ ~Fx)	2, UI
| 8.	Ax ⊃ ~Fx	7, 6, MP
| 9.	~Fx	8, 4, MP
| 10.	(~Cx · Dx) ⊃ Fx	3, UI
| 11.	~(~Cx · Dx)	10, 9, MT
| 12.	Cx ∨ ~Dx	11, DM, DN
13.	Ax ⊃ (Cx ∨ ~Dx)	4-12, CP
14.	(∀x)[Ax ⊃ (Cx ∨ ~Dx)]	13, UG

QED

9.

1.	(∃x)Gx ⊃ (∀x)(Fx ⊃ Dx)	
2.	(∃x)(Gx · ~Dx)	/ ~(∀x)Fx
3.	Ga · ~Da	2, EI
4.	Ga	3, Simp
5.	(∃x)Gx	4, EG
6.	(∀x)(Fx ⊃ Dx)	1, 5, MP
7.	Fa ⊃ Da	6, UI
8.	~Da	3, Com, Simp
9.	~Fa	7, 8, MT
10.	(∃x)~Fx	9, EG
11.	~(∀x)Fx	10, QE

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10.

1.	(∃x)Qx ⊃ (∀x)(Rx ⊃ Sx)	
2.	(∀x)~Qx ⊃ (∃x)Sx	
3.	(∀x) Rx	/ (∃x)Sx
| 4.	~(∃x)Sx	AIP
| 5.	~(∀x)~Qx	2, 4, MT
| 6.	(∃x)Qx	5, QE
| 7.	(∀x)(Rx ⊃ Sx)	1, 6, MP
| 8.	Rx ⊃ Sx	7, UI
| 9.	Rx	3, UI
| 10.	Sx	8, 9, MP
| 11.	(∃x)Sx	10, EG
| 12.	(∃x)Sx · ~(∃x)Sx	11, 4, Conj
13.	(∃x)Sx	4-12, IP, DN

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II. Invalidity

1. Invalid in a 1-member universe, where:
Aa: True; Ba: False; Ca: False.

2. Invalid in a 2-member universe, where:
Ea: True; Fa: False; Ga: False;
Eb: True or False; Fb: True; Gb: True

3. Invalid in a 3-member universe, where:
Pa: True; Qa: False; Ra: False
Pb: False; Qb: True; Rb: False
Pc: False; Qc: False; Rc: True or False

Alternate solutions are possible.