Philosophy 240: Symbolic Logic
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Reference Sheeet for *What Follows*
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Names of Languages

PL: Propositional Logic
M: Monadic (First-Order) Predicate Logic
F: Full (First-Order) Predicate Logic
FF: Full (First-Order) Predicate Logic with functors
S: Second-Order Predicate Logic

Basic Truth Tables

~	α
0	1
1	0

α	·	β
1	1	1
1	0	0
0	0	1
0	0	0

α	∨	β
1	1	1
1	1	0
0	1	1
0	0	0

α	⊃	β
1	1	1
1	0	0
0	1	1
0	1	0

α	≡	β
1	1	1
1	0	0
0	0	1
0	1	0

Rules of Inference

Modus Ponens (MP)
α ⊃ β
α / β

Modus Tollens (MT)
α ⊃ β
~β / ~α

Disjunctive Syllogism (DS)
α ∨ β
~α / β

Hypothetical Syllogism (HS)
α ⊃ β
β ⊃ γ / α ⊃ γ

Conjunction (Conj)
α
β / α · β

Addition (Add)
α / α ∨ β

Simplification (Simp)
α · β / α

Constructive Dilemma (CD)
(α ⊃ β) · (γ ⊃ δ)
α ∨ γ / β ∨ δ

Rules of Equivalence

DeMorgan's Laws (DM)

$\sim(\alpha \cdot \beta) \rightleftarrows \sim\alpha \lor \sim\beta$

$\sim(\alpha \lor \beta) \rightleftarrows \sim\alpha \cdot \sim\beta$

Association (Assoc)

$\alpha \lor (\beta \lor \gamma) \rightleftarrows (\alpha \lor \beta) \lor \gamma$

$\alpha \cdot (\beta \cdot \gamma) \rightleftarrows (\alpha \cdot \beta) \cdot \gamma$

Distribution (Dist)

$\alpha \cdot (\beta \lor \gamma) \rightleftarrows (\alpha \cdot \beta) \lor (\alpha \cdot \gamma)$

$\alpha \lor (\beta \cdot \gamma) \rightleftarrows (\alpha \lor \beta) \cdot (\alpha \lor \gamma)$

Commutativity (Com)

$\alpha \lor \beta \rightleftarrows \beta \lor \alpha$

$\alpha \cdot \beta \rightleftarrows \beta \cdot \alpha$

Double Negation (DN)

$\alpha \rightleftarrows \sim\sim\alpha$

Contraposition (Cont)

$\alpha \supset \beta \rightleftarrows \sim\beta \supset \sim\alpha$

Material Implication (Impl)

$\alpha \supset \beta \rightleftarrows \sim\alpha \lor \beta$

Material Equivalence (Equiv)

$\alpha \equiv \beta \rightleftarrows (\alpha \supset \beta) \cdot (\beta \supset \alpha)$

$\alpha \equiv \beta \rightleftarrows (\alpha \cdot \beta) \lor (\sim\alpha \cdot \sim\beta)$

Exportation (Exp)

$\alpha \supset (\beta \supset \gamma) \rightleftarrows (\alpha \cdot \beta) \supset \gamma$

Tautology (Taut)

$\alpha \rightleftarrows \alpha \cdot \alpha$

$\alpha \rightleftarrows \alpha \lor \alpha$

Six Derived Rules for the Biconditional

Rules of Inference

Biconditional Modus Ponens (BMP)

$\alpha \equiv \beta$

$\alpha \quad / \beta$

Biconditional Modus Tollens (BMT)

$\alpha \equiv \beta$

$\sim\alpha \quad / \sim\beta$

Biconditional Hypothetical Syllogism (BHS)

$\alpha \equiv \beta$

$\beta \equiv \gamma \quad / \alpha \equiv \gamma$

Rules of Equivalence

Biconditional DeMorgan's Law (BDM)

$\sim(\alpha \equiv \beta) \rightleftarrows \sim\alpha \equiv \beta$

Biconditional Commutativity (BCom)

$\alpha \equiv \beta \rightleftarrows \beta \equiv \alpha$

Biconditional Contraposition (BCont)

$\alpha \equiv \beta \rightleftarrows \sim\alpha \equiv \sim\beta$

Rules for Quantifier Instantiation and Generalization

Universal Instantiation (UI)

$\underline{(\forall\alpha)\mathscr{F}\alpha}$	for any variable α, any formula $\mathscr{F}$ containing α, and
$\mathscr{F}\beta$	any singular term β

Universal Generalization (UG)

$\underline{\mathscr{F}\beta}$	for any variable β, any formula $\mathscr{F}$ containing β, and
$(\forall\alpha)\mathscr{F}\alpha$	for any variable α

Never UG within the scope of an assumption for conditional or indirect proof on a variable that is free in the first line of the assumption.

Never UG on a variable when there is a constant present, and the variable was free when the constant was introduced.

Existential Generalization (EG)

$\underline{\mathscr{F}\beta}$	for any singular term β, any formula $\mathscr{F}$ containing β, and
$(\exists\alpha)\mathscr{F}\alpha$	for any variable α

Existential Instantiation (EI)

$\underline{(\exists\alpha)\mathscr{F}\alpha}$	for any variable α, any formula $\mathscr{F}$ containing α, and
$\mathscr{F}\beta$	any *new* constant β

Quantifier Equivalence (QE)

$(\forall\alpha)\mathscr{F}\alpha$	$\rightleftarrows$	$\sim(\exists\alpha)\sim\mathscr{F}\alpha$
$(\exists\alpha)\mathscr{F}\alpha$	$\rightleftarrows$	$\sim(\forall\alpha)\sim\mathscr{F}\alpha$
$(\forall\alpha)\sim\mathscr{F}\alpha$	$\rightleftarrows$	$\sim(\exists\alpha)\mathscr{F}\alpha$
$(\exists\alpha)\sim\mathscr{F}\alpha$	$\rightleftarrows$	$\sim(\forall\alpha)\mathscr{F}\alpha$

Rules of Passage

For all variables α and all formulas Γ and Δ:

RP1:	$(\exists\alpha)(\Gamma \vee \Delta)$	⇄	$(\exists\alpha)\Gamma \vee (\exists\alpha)\Delta$
RP2:	$(\forall\alpha)(\Gamma \bullet \Delta)$	⇄	$(\forall\alpha)\Gamma \bullet (\forall\alpha)\Delta$

For all variables α, all formulas Γ containing α, and all formulas Δ not containing α:

RP3:	$(\exists\alpha)(\Delta \bullet \Gamma\alpha)$	⇄	$\Delta \bullet (\exists\alpha)\Gamma\alpha$
RP4:	$(\forall\alpha)(\Delta \bullet \Gamma\alpha)$	⇄	$\Delta \bullet (\forall\alpha)\Gamma\alpha$
RP5:	$(\exists\alpha)(\Delta \vee \Gamma\alpha)$	⇄	$\Delta \vee (\exists\alpha)\Gamma\alpha$
RP6:	$(\forall\alpha)(\Delta \vee \Gamma\alpha)$	⇄	$\Delta \vee (\forall\alpha)\Gamma\alpha$
RP7:	$(\exists\alpha)(\Delta \supset \Gamma\alpha)$	⇄	$\Delta \supset (\exists\alpha)\Gamma\alpha$
RP8:	$(\forall\alpha)(\Delta \supset \Gamma\alpha)$	⇄	$\Delta \supset (\forall\alpha)\Gamma\alpha$
RP9:	$(\exists\alpha)(\Gamma\alpha \supset \Delta)$	⇄	$(\forall\alpha)\Gamma\alpha \supset \Delta$
RP10:	$(\forall\alpha)(\Gamma\alpha \supset \Delta)$	⇄	$(\exists\alpha)\Gamma\alpha \supset \Delta$

Here are versions of each of the rules that are less-meta-linguistic and maybe easier to read:

RP1:	$(\exists x)(Px \vee Qx)$	⇄	$(\exists x)Px \vee (\exists x)Qx$
RP2:	$(\forall x)(Px \bullet Qx)$	⇄	$(\forall x)Px \bullet (\forall x)Qx$
RP3:	$(\exists x)(\mathscr{F} \bullet Px)$	⇄	$\mathscr{F} \bullet (\exists x)Px$
RP4:	$(\forall x)(\mathscr{F} \bullet Px)$	⇄	$\mathscr{F} \bullet (\forall x)Px$
RP5:	$(\exists x)(\mathscr{F} \vee Px)$	⇄	$\mathscr{F} \vee (\exists x)Px$
RP6:	$(\forall x)(\mathscr{F} \vee Px)$	⇄	$\mathscr{F} \vee (\forall x)Px$
RP7:	$(\exists x)(\mathscr{F} \supset Px)$	⇄	$\mathscr{F} \supset (\exists x)Px$
RP8:	$(\forall x)(\mathscr{F} \supset Px)$	⇄	$\mathscr{F} \supset (\forall x)Px$
RP9:	$(\exists x)(Px \supset \mathscr{F})$	⇄	$(\forall x)Px \supset \mathscr{F}$
RP10:	$(\forall x)(Px \supset \mathscr{F})$	⇄	$(\exists x)Px \supset \mathscr{F}$

Rules Governing the Identity Predicate (ID)

IDr. Reflexivity: $\alpha=\alpha$

IDs. Symmetry: $\alpha=\beta$ ⇄ $\beta=\alpha$

IDi. Indiscernibility of Identicals

$\mathscr{F}\alpha$
$\alpha=\beta$ / $\mathscr{F}\beta$