Six Derived Rules for the Biconditional

Here are six rules governing the biconditional that you may use if you wish for the remainder of the term. Three of them are rules of inference and thus may be used only on whole lines. Three are rules of equivalence and may be used on parts of lines.

Rules of Inference

Biconditional Modus Ponens (BMP)
$$\alpha \equiv \beta$$
$$\alpha \qquad / \beta$$

Biconditional Modus Tollens (BMT)
$$\alpha \equiv \beta$$
$$\sim\!\alpha \qquad / \sim\!\beta$$

Biconditional Hypothetical Syllogism (BHS)
$$\alpha \equiv \beta$$
$$\beta \equiv \gamma \qquad / \alpha \equiv \gamma$$

Rules of Equivalence

Biconditional DeMorgan's Law (BDM)
$$\sim\!(\alpha \equiv \beta) \; \rightleftarrows \; \sim\!\alpha \equiv \beta$$

Biconditional Commutativity (BCom)
$$\alpha \equiv \beta \; \rightleftarrows \; \beta \equiv \alpha$$

Biconditional Contraposition (BCont)
$$\alpha \equiv \beta \; \rightleftarrows \; \sim\!\alpha \equiv \sim\!\beta$$

These six new rules do not replace, but supplement, the old rule of Material Equivalence (Equiv):

$$\alpha \equiv \beta \; \rightleftarrows \; (\alpha \supset \beta) \cdot (\beta \supset \alpha)$$
$$\alpha \equiv \beta \; \rightleftarrows \; (\alpha \cdot \beta) \vee (\sim\!\alpha \cdot \sim\!\beta)$$