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Truth Tables for Arguments (§6.4)

I. Logical equivalence and translation

The concept of logical equivalence allows us to make some observations, and clear up a few questions about translation.
First, we can see that the biconditional is a superfluous connective, since any statement made with the biconditional could be made, in slightly more complex form, with a conjunction of two conditionals.
That is, it is logically equivalent to a statement which uses only other connectives: '(α ⊃ β) · (β ⊃ α)'

α	≡	β		(α	⊃	β)	·	(β	⊃	α)
⊤	⊤	⊤		⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊥		⊤	⊥	⊥	⊥	⊥	⊤	⊤
⊥	⊥	⊤		⊥	⊤	⊤	⊥	⊤	⊥	⊥
⊥	⊤	⊥		⊥	⊤	⊥	⊤	⊥	⊤	⊥

Other connectives can be shown to be superfluous, in similar ways.
We will look at this topic in depth in Philosophy Friday #3: Adequate Sets of Connectives.

Unless and Exclusive Disjunction:
Previously, we translated 'unless; with a 'v'.
Consider the complex proposition: 'A car will not run unless there is gas in the tank.'
Let's think about what we want as the truth values of 'unless', and how those compare to the disjunction.

The car runs	**The car will not run unless it has gas**	The car has gas
⊤	⊤	⊤
⊤	⊥	⊥
⊥	⊤	⊤
⊥	⊤	⊥

In the first row, the car runs and has gas, so the complex proposition should be true.
In the second row, the car runs, but does not have gas.
In this case, the car runs on an alternative fuel source, or magic.
The complex proposition should thus be false.
In the third row, the car does not run, but has gas.
Perhaps the car is missing its engine.
This case does not falsify the complex proposition, which does not say what else the car needs to run.

The proposition gives a necessary condition for a car to run (having gas), but not sufficient conditions. Thus the statement should be considered true.
In the fourth row, the car does not run, but does not have gas, and so the proposition should be true.

The following truth table thus appropriately represents the complex proposition, translating 'unless' as '∨', since it is logically equivalent to the one we want.

~	R	∨	G
⊥	⊤	⊤	⊤
⊥	⊤	⊥	⊥
⊤	⊥	⊤	⊤
⊤	⊥	⊤	⊥

In contrast, consider: 'Carol will attend school full time unless she gets a job.'

Carol attends school	unless	C arol gets a job
⊤	?	⊤
⊤	⊤	⊥
⊥	⊤	⊤
⊥	⊥	⊥

In the second row, she attends school but doesn't get a job, and so the proposition should be true.
In the third row, she gets the job, and doesn't go to school, and so the proposition should be true.
In the last row, she doesn't get the job but doesn't go to school, and so the proposition should be false.
In the first row, she gets a job but attends school anyway.
If we take the proposition to be true, then we are using the standard truth table for inclusive disjunction.
If we say that the proposition is false in the first row, we arrive at a truth table for exclusive disjunction.
Unless is thus as ambiguous as 'or', and in the same way: there's an inclusive and exclusive 'unless'.

Either 'A ≡ J' or '~(A ≡ J)' will suffice for exclusive disjunction and exclusive unless in this case, since they are logically equivalent to the one we want.

~	A	≡	J
⊥	⊤	⊥	⊤
⊥	⊤	⊤	⊥
⊤	⊥	⊤	⊤
⊤	⊥	⊥	⊥

~	(A	≡	J)
⊥	⊤	⊤	⊤
⊤	⊤	⊥	⊥
⊤	⊥	⊥	⊤
⊥	⊥	⊤	⊥

We thus think of the exclusive unless as a biconditional: Carol will not attend school if, and only if, she gets a job.
Here is another interesting formulation: ‘(A ∨ J) · ~(A · J)’.

(A	∨	J)	·	~	(A	·	J)
⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥
⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊤
⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥

We could, if we wish, introduce a new symbol for exclusive disjunction, say ‘XOR’ or ‘‘⊕’.

Inclusive Disjunction

α	∨	β
⊤	⊤	⊤
⊤	⊤	⊥
⊥	⊥	⊤
⊥	⊥	⊥

Exclusive Disjunction

α	⊕	β
⊤	⊥	⊤
⊤	⊤	⊥
⊥	⊤	⊤
⊥	⊥	⊥

But, we will not use it.
If you have a sentence that you wish to regiment as an exclusive disjunction, just use a proposition of the form ‘~α ≡ β’.

Given two variables, there are 16 possible distributions of truth values.
We have labels for four.
We can define the other 12, using combinations of the five connectives.
(This is kind of a fun exercise. You might try it.)
As long as we can define all possibilities, it doesn’t matter which we take to be basic.
We just have to be careful to translate correctly.
When translating unless, use the wedge for inclusive senses, and as the default translation.
Use the biconditional (with one element negated) for exclusive senses.

II. Valid and Invalid Arguments

Compare the following arguments:

A. 1. If God exists then every effect has a cause.
2. God exists.
∴ Every effect has a cause.

B. 1. If God exists then every effect has a cause.
2. Every effect has a cause.
∴ God exists.

A is valid, and has the following form:

$\alpha \supset \beta$
α / β

This form is known as Modus Ponens.
Note that we write the premises on sequential lines, and the conclusion on the same line as the final premise, following a single slash.

B is invalid, and has the following form:

$\alpha \supset \beta$
β / α

Arguments of the form B commit the Fallacy of Affirming the Consequent.

Recall: In a **valid argument**, if the premises are true then the conclusion must be true.
Note, this definition says nothing about what happens if the premises are false.
An **invalid argument** is one in which it is possible for true premises to yield a false conclusion.
By focusing on valid arguments, we can make sure that if all our premises are true, so must our conclusions be.

III. A method for determining if an argument is valid

Step 1: Line up premises and conclusion horizontally, separating premises with a single slash and separating the premises from the conclusion with a double slash.
Step 2: Construct truth tables for each premise and the conclusion, using consistent assignments to component variables.
Step 3: Look for a counter-example: a row in which all premises are true and the conclusion is false.
If there is a counter-example, the argument is invalid. Specify a counter-example.
If there is no counterexample, the argument is valid.

Here is a valid argument:

P	⊃	Q	/	P	//	Q
⊤	⊤	⊤		⊤		⊤
⊤	⊥	⊥		⊤		⊥
⊥	⊤	⊤		⊥		⊤
⊥	⊤	⊥		⊥		⊥

Note: On no line are the premises true and the conclusion false. There is no counter-example.

Here is an invalid argument:

P	⊃	Q	/	Q	//	P
⊤	⊤	⊤		⊤		⊤
⊤	⊥	⊥		⊥		⊤
⊥	⊤	⊤		⊤		⊥
⊥	⊤	⊥		⊥		⊥

This third row is the *counterexample*:
The argument is invalid when P is false and Q is true.

Is this a valid argument?

1. P ⊃ (Q ⊃ P)
2. ~P   / Q

P	⊃	(Q	⊃	P)	/	~P	//	Q
⊤	⊤	⊤	⊤	⊤		⊥		⊤
⊤	⊤	⊥	⊤	⊤		⊥		⊥
⊥	⊤	⊤	⊥	⊥		⊤		⊤
⊥	⊤	⊥	⊤	⊥		⊤		⊥

Row 4 is a counter-example.
The argument is shown invalid when P is false and Q is false.

IV. Exercises. Determine whether each argument is valid. If invalid, specify the counter-example.

1.	$A \supset B$	
	$\sim A$	$/ \sim B$
2.	$C \vee D$	
	$\sim D$	$/ C$
3.	$E \equiv F$	$/ \sim E \vee F$
4.	$G \cdot H$	
	$H \supset I$	$/ \sim(G \cdot I)$
5.	$J \supset K$	
	$K \supset \sim J$	
	$\sim J \supset K$	$/ J \vee \sim K$

V. Solutions

1. Invalid, when A is false, and B is true
2. Valid
3. Valid
4. Invalid, when G, H, and I are all true
5. Invalid, when J is false and K is true