Philosophy 240: Symbolic Logic
Fall 2010
Mondays, Wednesdays, Fridays: 9am - 9:50am

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Derivations Using Identity II (§8.7)

I. A Couple of Longer Proofs

We're just going to do a couple of longer proofs using identity, today.

1. (∃x)Hx	
2. (x)(y)[(Hx · Hy) ⊃ x=y]	/ (∃x)[Hx · (y)(Hy ⊃ x=y)]
3. Ha	1, EI
│4. ~(∃x)[Hx · (y)(Hy ⊃ x=y)]	AIP
│5. (x)~[Hx · (y)(Hy ⊃ x=y)]	4, CQ
│6. (x)[~Hx ∨ ~(y)(Hy ⊃ x=y)]	5, DM
│7. ~Ha ∨ ~(y)(Hy ⊃ a=y)	6, UI
│8. ~(y)(Hy ⊃ a=y)	7, 3, DN, DS
│9. (∃y) ~(Hy ⊃ a=y)	8, CQ
│10. ~(Hb ⊃ a=b)	9, EI
│11. ~(~Hb ∨ a=b)	10, Impl
│12. Hb · ~a=b	11, DM, DN
│13. Hb	12, Simp
│14. ~a=b	12, Com Simp
│15. (y)[(Ha · Hy) ⊃ a=y]	2, UI
│16. (Ha · Hb) ⊃ a=b	15, UI
│17. Ha · Hb	3, 13, Conj
│18. a=b	16, 17, MP
│19. a=b · ~a=b	18, 14, Conj
20. (∃x)[Hx · (y)(Hy ⊃ x=y)]	4-19, IP

QED

This derivation could be shortened by using the Rules of Passage, which we discussed earlier this term.

1. (∃x)Hx	
2. (x)(y)[(Hx · Hy) ⊃ x=y]	/ (∃x)[Hx · (y)(Hy ⊃ x=y)]
3. (x)(y)[Hx ⊃ (Hy ⊃ x=y)]	1, Export
4. (x)[Hx ⊃ (y)(Hy ⊃ x=y)]	RP8 (Not a rule of our system!)
5. Ha	1, EI
6. Ha ⊃ (y)(Hy ⊃ a=y)]	3, UI
7. (y)(Hy ⊃ a=y)]	6, 5, MP
8. Ha • (y)(Hy ⊃ a=y)]	5, 7, Conj
9. (∃x)[Hx · (y)(Hy ⊃ x=y)]	8, EG

QED

But, unfortunatly, we don't have the rules of passage.
So, something like the first proof is necessary.

Here is a derivation of a longer argument using ID.

> There are at least two cars in the driveway.
> All the cars in the driveway belong to John.
> John has at most two cars.
> So, there are exactly two cars in the driveway.

1. (∃x)(∃y)(Cx • Dx • Cy • Dy • x≠y)	
2. (x)[(Cx • Dx) ⊃ Bxj]	
3. (x)(y)(z)[(Cx • Bxj • Cy • Byj • Cz • Bzj) ⊃ (x=y ∨ x=z ∨ y=z)]	
/ (∃x)(∃y){Cx • Dx • Cy • Dy • x≠y • (z)[(Cz • Dz) ⊃ (z=x ∨ z=y)]	
4. (∃y)(Ca • Da • Cy • Dy • a≠y)	1, EI
5. Ca • Da • Cb • Db • a≠b	4, EI
6. Ca • Da	5, Simp
7. (Ca • Da) ⊃ Baj	2, UI
8. Baj	7, 6, MP
9. Cb • Db	5, Simp
10. (Cb • Db) ⊃ Bbj	2, UI
11. Bbj	10, 9, MP
│ 12. ~(z)[(Cz • Dz) ⊃ (z=a ∨ z=b)]	AIP
│ 13. (∃z)~[(Cz • Dz) ⊃ (z=a ∨ z=b)]	12, CQ
│ 14. (∃z)~[~(Cz • Dz) ∨ (z=a ∨ z=b)]	13, Impl
│ 15. (∃z)[(Cz • Dz) • ~(z=a ∨ z=b)]	14, DM, DN
│ 16. Cc • Dc • ~(c=a ∨ c=b)	15, EI
│ 17. Ca	6, Simp
│ 18. Ca • Baj	17, 8 Conj
│ 19. Cb	9, Simp
│ 20. Cb • Bbj	19, 11, Conj
│ 21. Cc • Dc	16, Simp
│ 22. (Cc • Dc) ⊃ Bcj	2, UI
│ 23. Bcj	22, 21, MP
│ 24. Cc	21, Simp
│ 25. Cc • Bcj	24, 23, Conj
│ 26. Ca • Baj • Cb • Bbj • Cc • Bcj	18, 20, 25, Conj
│ 27. (y)(z)[(Ca • Baj • Cy • Byj • Cz • Bzj) ⊃ (a=y ∨ x=z ∨ y=z)]	3, UI
│ 28. (z)[(Ca • Baj • Cb • Bbj • Cz • Bzj) ⊃ (a=b ∨ a=z ∨ b=z)]	27, UI
│ 29. (Ca • Baj • Cb • Bbj • Cc • Bcj) ⊃ (a=b ∨ a=c ∨ b=c)	28, UI
│ 30. a=b ∨ a=c ∨ b=c	29, 26, MP
│ 31. a≠b	5, Simp
│ 32. a=c ∨ b=c	30, 31, DS
│ 33. ~(c=a ∨ c=b)	16, Com, Simp
│ 34. ~(a=c ∨ b=c)	33, ID
│ 35. (a=c ∨ b=c) • ~(a=c ∨ b=c)	32, 34, Conj
36. (z)[(Cz • Dz) ⊃ (z=a ∨ z=b)]	12-35, IP, DN
37. Ca • Da • Cb • Db • a≠b • (z)[(Cz • Dz) ⊃ (z=a ∨ z=b)]	6, 9, 31, 36, Conj
38. (∃y){Ca • Da • Cy • Dy • a≠y • (z)[(Cz • Dz) ⊃ (z=a ∨ z=y)]}	37, EG
39. (∃x)(∃y){Cx • Dx • Cy • Dy • x≠y • (z)[(Cz • Dz) ⊃ (z=x ∨ z=y)]}	38, EG

QED

II. Exercises

1.
1. $(x)[(Px \bullet Hjx) \supset Sx]$
2. $(\exists x)(Px \bullet Hjx)$
3. $(x)(y)[(Sx \bullet Hjx \bullet Sy \bullet Hjy) \supset x=y]$ / $(\exists x)\{Px \bullet Hjx \bullet (y)[(Py \bullet Hjy) \supset x=y] \bullet Sx\}$

2.
1. $(\exists x)(\exists y)(Px \bullet Rx \bullet Py \bullet Ry \bullet x \neq y)$
2. $(x)[(Px \bullet Rx) \supset Cx]$
3. $(x)(y)(z)[(Cx \bullet Cy \bullet Cz \bullet Rx \bullet Ry \bullet Rz) \supset (x=y \lor x=z \lor y=z)]$
/ $(\exists x)(\exists y)\{Px \bullet Rx \bullet Py \bullet Ry \bullet x \neq y \bullet (z)[(Pz \bullet Rz) \supset (z=x \lor z=y)]\}$

3.
1. $(\exists x)(\exists y)(\exists z)(Fx \bullet Fy \bullet Fz \bullet x \neq y \bullet x \neq z \bullet y \neq z)$
2. $(\exists x)\{Fx \bullet Gx \bullet (y)[(Fy \bullet Gy) \supset x=y]\}$
3. $(x)({\sim}Gx \supset Hx)$ / $(\exists x)(\exists y)(Hx \bullet Hy \bullet x \neq y)$

Warning! This last one may take over 75 steps!